APPROXIMATE DETERMINATION OF COOLING TOWER DIMENSIONS

By

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1. Introduction

Cooling tower dimensions are controlled by thermodynamic, aerodynamic, hydrodynamic, as well as by structural aspects.

World-wide boosting power demands require increased power stations thus ever bigger cooling towers.

These structures are, however, exposed to intricate forces and reactions and determination of the developing forces is still bound to uncertainties. Nevertheless, such structures are being designed and built, and like always in the course of centuries, casualties hint to correct solutions. Intensive research started in 1965 after the Ferrybridge cooling tower collapses. Another casualty in 1973 in Ardeer urged engineers to further contemplations and research. These research programs produced a lot of knowledge matter, that will be systematized from the aspect of cooling tower design data and presented below. Thus, essentially, the initial design stage relying on available knowledge will be presented.

2. Selection of the cooling tower shape

In 1917, Van Herson (Netherlands) was granted a patent on cooling tower shapes seen in Fig. 1 [1].

In the subsequent 1927 and 1931 patents of L. G. Mouchel and M. Gueritte, the wall directrix is about hyperbolic.

![Fig. 1]
Also cylindrical and bell-shaped towers have been built, but hyperboloids of revolution both behaved structurally better and were less material consuming. Further building technology advantages of this shape excluded anything else (Fig. 2).

![Diagram showing hyperboloid design]

### 3. Acting loads and effects

Loads and effects, internal forces or deformations to develop have been compiled in Table I. Loads and effects can be determined according to [2].

#### Table I

<table>
<thead>
<tr>
<th>Loads, effects</th>
<th>Stresses</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>Meridional compression, annular tension above, and compression below, the throat.</td>
<td>Minimum wall thickness has to be striven to.</td>
</tr>
<tr>
<td>Wind load</td>
<td>Meridionally, tension maxima arise in wind direction (0°), balancing dead load compression. Meridional compression maxima arise at ( \alpha = 65 - 75° ). Annular compressions and tensions, as well as local bendings arise mainly in the inner face. In wind gusts, dynamic effect, resonance, instability (critical force) have to be tested.</td>
<td>Wind load stresses may be reduced by applying ribbed surfaces.</td>
</tr>
<tr>
<td>Critical wind load</td>
<td>Top edge becomes oval.</td>
<td>Stability may be improved by horizontal rings and vertical ribs.</td>
</tr>
<tr>
<td>Thermal effect (operational; one-side insolation)</td>
<td>Bending moment develops, cracking has to be limited.</td>
<td>Top bracing ring may help.</td>
</tr>
<tr>
<td>Concrete swelling in wet operation</td>
<td>Bending moment similar to that due to thermal effect.</td>
<td>Of the same order as the thermal effect.</td>
</tr>
</tbody>
</table>
Table I (Continued)

<table>
<thead>
<tr>
<th>Loads, effects</th>
<th>Stresses</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidence, differential</td>
<td>Excess load on columns, excess tension in shell forward instability.</td>
<td>Reinforcement of sections over columns, guaranteed load distribution, prevention of no-strain deformation.</td>
</tr>
<tr>
<td>subsidence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earthquake</td>
<td>Stress excess both in columns and in shell.</td>
<td>Exact effect is unknown.</td>
</tr>
<tr>
<td>Faulty shape</td>
<td>Ring forces from previous loads may double (and reverse sign).</td>
<td>Extra ring reinforcement and adequate (meridional) reinforcement for distribution.</td>
</tr>
</tbody>
</table>

4. Preliminary dimensioning

Thermodynamic, aerodynamic and hydrodynamic analyses deliver diameters required at the tower bottom (air entry), at the narrowest cross section (throat) and at the air exit, as well as the needed tower height.

The structural designer relies on structural considerations in determining possible tower dimensions.

Valuable information for the preliminary dimensioning is given by experience with existing hyperboloid cooling towers. Table II has been compiled from geometries of recently erected cooling towers.

For letter symbols in Table II, see Fig. 2.

Preliminary dimensioning begins with determining the shell dimensions. Table II argues for the following proportions [3]:

- reduced tower height \( H/D_A \approx 1.25 - 1.50 \);
- reduced basic diameter \( D_A/D_A \approx 1.03 - 1.20 \);
- reduced throat diameter \( D_T/D_A \approx 0.55 - 0.65 \);
- reduced top diameter \( D_F/D_A \approx 0.61 - 0.73 \);
- reduced shell height \( H_H/D_A \approx 1.1 - 1.30 \);
- reduced throat height \( H_T/D_A \approx 0.92 - 1.02 \);
- reduced minimum shell thickness \( v/D_A \approx 0.0015 - 0.0020 \);
- reduced shell thickness at the lower edge \( v_0/D_A \approx 0.0060 - 0.0085 \).

In knowledge of the lower shell diameter \( D_A \), preliminary dimensioning may apply the relationships above (see the numerical example in Chapter 6).
<table>
<thead>
<tr>
<th>No.</th>
<th>Location or designer and surface configuration</th>
<th>$H$ (m)</th>
<th>$D_R$ (m)</th>
<th>Shell</th>
<th>$D_A$ (m)</th>
<th>$D_P$ (m)</th>
<th>$D_Y$ (m)</th>
<th>$H_T$ (m)</th>
<th>$H_S$ (m)</th>
<th>$e_R$ (cm)</th>
<th>$e$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ferrybridge torus and frustrum</td>
<td>114.3</td>
<td>91.42</td>
<td></td>
<td>88.52</td>
<td>50.29</td>
<td>54.56</td>
<td>108.2</td>
<td>89.89</td>
<td>6.1</td>
<td>12.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.291)</td>
<td>(1.032)</td>
<td></td>
<td>(1.0)</td>
<td>(0.658)</td>
<td>(0.616)</td>
<td>(1.222)</td>
<td>(1.015)</td>
<td>(0.069)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>2</td>
<td>Ilbenbüren hyperboloid and frustrum</td>
<td>101.02</td>
<td>73.0</td>
<td></td>
<td>65.7</td>
<td>52.5</td>
<td>55.5</td>
<td>81.52</td>
<td>59.75</td>
<td>19.5</td>
<td>70</td>
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<tr>
<td></td>
<td></td>
<td>(1.537)</td>
<td>(1.111)</td>
<td></td>
<td>(1)</td>
<td>(0.799)</td>
<td>(0.845)</td>
<td>(1.241)</td>
<td>(0.909)</td>
<td>(0.296)</td>
<td>(0.0107)</td>
</tr>
<tr>
<td>3</td>
<td>Carlton hyperboloid</td>
<td>82.0</td>
<td>62.2</td>
<td></td>
<td>61.75</td>
<td>37.0</td>
<td>39.6</td>
<td>79.40</td>
<td>63.42</td>
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<tr>
<td></td>
<td></td>
<td>(1.328)</td>
<td>(1.007)</td>
<td></td>
<td>(1)</td>
<td>(0.599)</td>
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<td>(1.027)</td>
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<td>(0.0049)</td>
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<td>M. Herzog hyperboloid</td>
<td>130.0</td>
<td>—</td>
<td></td>
<td>93.0</td>
<td>62.0</td>
<td>65.0</td>
<td>120.0</td>
<td>94.0</td>
<td>10.0</td>
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<tr>
<td></td>
<td></td>
<td>(1.398)</td>
<td>—</td>
<td></td>
<td>(1)</td>
<td>(0.666)</td>
<td>(0.699)</td>
<td>(1.290)</td>
<td>(1.011)</td>
<td>(0.107)</td>
<td>(0.0019)</td>
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<td>5</td>
<td>Hyperboloid</td>
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<td>87.0</td>
<td>49.5</td>
<td>—</td>
<td>107.0</td>
<td>89.0</td>
<td>9.0</td>
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<td>(1.333)</td>
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<td></td>
<td>(1)</td>
<td>(0.569)</td>
<td>—</td>
<td>(1.229)</td>
<td>(1.023)</td>
<td>(0.103)</td>
<td>(0.0017)</td>
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<tr>
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<td>W. Krätzig hyperboloid</td>
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<td>150.3</td>
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<td>80.0</td>
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<tr>
<td>7</td>
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<td>39.64</td>
<td>42.42</td>
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<td>7.5</td>
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<td>(0.578)</td>
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<tr>
<td></td>
<td></td>
<td>(1.317)</td>
<td>(1.119)</td>
<td></td>
<td>(1)</td>
<td>(0.777)</td>
<td>(0.783)</td>
<td>(1.045)</td>
<td>(0.907)</td>
<td>(0.272)</td>
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<td></td>
<td>101.8</td>
<td>69.0</td>
<td>70.2</td>
<td>112.5</td>
<td>94.8</td>
<td>15.0</td>
<td>70</td>
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<tr>
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<td>(1.055)</td>
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<td>(1)</td>
<td>(0.677)</td>
<td>(0.689)</td>
<td>(1.105)</td>
<td>(0.93)</td>
<td>(0.147)</td>
<td>(0.0063)</td>
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<tr>
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<td>M. Diver, A. C. Peterson (optimized shell)</td>
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<td>—</td>
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<td>128</td>
<td>76.0</td>
<td>84.0</td>
<td>160</td>
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<td>(1)</td>
<td>(0.594)</td>
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<td>(0.656)</td>
<td>(1.25)</td>
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<td>A. C. Peterson (optimized shell)</td>
<td>—</td>
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<td>145</td>
<td>84.0</td>
<td>94.0</td>
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<td>(1)</td>
<td>(0.579)</td>
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<td>(0.648)</td>
<td>(1.296)</td>
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<td>12</td>
<td>Symposium Konstruktiver Ingenieur 1977</td>
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<td>200</td>
<td>161.6</td>
<td></td>
<td>150.4</td>
<td>98.4</td>
<td>107.2</td>
<td>186</td>
<td>140</td>
<td>14.0</td>
<td>14</td>
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<tr>
<td></td>
<td></td>
<td>(1.329)</td>
<td>(1.074)</td>
<td></td>
<td>(1)</td>
<td>(0.6543)</td>
<td>(0.713)</td>
<td>(1.237)</td>
<td>(0.931)</td>
<td>(0.093)</td>
<td>(0.0073)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>—</td>
<td>—</td>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(0.0015)</td>
</tr>
</tbody>
</table>
5. Determination of approximate dimensions

The next stage of design — preceding the detailed analysis — involves the assertion of the preliminary dimensions. To this aim, principal stresses are determined by a simplified interpretation — approximating on the side of safety — of detailed structural analyses. Preliminary dimensions coping with these stresses may be accepted as approximate dimensions. Else they have to be corrected.

Main checking steps and applicable approximate relationships will be presented below, with references.

5.1 Shell directrix

Advisably, the shell directrix equation is assumed as indicated in Fig. 3:

\[ R = c + a \sqrt{1 + \frac{z^2}{b^2}} \]  \hspace{1cm} (1)

where \( a \) and \( b \) are hyperbola axes, and \( c \) is the distance between hyperbola and axis of revolution. The \( b \) value results from:

\[ b = \frac{H_T}{\sqrt{\left(\frac{R_A - c}{R_T - c}\right)^2 - 1}} \]  \hspace{1cm} (2)

The most favourable shifting value \( c \) is an optimum problem, it may be chosen at about \((0.5 \div 0.75)\ R_T\). Increasing the \( c \) value means an increase of the curvature about the throat and the reduction of curvature in the lower part.

Shifting not only affects the internal forces but also improves the shell stability, advantages to be confirmed by detailed analyses. In determining the shell directrix, basic angle of the meridian curve has to be kept at about 70°.

After having established proportions under 4, the shell directrix may be fitted and its equation established.
5.2 Determination of internal forces

Maximum values of internal forces will be determined for principal loads — such as dead load, wind, earthquake — and for other effects — thermal, differential subsidence and faulty shape.

Critical grouping of internal forces has to comply with Hungarian standard MSz 15021, taking also the location of the cross section into account [4].

5.2.1 Membrane forces

Membrane forces $N_\varphi$, $N_\theta_\varphi$, and $N_\theta$ develop in the shell (Fig. 4).

![Fig. 4](image)


These tables contain the following parameters for dead load, earthquake and wind load:

$$k^2 = 1 + \frac{a^2}{b^2} \quad \text{values: 1.05; 1.06; 1.08; 1.10; 1.15; 1.25; 1.50.}$$

$$\varphi = \frac{\varphi_F - \varphi}{\varphi_F - \varphi_A} \quad \text{values: 0.1; 0.2; 0.3 . . . , 0.9; 1.0.}$$

$$R_T \quad \text{values: 0.45; 0.55; 0.65.}$$

$$R_T \quad \text{values: 0.85; 0.90; 0.95.}$$

with specific forces:

$$n_\varphi = \frac{N_\varphi}{PR_T} ; \quad n_\theta = \frac{N_\theta}{PR_T} ; \quad n_\theta_\varphi = \frac{N_\theta_\varphi}{PR_T} . \quad (3)$$

$P$ in (3) is a substitutive load acting on the shell middle surface (MN/m²).
One among the tables related to the numerical example will be presented (Table III).

Table III

<table>
<thead>
<tr>
<th>$k^2$</th>
<th>$\phi = \ldots$</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05</td>
<td>...</td>
<td>-4.172</td>
<td>-4.838</td>
<td>-5.848</td>
</tr>
<tr>
<td>1.06</td>
<td>...</td>
<td>-3.814</td>
<td>-4.429</td>
<td>-5.365</td>
</tr>
<tr>
<td>1.08</td>
<td>...</td>
<td>-3.312</td>
<td>-3.857</td>
<td>-4.693</td>
</tr>
<tr>
<td>1.10</td>
<td>...</td>
<td>-2.970</td>
<td>-3.469</td>
<td>-4.239</td>
</tr>
<tr>
<td>1.15</td>
<td>...</td>
<td>-2.441</td>
<td>-2.870</td>
<td>-3.546</td>
</tr>
<tr>
<td>1.25</td>
<td>...</td>
<td>-1.914</td>
<td>-2.280</td>
<td>-2.877</td>
</tr>
<tr>
<td>1.50</td>
<td>...</td>
<td>-1.388</td>
<td>-1.704</td>
<td>-2.262</td>
</tr>
</tbody>
</table>

In the case of wind loading, among meridional forces $N_\phi$, tension maxima occur at $\theta = 0^\circ$, compression maxima at $\theta = 70^\circ$. Among shear forces $N_\phi$, maximum is at $\theta = 45^\circ$. Annular force $N$ maxima are at $\theta = 0^\circ$.

After having determined the basic values, design stresses will be determined according to standard specifications MSz 15021.

Among membrane forces a faulty shape may produce significant increase and sign reverse of annular forces, to be reckoned with by doubling the design annular force and assuming it to act also with the opposite sign.

For the determined membrane forces, the shell wall thickness, the meridional tension and compression reinforcement as well as the annular tension reinforcement should be checked.

5.22 Moments

Shell moments are mainly due to wind load, imposing exemptness from cracks or limited crack width in the shell. Against moments due to earthquake as extraordinary load, shell stability and safety from life hazard or important material losses have to be ensured.

Approximate meridional moments due to wind load [7]:

$$M_\phi = \pm 0.001 \, pR^2$$  \hspace{1cm} (4)

where $p$ is the pressure at design wind velocity, and $R$ is the shell radius at the tested level.

Approximate annular moments:

$$M_\phi = \pm 0.004 \, pR^2.$$
Further meridional and annular moments are due to thermal effects, to be determined reckoning with cracked condition.

Now, cross-sectional dimensions and reinforcement may be determined for the resulting stresses.

Let us remark that [2] specifies min. 0.4% of reinforcement in either direction. Additional reinforcement of about 0.1% is required to balance thermal effects.

5.3 Shell stability

Checking the shell stability means in fact confirmation of the wall thickness chosen. Most of the available theories of stability reckon with homogeneous, crack-free cross sections, although shells are mostly cracked under loads and various effects. These cracks may propagate upon shell buckling. Thus, tensions in the concrete cross section are advisably limited to possibly little exceed the ultimate tensile stress.

The following relationships serve for determining critical dynamic pressure $p_{cr}$ under wind load (Table IV) [7, 9, 10].

<table>
<thead>
<tr>
<th>Author</th>
<th>Design load or stress</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Der and Fiedler</td>
<td>$p_{cr} = 0.07 E_0 \left( \frac{v}{R_t} \right)^{7/3}$</td>
<td>$v$ = mean wall thickness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_t$ = throat circle radius</td>
</tr>
<tr>
<td>ACI—ASCE</td>
<td>$p_{cr} = 0.032 E_0 \left( \frac{v}{R_t} \right)^{7/3}$</td>
<td></td>
</tr>
<tr>
<td>Herzog</td>
<td>$p_{cr} = 0.158 E_{red} \left( \frac{v}{R} \right)^{4/5}$</td>
<td>$E_{red} \approx \frac{E_0}{2}$</td>
</tr>
<tr>
<td>Krätzig, Zerna</td>
<td>$\sigma_0 = \frac{0.965 E_0}{\sqrt{1 - \mu^2}} \left( \frac{v}{R_t} \right)^{4/3} k_0$</td>
<td>$k_0 = 0.105 (1 - x) (1 - y) +$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\varphi = \frac{0.612 E_0}{\sqrt{1 - \mu^2}} \left( \frac{v}{R_t} \right)^{4/3} k_\varphi$</td>
<td>$+ 0.222 (1 - x)y +$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 0.056 (1 - y)x +$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 0.151xy$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_\varphi = 1.28 (1 - x) (1 - y) +$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 1.13 (1 - x)y +$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 1.35 (1 - y)x +$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 1.82xy$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x = \left( \frac{R_t^{0.571}}{R_A} \right) \frac{1}{0.262}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y = \left( \frac{R_t^{0.28}}{H_T} \right) \frac{1}{0.166}$</td>
</tr>
</tbody>
</table>
Authors of the formulae suggest a minimum 2.0 quotient of critical by effective load or stress!

Critical load in the top shell cross section under dead load and wind load is given by:

\[ N_{p \text{ crit}} = 0.079 \frac{E' \cdot z^2}{D_A}, \]  \hspace{1cm} (5)

again with a safety factor of 2.0.

[7] suggests a design load (MN/m²)

\[ P_{\text{crit}} = \frac{2}{3} E' \cdot \left( \frac{v^1}{D^1} \right) \cdot (m^2 - 1) \]  \hspace{1cm} (6)

during construction, where \( E' = 0.65 \cdot 6640 \sqrt{0.68K_{25}} \cdot 0.85 \) (MN/m²); with notations in Fig. 5, \( H^1 \) is the so-called effective ring height, \( H^1 = 7/8H - H_4 \); \( v^1 \) and \( D^1 \) are wall thickness and diameter, resp., at the tested height.

5.4 Accessory examination

In addition to the possibility of overall failure, the shell has also to be checked for local effects.

Two local effects of major importance for failure are "top edge becoming oval" as well as stress excess in the lower part of the shell due to column subsidence.

To prevent the top edge from becoming elliptic, a bracing ring has to be applied (Fig. 6).
According to [11], the top edge becomes elliptic if the design wind velocity is higher than critical, to be determined as:

$$v_{\text{crit}} = \frac{2.14}{R_F} \cdot \sqrt{\frac{E_{bo} I}{\mu}},$$

(7)

where $I$ is the moment of inertia referred to the vertical centroidal axis of the circular ring. Determination of the circular ring cross section is allowed to involve the interacting plate width; $\mu$ is the specific mass of the ring and $R_F$ its radius; $E_{bo}$ is the initial modulus of elasticity of the concrete.

The load acting on the bracing ring is obtained from the design wind velocity as:

$$p_r = 0.45 p_t \cdot \left[ \frac{1}{1 - \frac{f_0^2}{0.466}} \right],$$

(8)

where $f_0$ is frequency of the transversal eddy separation calculated from the design wind velocity:

$$f_0 = 0.1 \frac{v}{R_F}.$$

(9)

Lower part of the shell has to be examined as a deep beam. Lower part of the shell wall has to be gradually thickened according to Eq. (5).

6. Numerical example

Numerical examples follow the order of items in this paper.

6.1 Preliminary dimensioning

Let the shell bottom diameter be given: $D_A = 100$ m. Further dimensions follow item 5.1 (Fig. 2):

- $H_T = 1 \cdot 100 = 100$ m
- $D_F = 0.68 \cdot 100 = 68$ m
- $v = 0.002 \cdot 100 = 0.20$ m

- $H_H = 1.2 \cdot 100 = 120$ m
- $D_T = 0.59 \cdot 100 = 59$ m
- $v_0 = 0.007 \cdot 100 = 0.70$ m.

6.2 Shell directrix

Starting from the preliminary dimensioning, shell directrix equation is derived from Eqs (1) and (2):

$$b = \frac{100}{\sqrt{(\frac{50 - 20}{29.5 - 20})^2 - 1}} = 33.33 \text{ m}; \quad c = 20.0 \text{ m}; \quad a = 29.5 - 20.0 = 9.5 \text{ m}

R = 20 + 9.5 \sqrt{1 + \frac{z^2}{33.33^2}}.$$
COOLING TOWER DIMENSIONS

Basic angle of the meridian at the bottom edge:

$$\varphi_A = 90^\circ - \arctg \frac{a \cdot z}{b \sqrt{b^2 + z^2}} = 90^\circ - \arctg \frac{9.5 \cdot 100}{33.33 \sqrt{33.33^2 + 100^2}} = 74.9^\circ.$$ 

6.3 Loads

- Shell dead load: $0.2 \cdot 25 \text{ kN/m}^2 = 5 \text{ kN/m}^2$.
- Seismic load: horizontal acceleration \([9]\) $b_z = 0.02 \text{ g}$

$$P = m \cdot a = m \cdot 0.02 \cdot g \approx 0.2 \cdot m$$

hence, 0.2 times the dead load is assumed to act horizontally:

$$g_f = \pi \cdot 0.2 \cdot 5 = 3.14 \text{ kN/m}^2.$$ 

- Wind load:

average wind velocity $\bar{v}_{10}$ is assumed at 130 km/h, yielding the design wind velocity with respect to dynamic and resonance effects, and to variations along the height \([3]\):

$$v_z^* = \left( \frac{z^*}{10} \right)^{0.16} \cdot \bar{v}_{10} (1 + 4 \cdot 0.18) \varphi$$

where $z^*$ is interpreted beginning from the lower shell edge upwards:

$Fz^* = 10 \text{ m}; v_z^* = 1 \cdot 36.1 (1 + 0.72) 1.1 = 68.3 \text{ m/sec};$

the dynamic pressure:

$$P_{10} = \frac{(v_z^*)^2}{16} = \frac{68.3^2}{16} = 292 \text{ kp/m}^2 = 2.92 \text{ kN/m}^2;$$

$$z^* = 100 \text{ m}; v_z^* = \left( \frac{100}{10} \right)^{0.16} \cdot 36.1 (1 + 0.72) 1.1 = 98.72 \text{ m/sec},$$

$$P_{100} = \frac{98.72^2}{16} = 609 \text{ kp/m}^2 = 6.09 \text{ kN/m}^2;$$

$$z^* = 120 \text{ m}; v_z^* = \left( \frac{120}{10} \right)^{0.16} \cdot 68.3 = 101.65 \text{ m/sec},$$

$$P_{120} = \frac{101.65^2}{16} = 646 \text{ kp/m}^2 = 6.46 \text{ kN/m}^2.$$ 

6.4 Membrane forces

- Parameters

\([5,6]\) suggest the following parameters to be required for determining the stresses:

$P$ (substitutive load) = for dead load = $g = 5 \text{ kN/m}^2$ 

= for seismic load = $g_f = 3.14 \text{ kN/m}^2$

= for wind load = $P_{16} = 2.92 \text{ kN/m}^2$

$$k^2 = 1 + \frac{a^2}{b^2} = 1 + \frac{9.5^2}{33.33^2} = 1.0812 \ (\text{see Table III})$$

$$\Phi = \frac{\varphi_F - \varphi}{\varphi_F - \varphi_A} = 1.0 \ (\text{at bottom edge})$$

$$\frac{R_T}{R_A} = \frac{29.5}{50.0} = 0.59; \quad \frac{R_T}{R_F} = \frac{29.5}{31.06} = 0.9492$$
— from dead load: $\dot{\theta} = 0^\circ - 180^\circ$

\[
\begin{align*}
N_0^{\varphi,g} &= n_\varphi \cdot P \cdot R_T = -4.436 \cdot 5 \cdot 29.5 = -654.31 \text{ kN/m (compression)} \\
N_0^{\varphi,f,g} &= -n_\varphi \cdot P \cdot R_T = -0.5146 \cdot 5 \cdot 29.5 = -75.9 \text{ kN/m (compression)} \\
N_0^{\varphi,g} &= 0
\end{align*}
\]

— from seismic load: $\dot{\theta} = 0^\circ$

\[
\begin{align*}
N_0^{\varphi,gf} &= n_\varphi \cdot P \cdot R_T = 13.9 \cdot 3.14 \cdot 29.5 = 1287.4 \text{ kN/m (tension)} \\
N_0^{\varphi,f,gf} &= -n_\varphi \cdot P \cdot R_T = -1.33 \cdot 3.14 \cdot 29.5 = -75.9 \text{ kN/m (compression)} \\
N_0^{\varphi,gf} &= -n_\varphi \cdot P \cdot R_T = -5.54 \cdot 3.14 \cdot 29.5 = -513.1 \text{ kN/m (compression)}
\end{align*}
\]

with unit Fourier coefficients because of $\dot{\theta} = 0^\circ$.

— from wind load:

(only maxima will be determined, leading to different angles $\dot{\theta}$ for each force).

Circumferential distribution of the wind load will be assumed according to [2].

\[
\begin{align*}
\dot{\theta} &= 0^\circ; \ c_p = 1.0 \\
\overline{N}_{p,10}^{\varphi} &= n_\varphi \cdot PR = 23.1 \cdot 2.92 \cdot 29.5 = 1989.8 \text{ kN/m (tension)}
\end{align*}
\]

Here the Fourier coefficient is $\alpha_0 = 0.35$

\[
\begin{align*}
N_{p,10}^{\varphi} &= \alpha_0 \cdot \overline{N}_{p,10}^{\varphi} = 0.35 \cdot 1989.8 = 696 \text{ kN/m} \\
N_{p,10}^{\varphi} &= \alpha_0 \cdot n_\varphi \cdot P \cdot R = 0.35 \cdot (-2.25) \cdot 2.92 \cdot 29.5 = -67.8 \text{ kN/m} \\
\dot{\theta} &= 70^\circ; \ c_p = 1.2 \\
N_{p,10}^{\varphi} &= c_p \cdot \alpha_0 \cdot (-n_\varphi) \cdot P \cdot R = 1.2 \cdot 0.35 \cdot (-20.94) \cdot 2.92 \cdot 29.5 = -757.5 \text{ kN/m} \\
N_{p,10}^{\varphi} &= \alpha_0 \cdot n_\varphi \cdot P \cdot R = 0.35 \cdot 0.409 \cdot 2.92 \cdot 29.5 = 12.3 \text{ kN/m} \\
\dot{\theta} &= 45^\circ; \ c_p = 0.56 \\
N_{p,10}^{\varphi} &= \frac{1}{\pi} \sum_1 \alpha_n \cdot (-n_\varphi \cdot n) \cdot \sin n\dot{\theta} \cdot P \cdot R = -253.6 \text{ kN/m}
\end{align*}
\]

$n_\varphi$, $n_\varphi$ and $n_\varphi$ in the above calculations have been determined according to [5, 6].

6.5 Design membrane forces

Meridional:

Load group I:

\[
\begin{align*}
\dot{1}N_{\varphi,M} &= N_{\varphi,g} + 1.2 N_{\varphi,p,10} \\
\dot{1}N_{\varphi,M} &= -654.31 + 1.2 \cdot 696 = 180.89 \text{ kN/m (tension)} \\
\dot{1}N_{\varphi,M} &= 654.31 + 1.2 \cdot 757.5 = -1563.31 \text{ kN/m (compression)}
\end{align*}
\]

Load group II:

\[
\begin{align*}
\dot{2}N_{\varphi,M} &= N_{\varphi,g} + N_{\varphi,gf} \\
\dot{2}N_{\varphi,M} &= -654.31 + 1287 = 632.69 \text{ kN/m (tension)}
\end{align*}
\]

Annular: (load groups as before)

\[
\begin{align*}
\dot{1}N_{\varphi,M} &= -75.9 + 1.2 \cdot (-67) = -157.26 \text{ kN/m (compression)} \\
\dot{1}N_{\varphi,M} &= -75.9 + 1.2 \cdot (-67.8) = -157.26 \text{ kN/m (compression)}
\end{align*}
\]

Shear force:

\[
\begin{align*}
\dot{1}N_{\varphi,\theta,M} &= 0 + 1.2 \cdot (-253.6) = -304.32 \text{ kN/m} \\
\dot{1}N_{\varphi,\theta,M} &= 0 + (-513.1) = -513.10 \text{ kN/m}
\end{align*}
\]
6.6 Moments

Moments are calculated according to 5.22

\( \theta = 70^\circ, \ c_p = 1.2 \)

\[
M_{\varphi, p10} = \pm 0.001 \cdot 2.92 \cdot 1.2 \cdot 50^2 = 8.76 \text{ kNm/m}
\]

\[
M_{\varphi, p100} = \pm 0.001 \cdot 6.09 \cdot 1.2 \cdot 29.5^2 = 6.36 \text{ kNm/m}
\]

\[
M_{\varphi, p10} = \pm 0.004 \cdot 2.92 \cdot 1.2 \cdot 50^2 = 35.04 \text{ kNm/m}
\]

\[
M_{\varphi, p100} = \pm 0.004 \cdot 6.09 \cdot 1.2 \cdot 29.5^2 = 25.44 \text{ kNm/m}
\]

6.7 Critical shell force

In determining the critical shell force, the assumed concrete grade is B280 and

\( E_{bo} = 2.80 \cdot 10^6 \text{ MN/m}^2 \).

Calculation relies on Table IV.

<table>
<thead>
<tr>
<th>Author</th>
<th>Critical load or stress</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind load (about the throat)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Der and Fiedler</td>
<td>( P_{cr} = 17.3 \text{ kN/m}^2 )</td>
<td>( v = 0.2 \text{ m} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E_{bo} = 2.8 \cdot 10^6 \text{ MN/m}^2 )</td>
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<tr>
<td></td>
<td></td>
<td>( R_T = 29.5 \text{ m} )</td>
</tr>
<tr>
<td>ACI-ASCE</td>
<td>( P_{cr} = 12.9 \text{ kN/m}^2 )</td>
<td>( v = 0.2 \text{ m} )</td>
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<tr>
<td></td>
<td></td>
<td>( E_{bo} = 2.8 \cdot 10^6 \text{ MN/m}^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( R_T = 29.5 \text{ m} )</td>
</tr>
<tr>
<td>Herzog</td>
<td>( P_{cr} = 10.7 \text{ kN/m}^2 )</td>
<td>( v = 0.2 \text{ m} )</td>
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<tr>
<td></td>
<td></td>
<td>( E_{bo} = 2.8 \cdot 10^6 \text{ MN/m}^2 )</td>
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<tr>
<td></td>
<td></td>
<td>( R_T = 29.5 \text{ m} )</td>
</tr>
<tr>
<td>Zerna-Krätzig</td>
<td>( \sigma_p = 4.011 \text{ MN/m}^2 )</td>
<td>( E_{bo} = 2.8 \cdot 10^6 \text{ MN/m}^2 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_p = 29.779 \text{ MN/m}^2 )</td>
<td>( v = 0.20 \text{ m}; \ R_T = 29.5 \text{ m} )</td>
</tr>
<tr>
<td></td>
<td>( \mu = 0.2 )</td>
<td></td>
</tr>
<tr>
<td>Dead + wind load (near the bottom edge)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Herzog</td>
<td>( N_{\varphi, cr} = 347.6 \text{ kN/m} )</td>
<td>( v = 0.20 \text{ m} )</td>
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<tr>
<td></td>
<td></td>
<td>( E_{bo} = 2.8 \cdot 10^6 \text{ MN/m}^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D_A = 100 \text{ m} )</td>
</tr>
<tr>
<td></td>
<td>( N_{\varphi, cr} = 4358.0 \text{ kN/m} )</td>
<td>( D_A = 100 \text{ m} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( v_o = 0.70 \text{ m} )</td>
</tr>
<tr>
<td>During construction (built to height ( H = 80 \text{ m} ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chambaud [7]</td>
<td>( P_{cr} = 1.27 \text{ kN/m}^2 )</td>
<td>( E' = 1.74 \cdot 10^4 \text{ MN/m}^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( m = 3; \ p^1 = 0.2 \text{ m} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D^1 = 83.42 \text{ m} )</td>
</tr>
</tbody>
</table>
6.8 Checking the cross sections

Wall thickness may be checked by confronting effective and critical loads.

Effective loads:

Wind load: \( p_{10} = 2.92 \text{kN/m}^2; \ p_{100} = 6.09 \text{kN/m}^2 \).

Safety:

\[
\begin{align*}
 k_1 &= \frac{17.3}{6.09} = 2.84; \quad k_2 &= \frac{12.9}{6.09} = 2.12; \quad k_3 = \frac{10.7}{6.09} = 1.76.
\end{align*}
\]

Effective stresses:

Wind + dead load:

\[
\begin{align*}
\sigma_{p} &= \frac{157.26 \cdot 10^{-3}}{0.2 \cdot 1.0} = 0.782 \text{MN/m}^2 \\
\sigma_{\varphi} &= \frac{1563.31 \cdot 10^{-3}}{0.2 \cdot 1.0} = 7.81 \text{MN/m}^2
\end{align*}
\]

\( (v_0 = 0.7 \text{m}; \ \sigma_{\varphi} = 2.23 \text{MN/m}^2) \).

Safety according to Dunkerley:

\[
\begin{align*}
 v_\varphi &= \frac{4.011}{0.782} = 5.13; \quad v_\varphi &= \frac{29.779}{7.81} = 3.81 \\
k_4 &= \frac{1}{v_\varphi} + \frac{1}{v_0} = \frac{1}{0.19} \cdot \frac{1}{0.26} = 2.22.
\end{align*}
\]

All safety factors \( k_i \) but one are as high as 2.0, thus, 20 cm wall thickness in the upper shell part is adequate for stability.

Bottom safety factor:

\[
k_3 = \frac{N_{\varphi,cr}}{N_{\varphi,M}} = \frac{4358.0}{1563.31} = 2.79.
\]

Constructional load amounts to 1.30 kN/m², nearly the critical load.

Reinforcement needed (from steel B. 60.40):

meridional, in the lower shell part:

\[
F_{\varphi} = \frac{632.69}{34} = 18.61 \text{ cm}^2/\text{m} \quad (2 \times \varnothing 16/20)
\]

annular, in the lower shell part (\( \pm \) double value because of faulty shape):

\[
F_{\varphi} = \frac{2 \cdot 157.26}{34} = 9.26 \text{ cm}^2/\text{m} \quad (2 \times 12/20)
\]

Minimum reinforcement is 0.4% according to [2].

\[
0.4 \cdot 70 = 28 \text{ cm}^2/\text{m}; \quad \text{and} \quad 0.4 \cdot 20 = 8 \text{ cm}^2/\text{m}.
\]

In conclusion, the assumed shell geometry and wall thicknesses may be stated satisfactory.

6.9 Analysis of the top edge ring

The ring has to match the case seen in Fig. 6/b. The calculation follows item 5.4.

\[
\begin{align*}
F_r &= 11 \ 200 \text{ cm}^2; \ S_X = 352 \ 000 \text{ cm}^2; \ x = 31.4 \text{ cm}; \\
I_x &= 27 \ 859 \ 418 \text{ cm}^4; \ E_{\varphi} = 2.8 \cdot 10^4 \text{ MN/m}^2
\end{align*}
\]
weight of the ring per linear meter: \( p = F_r \cdot \gamma = 28 \text{ kN/m} \):

ring mass referred to gravity: \( \mu = \frac{p}{g} = \frac{28}{9.8} = 2.854 \text{ kNm}^{-2} \text{sec}^2 \).

Critical wind velocity (see Eq. (6)):

\[
v_{\text{crit}} = \frac{2.14 \cdot \sqrt{2.8 \cdot 10^6 \cdot 0.27859}}{2.554} = 103.2 \text{ m/sec},
\]

exceeding the design wind velocity \( (v_{120} = 101.65 \text{ m/sec}), \) thus, the bracing ring is of adequate size.

For an eddy separation frequency

\[
f_\phi = 0.1 \frac{v_{120}}{R_F} = 0.1 \frac{101.65}{34.28} = 0.297 \text{ 1/sec}
\]

\( f_\phi = 0.265 \) and \( f_\phi = 0.265 \)

\[
\text{dynamic pressure acting on the ring:}
\]

\[
p_r = 0.45 \cdot 6.46 \left[ \frac{1}{1 - \left( \frac{0.297}{0.466} \right)^2} \right] = 4.89 \text{ kN/m}^2
\]

applying tension and bending on the ring (Fig. 8)

\[
M = \frac{h_r \cdot p_r \cdot R^2}{4} = \frac{2.6 \cdot 4.89 \cdot 34^2}{4} = 3674 \text{ kNm}
\]

\[
H = h_r \cdot p_r \cdot R = 2.6 \cdot 4.89 \cdot 34 = 432 \text{ kN}.
\]

The reinforcement required:

\[
F_r = 112 \text{ cm}^2 \quad (18 \varnothing 28; \text{ or } 23 \varnothing 25; \text{ or } 36 \varnothing 20 \text{ B.60.40}).
\]

\[
\text{Fig. 7}
\]

Summary

Fundamental relationships and experience accumulated in research on, and in designing cooling towers have been complied as an aid to approximate dimensioning. The analysis method is completed by numerical examples.
References

3. Recapitulative Evaluation of the Special Literature.* Expertize developed at the Department of Reinforced Concrete Structures, Technical University, Budapest 1977.
4. Loads on, and Special Requirements for, Building Structures.* Hungarian Standard MSz 15021.
11. Analysis of the Edge Ring of Cooling Towers.* Expertize developed (by Gy. Vértes) at the Department of Reinforced Concrete Structures, Technical University, Budapest 1977.

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* In Hungarian.