DYNAMIC ASPECTS OF REPEATED GEODETIC LEVELLINGS

By

P. Biró

Department of Geodesy, Institute of Geodesy, Surveying and Photogrammetry;
Technical University, Budapest

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Geodetic levelling is the usual method of determining height differences between neighbouring bench marks. Repeated geodetic levellings are the common method of determining vertical surface displacements (so-called "recent crustal movements"). In both cases the equipotential surfaces of the Earths' gravity field serve as the reference system of heights. Basic assumption in determining heights, in particular, vertical surface displacements is the stability of our reference system, that is,

- the distance of any two level surfaces; and
- the location of all level surfaces in space

are supposed to be unchangeable (i.e. they do not vary in time).

Nowadays we have more and more theoretical and practical information about variations of the Earth's gravity field. No doubt, such variations do have a great influence on geodetic measurements.

Anyway we have to check

- the influence of variation with time of the Earth's gravity field on the heights of bench marks;
- the interpretation of the observed changes of heights and of gravity, i.e. to explain their true physical meaning.

Author's previous theoretic investigations led to the following basic relationships for two models of different complexities and for the Earth [2—5].

In the simple case of a *perfectly rigid* Earth, unable to deformation, no vertical surface movement could arise. Any change of the potential of our bench marks would unambiguously show the variation with time of the gravity field, or vice versa, changes of the surface gravity would lead to unambiguous conclusions on the change of the potential. The displacement of the equipotential surfaces of the gravity field is expressed by

$$a = \frac{\delta W}{g} \tag{1}$$

where a is the displacement of the equipotential surface of the rigid Earth of potential W, δW being the potential change at the tested point (i.e. bench mark), and g the gravity.

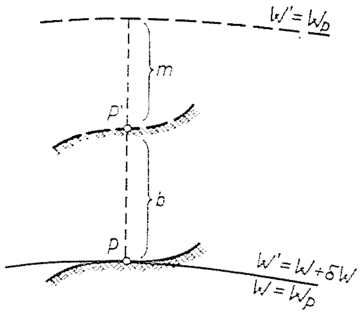

Fig. 1

In reality, however, our Earth is not rigid but *elastic*, deformable upon gravity field changes. Consequently, field changes not only entrain the displacement of the equipotential surfaces, but this displacement is followed by the elastic deformation of the Earth surface (Fig. 1). Accordingly, the position of our bench marks relative to the equipotential surfaces of the gravity field, i.e. their height above the sea level H_P will change by

$$\delta H_P = -m_P = -\frac{\delta W_{P'}}{g} = B\delta\, g_{P'}$$

where

$$B = \left(\frac{\partial g}{\partial H}\right)^{-1} \approx -\frac{W}{2g^2} \text{ theoretically.} \tag{2}$$

In general, it has to be assumed that the changes of the gravity field at different points of the Earth surface are different ($\delta g_{P'} \neq \delta g_{Q'}$). Let $\delta g_{P'}$ and $\delta g_{Q'}$ be the variation of the surface gravity at shifted points P' and Q', resp., the height difference H_Q^P between points P and Q will change by:

$$\delta H_Q^P = -(m_P - m_Q) = B\,(\delta g_{P'} - \delta g_{Q'})\,. \tag{3}$$

It should be noticed that this is the difference of displacements of the tested bench marks referred to the equipotential surfaces of the gravity field. Since, however, the equipotential surfaces themselves are shifted in the space, (3) fails to indicate the absolute vertical movements of the points. In our simple model, beside elastic deformations following gravity field variations, no other, e.g. geological surface movements have been assumed to occur.

Using *Love*'s theory of the Earth's elasticity, the difference between the absolute vertical displacements of points P and Q on the Earth surface can be expressed by

$$b_P - b_Q = h(a_P - a_Q) = \frac{h}{g}\,\frac{\delta(W_{P'} - W_{Q'})}{D} = -\frac{h}{D}\,B\,(\delta g_{P'} - \delta g_{Q'}) \tag{4}$$

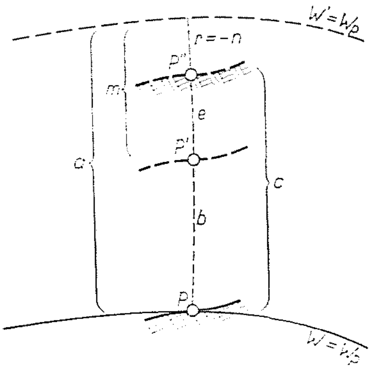

Fig. 2

where

$$D = 1 - h + k$$

h and k being *Love*'s numbers. As the difference of Eqs (3) and (4) expressing the difference between the absolute and the relative displacement of the Earth surface points, we obtain

$$d = b_P - b_Q - \delta H_Q^P = - \left(\frac{h}{D} + 1 \right) B \left(\delta g_{P'} - \delta g_{Q'} \right) \tag{5}$$

that would be zero only for quite exceptional values of h and k; in general, however, it is *significantly non-zero*. The difference depends on the real values of the *Love*'s numbers. Their empirical values being known only from observations of relatively short-period phenomena, determination of *Love's* numbers for long-period or secular variations of the gravity field needs further investigations.

Anyhow, from the available data it can be stated that both *the relative height difference and the absolute location in space of the Earth surface points in a gravity field varying in time are in general time-dependent, even if no kind of geological movement intervened.* Thus, a due circumspection is needed when interpreting observed changes of height differences.

In the *general case*, vertical displacements of the Earth surface are entrained, — in addition to elastic deformations following time variations of the gravity field, — also by other movements, e.g. those of geological origin (Fig. 2). The complete Earth surface displacement is thus a resultant of different effects, hence also the observed changes of the surface gravity are composed both of the effect of surface displacements and of field changes.

In this general case, too, geodetic levellings lead only to displacements of the Earth surface referred to the equipotential surfaces of the gravity field:

$$n_P = - B \, \delta g_{P''} = - \delta H_P \tag{6}$$

where $\delta g_{P''}$ is the change of gravity observed on the Earth surface displaced from several causes. In calculating the absolute vertical displacement of the Earth surface c, the relative displacement n ought to be reduced by the absolute displacement of the equipotential surfaces of the gravity field:

$$c_P = a_P - n_P = a_P + \delta H_P . \tag{7}$$

Unfortunately, however, no exact method can be suggested for calculating a from the gravity change, like in case of a rigid Earth body or of purely elastic deformations, namely now it contains various unknown effects. This is why research to determine the absolute displacement of equipotential surfaces by other means (e.g. by satellite altimetry), or to determine the secular variation of the gravity field from the observation of that of some other physical field (e.g. the Earth's magnetic field) is of great importance. Until no relevant research result is available, one must be aware that *the vertical surface movements observed by usual geodetic levellings are but relative values referred to the equipotential surface which may significantly differ from the absolute surface displacements* ($c \neq n$). Denoting them by r_P and $r_P - r_Q$ (relative surface movement), resp.,

$$r_P = - n_P = \delta H_P = B\, \delta g_{P''} \tag{8}$$

and

$$r_P - r_Q = -(n_P - n_Q) = \delta H_Q^P = B\,(\delta g_{P''} - \delta g_{Q''}) = B\, \delta(g_{P''} - g_{Q''}) . \tag{9}$$

Nevertheless the changes of gravity on the Earth surface are resultants of several effects themselves, and do not let conclude on the separate causes (e.g. surface movements, secular variations of the field) without accessory examinations. At the same time, however, besides of the usual methods, involving the observation of changes of the surface gravity, the determination of surface movements referred to the equipotential surfaces is likely to be advantageous and opens new possibilities.

Our previous research has led to the conclusion that instead of true (absolute) surface displacements c, only relative surface movements r_P or their difference $r_P - r_Q$ (referred to the equipotential surfaces of the gravity field) can be practically determined. Relationships (8) and (9) show two possible methods to exist for this purpose. Relative surface movements can be determined either by usual repeated geodetic levellings (δH_Q^P or δH_P if connected to the current mean sea level) or by repeated precise absolute or relative gravity observations $\delta g_{P''} - \delta g_{Q''}$ and $\delta(g_{P''} - g_{Q''})$. Both kinds of observations yield the change of the potential difference of the bench marks.

This conclusion has led to the suggestion [4, 5, 6] that extended levelling lines for determining the relative crustal movements can be replaced or controlled by repeated precise gravity measurements. Considering the accuracy of modern absolute or relative precise gravity observations (a few microgals) they are seen to be safely comparable to the reliability of a levelling of several hundred or 1000 to 1500 km length.

Therefore a new possible scheme of vertical crustal movement control net presented in Fig. 3 has been suggested [6].

Relative crustal (surface) movements can be determined by usual repeated geodetic levellings in local systems related to the level surface of local reference stations Q_i. These reference stations can be connected to each other and to regional control points S_i by regional geodetic levelling lines. Some of these on the shore should be connected to tide gauges (i.e. to the current mean sea level). Regional control points should be connected to each other and to some super control points with known absolute gravity by relative precise gravity measurements.

National vertical control nets could serve for local systems. Average distance of local reference stations (bench marks) can be about some hundred kilometers, average space of regional control points could be suggested as 1000 to 2000 km. On this way the reliability of relative crustal movements can be increased very economically.

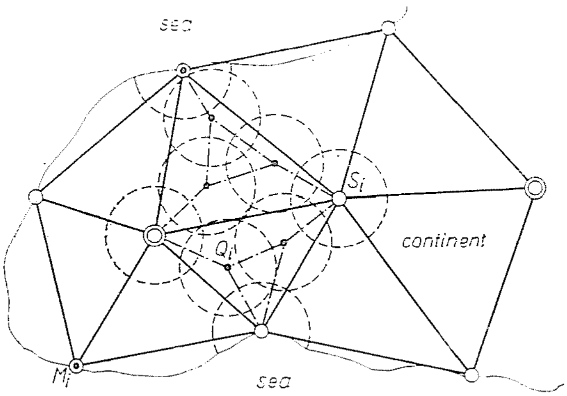

Fig. 3. Suggested scheme of a continental network for studying recent relative crustal movements

Symbols:

◎	super control point (with absolute gravity determination)
○ S_i	regional control point
⊙ M_i	regional control point with tide gauge
• Q_i	local reference station
—	gravimetric connections
—·—	regional levelling lines
◌	local systems for studying crustal movements

Formula (9) is an exact relationship between $\delta H_Q^P = \delta(H_P - H_Q)$ and $\delta(g_P - g_Q)$ *for any case, involving* no assumption. It can be checked empirically with the conclusions deduced therefrom by simultaneously observing both quantities. Special precise gravity lines (as for example the Fennoscandian land uplift gravity line [13, 14, 15], or a similar one in the GDR and others) will give very important information also for this purpose *if also height differences will be simultaneously observed by repeated geodetic levellings between the stations.*

Considerations deduced theoretically from formula (9), namely, that repeated precise levelling for investigating recent crustal movements can be replaced by repeated precise gravity observations, are in complete agreement with the recent practical experience and conclusions by Professor NAKAGAWA and Mr. SATOMURA [16]. They succeeded in making repeated simultaneous gravity observations and precise levellings around the *Lake Biwa* (Japan) between 1971 and 1976. They have 15 bench marks on a line of about 90 km long and made observations every year by *LaCoste—Romberg G* gravimeters. In their paper they have graphically shown gravity change and changes of elevations of bench marks and stated the obvious correlation of both measurements.

With kind allowance of the authors [17] we checked numerically this correlation and got a correlation factor of —0.63, which is not too bad, but also not very good. But gravity change and changes of elevations above the sea level are loaded by many uncertainties (i.e. unknown changes of gravity and of the elevation of the reference station, inhomogeneous reliability of data etc.). Therefore it is more useful to check the correlation of changes of gravity differences and of height differences between neighbouring bench marks. Observations (gravity measurement and geodetic levelling) directly result in these changes without making any assumption, and most probably with near the same accuracy for any couple of stations. Between 15 bench marks one gets 14 differences. Gravity observations are available from 4 or 5 years and levellings from 1971, and 1975/1976. According to this fact we had a set of data for the change of height differences $\delta(H_P - H_Q)$ between 1971 and 1975/1976 and another set of data for the change of gravity differences of neighbouring bench marks for the same time interval. This latter has been calculated according to two alternatives:

a) as the difference of the observed gravity difference of 1971 and 1975/1976, when also levellings were completed *(observed changes of gravity differences)*;

b) for the time interval of the levellings, fitting a straight line, by the least squares method to the observed gravity differences of each couple of neighbouring bench marks *in each of the years* between 1971 and 1975/1976 *(predicted changes of gravity differences).*

Applying both sets of changes of gravity differences $\delta(g_P - g_Q)$ the correlation of them with the changes of height differences

$$\delta(H_P - H_Q) = A + B\ \delta(g_P - g_Q)$$

has been tested, yielding the *correlation factors* of $r = -0.84$ and $r = -0.70$ for observed and for predicted changes, respectively, of gravity differences, showing a significant correlation of the analyzed quantities.

Correlation factor for predicted gravity changes is less significant than that for observed ones. This may be attributed to the fact that gravity changes of all the bench marks are not really linear! The deviations of observations from the "least squares" straight line are only partly due to errors of observations, partly they show real differences of gravity changes between the different years. It means that *gravity changes are most probably "rapid" (irregular), local variations*, and in most part not regional, secular ones (trends have different signs!).

The "least squares" fitting made it possible to calculate the mean square error of one gravity difference to be ± 12 μgal. Accordingly, the m.s.e. of the change of gravity difference is $\sqrt{2}\,.\,12 = \pm 17$ μgal.

For an average spacing of bench marks $s = 6.5$ km and accepting $0.8\sqrt{s^{\mathrm{km}}}$ for the *m.s.e.* of precise levelling, we get for the *m.s.e.* of one difference of height ± 2.04 mm and for the *m.s.e.* of the change of each height difference $\sqrt{2} \cdot 2.04 = \pm 2.9$ mm. These mean square errors were accepted as measure of the reliability of the sets of data. The sets of data have been homogenized by dividing each data by its proper mean square error [20]. In this way homoceneous sets of data resulted with uniform reliability (with unit weight). They could be used to fit a "mean adjusting straight line" (p. 419—421 in [21])

$$\delta(H_P - H_Q) = A + B\ \delta(g_P - g_Q)$$

expressing the correlation of changes of gravity differences and of differences of heights under the condition of

$$\Sigma(v_{\delta g}^2 + v_{\delta H}^2) = \text{min}, \ v \text{ being the residuals.}$$

The same correlation has been checked also by regression analysis in both alternatives:

$$\delta(H_P - H_Q) = A_1 + B_1\ \delta(g_P - g_Q)$$

under the condition $\Sigma v_{\delta H}^2 = \min$ (i.e. $m_{\delta g} = 0$) and

$$\delta(g_P - g_Q) = A_2^x + B_2^x\ \delta(H_P - H_Q)$$

under the condition $\Sigma v_{\delta g}^2 = \min$ (i.e. $m_{\delta H} = 0$). Solving this latter for $\delta(H_P - H_Q)$, one gets

$$\delta(H_P - H_Q) = \frac{A_2^x}{B_2^x} + \frac{1}{B_2^x}\,\delta(g_P - g_Q) = A_2 + B_2\,\delta(g_P - g_Q)\,.$$

These analyses have resulted in

$A_1 = +0.57$ mm $B_1 = -0.61$ mm/μgal; ($m_{\delta g} = 0$)
$A_2 = +0.80$ mm $B_2 = -0.87$ mm/μgal; ($m_{\delta H} = 0$)
$A\ = +0.79$ mm $B\ = -0.85$ mm/μgal; ($m_{\delta g} = \pm 17\mu$gal, $m_{\delta H} = \pm 2.9$ mm)

for observed changes of gravity differences.

Theoretical investigation has led to $A = 0$ and $B = -W/2g^2$; for the investigated territory $B = -3.3$ mm/μgal.

All values of A determined experimentally are small enough (related to their *m.s.e.*) to be neglected but experimental values of coefficient B are about 1/5—1/4 part of the theoretical one. (This confirms numerically the earlier experience in [16]).

This contradiction needs further investigation by repeated simultaneous precise gravity measurements and geodetic levellings along several test lines.

Anyway the theoretical value $B = -W/2g^2$ is exactly valid only for the external gravity field of the Earth on one hand; on the other hand, all observed changes of gravity differences are $\delta(g_P - g_Q) < 1.7\, m_{\delta g}$ and half of the observed changes of height differences are $\delta H < 2\, m_{\delta_H}$.

Conclusions

— Theoretical investigations and practical experience by several authors [2—7, 8, 9, 10, 11, 12, 18, 19] confirm the establishment of time-changes of geopotential differences of stations on the earth's surface to characterize true vertical surface (crustal) movements *only in exceptional cases, where no changes of the earth's gravity field occur* (Eq. 9).

— *Changes of gravity differences have been shown* [16], [4—6] *to be related to changes of height differences.* Theoretical investigations [4 to 6] show a linear relationship, in the form

$$\delta(H_P - H_Q) = B\,\delta(g_P - g_Q)$$

where theoretically:

$$B = -\frac{W}{2g^2} \quad (\approx -3.3 \text{ for Japan})$$

and

— $0.85 < B < -0.61$ calculated statistically from the observations around *Lake Biwa*.

— The same conclusion was drawn experimentally [16] and theoretically [4 to 7] that time changes of geopotential differences (i.e. changes of elevation differences) of bench marks can be observed either by repeated geodetic levellings or by repeated gravity observations. Authors share the view that repeated precise gravity measurements may be substituted for repeated geodetic levelling as a more rapid and economical although less accurate method for short distances at the actual stade of techniques.

— Present accuracy of gravity measurements ($\pm 10-20$ μgals) does not allow yet to numerically determine changes of height differences to one or two millimeters, but they are accurate enough to show the trends and they are comparable to the reliability of extended geodetic levelling.

— A new scheme of an extended (continental) network (Fig. 3) has been suggested for studying relative vertical crustal movements using gravity determinations for economically increasing the reliability of the net.

— Present analysis of the observations reported in [16] call the attention that beside secular and regional gravity changes, significant and *considerable irregular local changes* of the Earth's gravity field may occur within restricted ranges (few kilometers).

— In accordance with page 217 in [16] let us suggest to perform repeated *simultaneous precise gravity measurements and geodetic levellings* along several test lines and in different areas for studying the relationship between the results of both kinds of measurements.

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Summary

Theoretical investigations show time changes of geopotential or elevation differences of bench marks to characterize true vertical movements of the Earth's surface (or crust) only in exceptional cases. Theoretical and practical experiences show the changes of gravity differences and of height differences to be in linear correlation. Thereby repeated precise gravity measurements may be substituted for repeated geodetic levelling as a more economical and rapid method especially for long distances. A new continental network scheme has been suggested for studying recent vertical movements. Attention is called to possible considerable irregular local changes of Earth's gravity field within restricted ranges. It is recommended to carry out repeated simultaneous precise gravity measurements and geodetic levellings along several test lines and areas.

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Professor Peter Biró, H-1521 Budapest