DISCRETE ANALYSIS OF MULTISTOREY BUILDINGS, WITH RESPECT TO JOINT ELASTICITY

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1. Introduction

In recent decades, world-wide generalization of in-situ assembled, precast large-slab buildings highlighted relevant structural problems. A major problem is to determine the displacement of structural units, the developing stresses and ultimate load capacities. Quite a number of design methods are available both for theoretical research and for practical design [1, 2].

Among them, algorithm of the finite elements method relying on principles of the displacement method is felt to be the most convenient. In the last years, comprehensive research has been made at the Department of Civil Engineering Mechanics on the application possibilities of the finite elements method, and on the development of effective computer programs involving these algorithms, in particular, for taking the elastic deformations of junctions into consideration.

2. Unit types with rigid nodal joints

Two, essentially different considerations were underlying our models two program sets have been developed for, both suiting either plane walls (independent, detached from the building) or complex spatial buildings, as the case may be, under arbitrary boundary conditions (various soil models, symmetry etc.).

2.1 *Elastic plate model*

The first alternative involved rectangular units with corner nodes to model walls or floors. It is decisive for the model that units join each other at nodes with an infinite rigidity. Obviously, assembly of large-slab buildings must absolutely strive to in-situ joints of a rigidity as high as possible (welded and grouted) nevertheless both theoretical considerations and conclusions drawn from laboratory and full-scale model tests [3] argue against the excessive requirement of such a rigidity from prefabricated buildings, if not within certain load limits and even then, only from certain joint types.

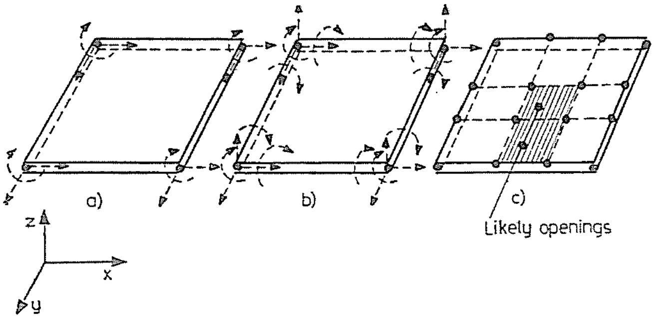

Fig. 1

Remark that units affected by the program set may have various characteristics.

The unit of twelve degrees of freedom seen in Fig. 1a suits only consideration of the plate effect. Type 1b reckons also with the effect of bending, it behaves like a "plane" shell unit (with 24 degrees of freedom). Type 1c is a so-called substructural unit. Here only the so-called global nodes at the corners join other units, displacements of which help of determine those of the other edge points. (By the moment, only its alternative with 12 degrees of freedom is effective.)

Any of these alternatives suits the analysis of either plane walls or spatial structures. By nature of the displacement method, unknown parameters of the problem are nodal displacements, that can be calculated in knowledge of the external load and of the structure geometry and physical data, while in knowledge of displacements, unit stresses may be indicated.

2.2 *Rigid panel model*

The other model type relies on absolutely different principles. Here every unit (wall slab) is modelled by an independent panel considered to be infinitely stiff in its plane [4, 5].

Units are connected by springs fit to take tension and compression at the corners, and shear along the edges (Fig. 2). Springs represent elastic characteristics of the slabs, in knowledge of the unit geometry and material characteristics their constants can unambiguously be determined by means of forces developing from unit displacements at the corners.

A spatial alternative may be created by analogy to the plane model in Fig. 2. Here a unit is connected by five springs on each edge to the other ones (Fig. 3).

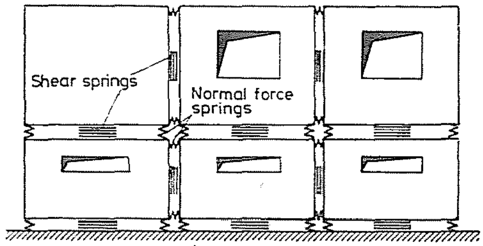

Fig. 2

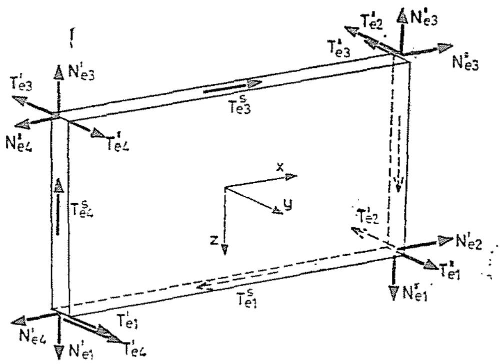

Fig. 3

In the figure, spring forces considered to be positive are represented. Forces normal to the plane are taken by shear springs. Determination of spring constants follows the same principles as in the plane case.

The presented method is rather advantageous. Since rigid panels can only perform rigid-body motion (e.g., in the plane case, two normal displacements and a rotation) the degrees of freedom of the tested system hence also the size of the coefficient matrix of the set of equations to be solved markedly decrease, the problem can be solved on a smaller computer or in less running time.

This method especially suits decription of the nodal behaviour, of particular importance, nodes being critical parts of large-slab buildings.

It should be noted that also the plastic behaviour of units can be described by this method, just as the model under 2.1 may be modified for the non-linear-plastic variety of rigid panels, affording a rather simple ultimate plastic analysis of panel-skeleton buildings [5]. Rigid panel models permit to take arbitrary boundary conditions into consideration. The soil behaviour is advisably simulated by the shear model, it being easy to fit to the stiff deep beam model [7].

3. Unit types with elastic nodal joints

In creating numerical models coping with the design practice, knowledge of the effective stiffness is imperative, alongside with the application of a structural skeleton fit to take joint elasticity into consideration.

Theoretical and experimental research has been made in this scope abroad [6]. The nodal joint model in Fig. 4 is generally considered as a close approximation.

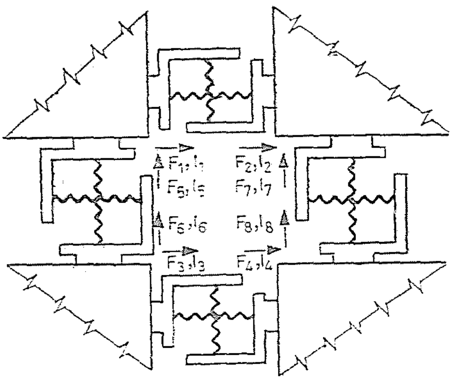

Fig. 4

The diagram represents the junction between corners of four units. Every corner suits to take normal displacements and forces, increasing the degrees of freedom of the complete "node" from two to eight.

Formulae are given in [6] for calculating the spring constants as a function of joint design (closed, open, ribbed etc.), reinforcement and cross section geometry.

This method has the inconvenients of much more unknowns than originally, and of the rather difficult consideration of the essentially separate nodal and joint rigidities, arguing against its application in our problems.

3.1 *Elastic unit — elastic joint*

3.11 Description of the model

Let us consider the panel in Fig. 5, joining adjacent units via corner springs.

Be $\mathbf{K}_e$ the elementary stiffness matrix of the panel unit with rigid nodes. Unit displacement (shift or rotation) at a spring end joining the panel will induce a displacement $\mathbf{u}$ of panel corners. In this case, obviously,

$$\mathbf{K}_e\mathbf{u} = \mathbf{s} \tag{1}$$

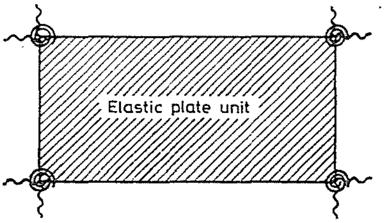

Fig. 5

for the panel, and

$$\mathbf{D}(\mathbf{e}_i - \mathbf{u}) = \mathbf{s} \tag{2}$$

for the springs, where $\mathbf{s}$ is the spring force vector, $\mathbf{D}$ the diagonal matrix of spring constants, and $\mathbf{e}_i$ a unit vector for the i-th displacement. Expressing vector $\mathbf{u}$ from (2):

$$\mathbf{u} = \mathbf{D}^{-1}(\mathbf{D}\mathbf{e}_i - \mathbf{s}) \tag{3a}$$

or

$$\mathbf{u} = \mathbf{e}_i - \mathbf{D}^{-1}\,\mathbf{s}. \tag{3b}$$

Substituting (3b) into (1):

$$\mathbf{K}_e(\mathbf{e}_i - \mathbf{D}^{-1}\,\mathbf{s}) = \mathbf{s}. \tag{4}$$

Arranged:

$$\mathbf{s} = (\mathbf{E} + \mathbf{K}_e\mathbf{D}^{-1})^{-1}\,\mathbf{K}_e\,\mathbf{e}_i. \tag{5}$$

(5) determines spring forces from unit displacement. According to principles of the displacement method, the determined vector yields the i-th column of a modified elementary stiffness matrix embracing elastic properties of both the unit and the joint. Accordingly, from (5):

$$\mathbf{K}_{\mathrm{mod}} = (\mathbf{E} + \mathbf{K}_e\,\mathbf{D}^{-1})^{-1}\mathbf{K}_e. \tag{6}$$

This stiffness matrix directly fits the algorithm of plate programs, no compilation change from the plate unit with rigid nodes is needed, neither the number of unknowns, thus, neither running time nor storage space are increased.

By nature of the equation, obviously, for infinitely high spring constants, $\mathbf{K}_{\mathrm{mod}}$ tends to $\mathbf{K}_e$, and for infinitely high $\mathbf{K}_e$, it tends to the spring constant matrix.

With the global stiffness matrix of the structure established, and fictitious nodal displacements $\mathbf{u}_f$ calculated, spring forces are obtained from:

$$\mathbf{s} = \mathbf{K}_{\mathrm{mod}}\,\mathbf{u}_f \tag{7}$$

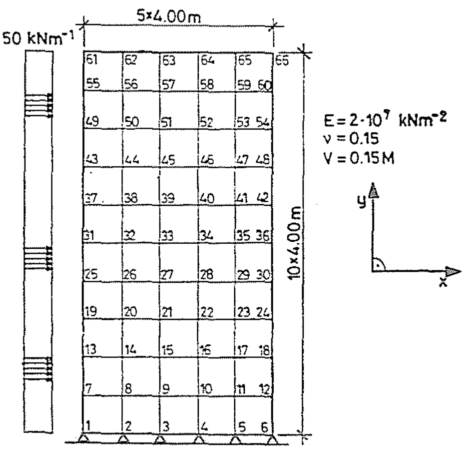

Fig. 6

leading, in turn, to corner displacements of elastic deep beams:

$$\mathbf{u} = \mathbf{u}_f - \mathbf{D}^{-1}\mathbf{s}. \tag{8}$$

In possession of $\mathbf{u}$, plate stresses can be determined.

Application of the model will be illustrated on a simple structure, e.g. the wall seen in Fig. 6. (Similar observations were made with spatial models, the plane model was chosen for the sake of easy surveying.)

The obtained horizontal displacement of a point on the top edge *vs.* spring constants is seen in Fig. 7. Displacements of either plane or spatial units were observed to about equal those for the stiff node model for spring constants of the order of 10^8 to 10^9 kNm^{-1}.

Vertical forces acting on units at nodes on the lower wall edge (considered as restrained at a close approximation) are seen in Fig. 8.

The diagram points out the increase of force differences due to node "relaxation".

3.12 Reckoning with the effect of shear deformations

The model seen in Fig. 5 is simple and easy to manage. In designing large-slab buildings, however, reckoning with shear deformations between units may be required. (This effect may be taken into consideration by means of the rigid panel model under 2.2.)

Alternative under 3.11 can only describe this phenomenon intermediating nodal springs but it is a rather unreliable method, likely to bias forces acting on the plate. A closer approximation of connection forces between units is offered by the model seen in Fig 9a. Here further (again elastic) nodes are assumed at mid-edges permitting to reckon with shear deformation effects.

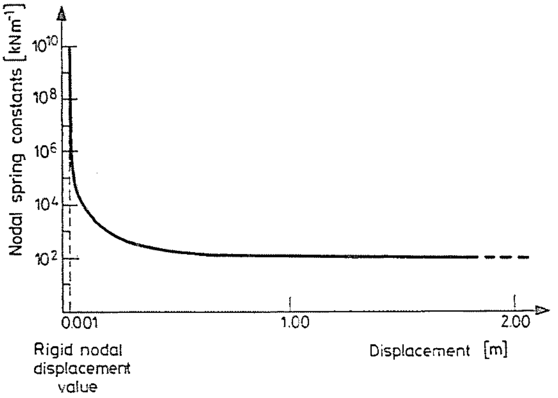

Fig. 7

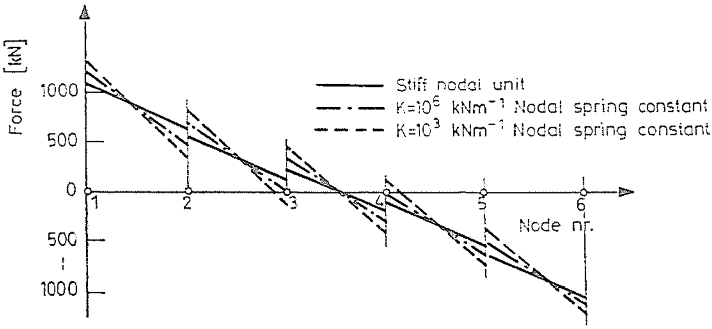

Fig. 8

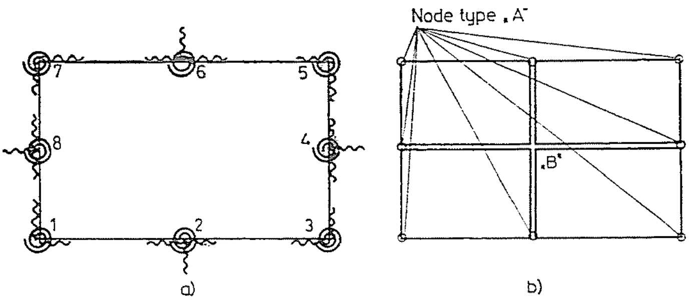

Fig. 9

Rather than to analytically establish the new stiffness matrix for the new unit (with 24 degrees of freedom), it is easy to produce by means of the method of substructures.

Partitioning the equilibrium equation of the elastic unit set (of momentarily perfectly stiff nodal junctions) assuming no force at the middle node:

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{F}_a \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{ba} \\ \mathbf{K}_{ab} & \mathbf{K}_{aa} \end{bmatrix} \begin{bmatrix} \mathbf{e}_b \\ \mathbf{e}_a \end{bmatrix} \tag{9}$$

where $\mathbf{F}$ and $\mathbf{e}$ are vectors of nodal forces and displacements, $\mathbb{K}$ being the stiffness matrix of the set. Expressing displacements of the inner point in terms of $\mathbf{e}_a$:

$$\mathbf{e}_b = -\mathbb{K}_{bb}^{-1}\,\mathbb{K}_{ba}\,\mathbf{e}_a \tag{10}$$

$$\mathbb{F}_a = \underbrace{(\mathbb{K}_{aa} - \mathbb{K}_{ab}\,\mathbb{K}_{bb}^{-1}\,\mathbb{K}_{ba})}_{\mathbb{K}}\,\mathbf{e}_a. \tag{11}$$

The resulting matrix $\mathbb{K}$ may be used in the following as stiffness matrix of the new unit (with eight nodes and 24 degrees of freedom). Deduction under 2.11 may help dissolving the nodal stiffness. This unit type has the inconvenient to require modifications in the computer program (a different compilation procedure etc.).

3.2 *Generalized rigid panel model*

This model permits a rather simple and efficient consideration of joint deformations, by simply complementing the matrix of unit elastic characteristics

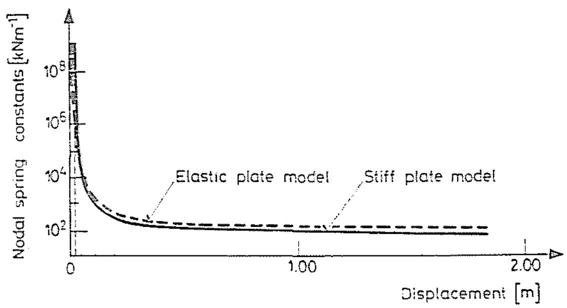

Fig. 10

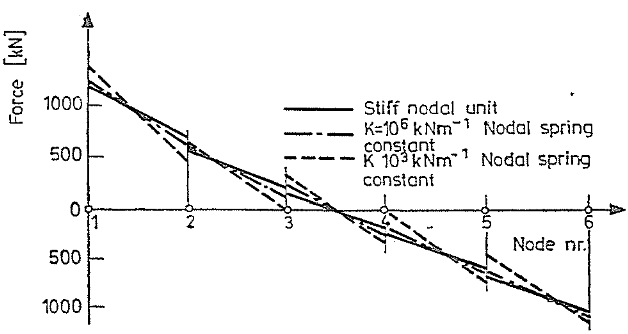

Fig. 11

by the effect of elastic joints [4, 5], yielding for the adjacent edges:

$$\mathbf{K}_{\text{mod}} = (\mathbf{K}_{\text{unit el.}}^{-1} + \mathbf{K}_{\text{node el.}}^{-1})^{-1} \tag{12}$$

needing no modification whatever of the available computer algorithm.

Outcomes of numerical analyses on different structures equal results obtained on the model under 3.11. Horizontal displacements at top mid-edge of the wall structure seen in Fig. 6 have been plotted *vs.* nodal junction spring constants in Fig. 10 (in smooth line). For the sake of comparison, outcomes with the elastic plate model have been represented in dashed line.

Figure 11 represents vertical forces at the lower nodes. The results show this model to suit simple and reliable solution of the problem.

4. Application of reduced substructure units

Voluminous problems may advantageously be solved by so-called "reduced" substructure units (Fig. 12). Essential of the method is to determine displacements of the so-called inner edge points (type "b") from those of the "global" points type "a" by means of matrix equation

$$\mathbf{e}_b = \mathbf{G}_{ab}\, \mathbf{e}_a \tag{13}$$

where matrix $\mathbf{G}_{ab}$ contains displacement constraint conditions we prescribed. Thereby even for very many inner units, an elementary stiffness matrix $\mathbf{K}_{\text{red}}$

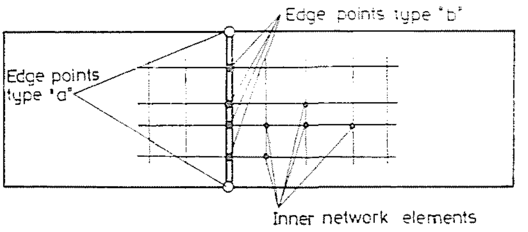

Fig. 12

can always be established, valid only to edge points type "a". Cyclic application of this method permits to analyse extensive systems by means of a few units.

Provided all nodal junctions along the edge are considered as elastic, constraint condition matrix $\mathbf{G}_{ab}$ in (13), thus, nodal stiffness in reduced substructures can only be dissolved if only nodes type "a" are assumed to be elastic connections, while displacements of inner edge points type "b" are treated as independent.

Summary

Simple methods have been presented for the design of units with elastic nodes. Integrating the models with two kinds of programs, analysis of plane and spatial structures showed them to be effective, to require no important modification of the available programs or an important increase of the running time.

These methods have been applied in two problems:

a) Numerical analysis of the effect of varying nodal spring characteristics by vertical and horizontal parameters on the displacements.

b) Theoretical relationships for determining the nodal spring constants for existing structures.

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