LIMIT ANALYSIS OF LARGE PANEL BUILDINGS AND THE SUBGRADE

By

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1. The model of the structure

Since the joints made in situ are generally the weakest parts of a prefabricated structure, in the present analysis of large panel (LP) buildings it is assumed that in plastic limit state merely the joints of the panels undergo plastic deformation. Thus, in the model of the LP structure the panels can be considered as rigid elements, while the joints are replaced by plastic springs acting in tension, compression and shear (Fig. 1) [1]. Considering a wall of a LP building, the equilibrium of the ith rigid element (Fig. 2a) subject to external loads and spring forces

$$\mathbf{q}_i^* = [\mathrm{P}_{ix}\,\mathrm{P}_{iy}\,\mathrm{M}_i]; \qquad \mathbf{s}_{ik}^* = [\mathrm{N}'_{ik}\,\mathrm{N}''_{ik}\,\mathrm{T}_{ik}]$$

resp., can be expressed as

$$\mathbf{G}_i^*\,\mathbf{s}_i + \mathbf{q}_i = \mathbf{0}. \tag{1}$$

Here $k = 1, 2, 3, 4$ refers to the edges of the element, and

$$\mathbf{G}_i^* = \left[\begin{array}{ccc:ccc:ccc:ccc} 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & -1 \\ -\mathrm{b}_i & \mathrm{b}_i & \mathrm{a}_i & \mathrm{a}_i & -\mathrm{a}_i & \mathrm{b}_i & \mathrm{b}_i & -\mathrm{b}_i & \mathrm{a}_i & -\mathrm{a}_i & \mathrm{a}_i & \mathrm{b}_i \end{array}\right], \quad \mathbf{s}_i = \begin{bmatrix} \mathbf{s}_{i1} \\ \mathbf{s}_{i2} \\ \mathbf{s}_{i3} \\ \mathbf{s}_{i4} \end{bmatrix}.$$

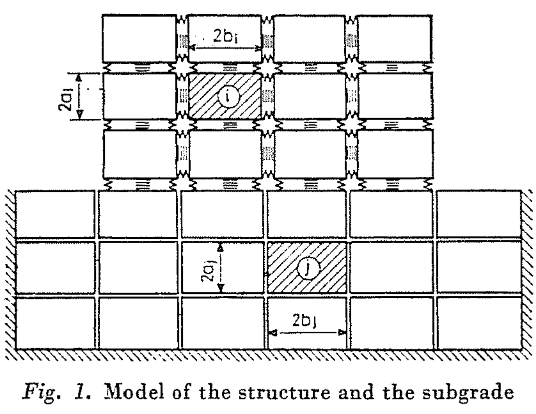

Fig. 1. Model of the structure and the subgrade

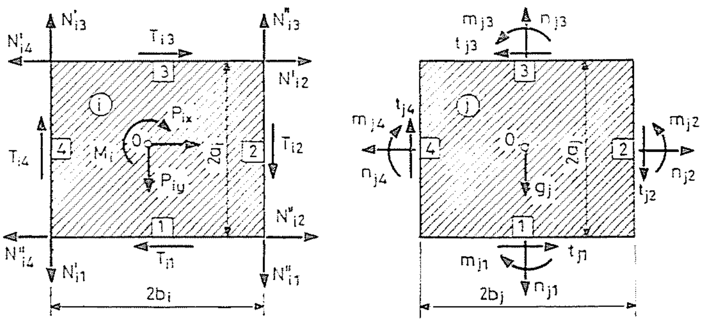

Fig. 2. Discrete elements a) structure, b) subgrade

Denoting the yield forces of the springs in tension, compression and shear by N_{ik}^{+}, N_{ik}^{-} and T_{ik} resp., and neglecting the interaction between the springs, the yield conditions of the edges have the form

$$\mathbf{H}_{ik}\,\mathbf{s}_{ik} - \mathbf{r}_{ik} \leq \mathbf{0}; \qquad k = 1, 2, 3, 4. \tag{2}$$

Here

$$\mathbf{H}_{ik}^{*} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \qquad \mathbf{r}_{ik}^{*} = [N_{ik}^{+}\; N_{ik}^{-}\; N_{ik}^{+}\; N_{ik}^{-}\; T_{ik}\; T_{ik}].$$

2. The model of the subgrade

The ideal plastic subgrade (soil) is subdivided into discrete prismatical elements which transmit normal, shear and couple forces to each other along their contact surfaces (Fig. 1) [2]. Considering the jth element (Fig. 2b) subject to its own weight and to contact forces

$$\mathbf{g}_j^{*} = 4\mathrm{a}_j\mathrm{b}_j c\gamma_j\,[0 \quad 1 \quad 0], \quad \mathbf{p}_{jk}^{*} = [\mathbf{n}_{jk}\mathbf{t}_{jk}\mathbf{m}_{jk}]$$

resp., the equilibrium equations can be written as

$$\mathbf{B}_j^{*}\,\mathbf{p}_j + \mathbf{g}_j = \mathbf{0}. \tag{3}$$

Here $k = 1, 2, 3, 4$ refers to the boundary surfaces, c and γ_j denote the thickness and the specific weight, resp., of the element, and

$$\mathbf{B}_j^{*} = \left[\begin{array}{ccc:ccc:ccc:ccc} 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & -\mathrm{a}_j & 1 & 0 & \mathrm{b}_j & -1 & 0 & -\mathrm{a}_j & -1 & 0 & \mathrm{b} & 1 \end{array}\right], \qquad \mathbf{p}_j = \begin{bmatrix} \mathbf{p}_{j1} \\ \mathbf{p}_{j2} \\ \mathbf{p}_{j3} \\ \mathbf{p}_{j4} \end{bmatrix}.$$

Applying the linearized *Tresca* yield condition to the stress state at the centre 0 of the element we obtain the inequality

$$\mathbf{C}_j \mathbf{p}_j - \boldsymbol{\sigma}_j \leq \mathbf{0}. \tag{4}$$

Here

$$\mathbf{C}_j = \left[\begin{array}{ccc|ccc|ccc|ccc}
-1 & \frac{3}{8} & \frac{9}{8a_j} & \frac{b_j}{a_j} & \frac{3b_j}{8a_j} & \frac{9}{8a_j} & -1 & \frac{3}{8} & -\frac{9}{8a_j} & \frac{b_j}{a_j} & \frac{3b_j}{8a_j} & -\frac{9}{8a_j} \\
-1 & -\frac{3}{8} & -\frac{9}{8a_j} & \frac{b_j}{a_j} & -\frac{3b_j}{8a_j} & -\frac{9}{8a_j} & -1 & -\frac{3}{8} & \frac{9}{8a_j} & \frac{b_j}{a_j} & -\frac{3b_j}{8a_j} & \frac{9}{8a_j} \\
1 & -\frac{3}{8} & -\frac{9}{8a_j} & -\frac{b_j}{a_j} & -\frac{3b_j}{8a_j} & -\frac{9}{8a_j} & 1 & -\frac{3}{8} & \frac{9}{8a_j} & -\frac{b_j}{a_j} & -\frac{3b_j}{8a_j} & \frac{9}{8a_j} \\
1 & \frac{3}{8} & \frac{9}{8a_j} & -\frac{b_j}{a_j} & \frac{3b_j}{8a_j} & \frac{9}{8a_j} & 1 & \frac{3}{8} & -\frac{9}{8a_j} & -\frac{b_t}{a_j} & \frac{3b_j}{8a_j} & -\frac{9}{8a_j}
\end{array}\right]$$

$$\boldsymbol{\sigma}_j^* = 4b_j c\sigma_j^\circ \, [1 \;\; 1 \;\; 1 \;\; 1]$$

and σ_j° denotes the yield stress of the jth element of the subgrade.

3. Limit analysis

Using the models described above, statically admissible stress fields can be constructed for the LP structure and the subgrade which makes possible the unified limit analysis of the whole system. Considering a wall subject to a one-parameter load $\mathbf{q} = \mathrm{m}\mathbf{q}_0$, the equilibrium and the yield conditions have the form

$$\mathbf{G}^* \mathbf{s} + \mathrm{m}\mathbf{q}_0 = \mathbf{0}, \qquad \mathbf{H}\mathbf{s} - \mathbf{r} \leq \mathbf{0}, \tag{5}$$

resp., while the same conditions for the subgrade are

$$\mathbf{B}^*\mathbf{p} + \mathbf{q} = \mathbf{0}, \qquad \mathbf{C}\mathbf{p} - \boldsymbol{\sigma} \leq \mathbf{0}. \tag{6}$$

Since $\mathbf{s}$ and $\mathbf{p}$ are related by the statical boundary conditions of the contact line between the structure and the subgrade, (5) and (6) make up a system of linear equations and inequalities the solution of which, together with the extremal condition

$$\mathrm{m} = \max! \tag{7}$$

leads to linear programming.

4. Applications

The method described suits the analysis of the structural response of LP buildings under abnormal loading conditions. Assuming ineffective (missing) panels or subgrade elements the effect of gas explosion or of cavitation of the soil can be investigated. Besides, the discrete models can be applied to the approximate limit analysis of LP structures subject to dynamic pressure.

Summary

Simple discrete models are presented for the limit analysis of prefabricated large panel buildings resting on a plastic subgrade. The model of the structure consists of rigid panels interconnected along their edges by plastic springs, while the subgrade is subdivided into prismatical elements which transmit normal, shear and couple forces to each other. Using these discrete models and linear(ized) yield conditions, statically admissible stress fields can be constructed for the structure and the subgrade. This makes possible the unified limit analysis of the whole system by the use of linear programming.

References

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