PRESENTATION OF VERTICAL MOVEMENTS OF EARTH'S SURFACE ON COMBINED VELOCITY MAPS

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Received: 27th November, 1978

In recent decades, vertical movement investigations have become routine in geodesy, often comprising not only the observation of vertical movements of different engineering structures but also of smaller or greater parts of the Earth's surface. Such investigations may last several years or even decades. (What is more, the greatest such project of our days, investigation of recent crustal movements, has a perspective of centuries.)

Below, representation of results obtained for the vertical movements of some part of the Earth's surface will be discussed, pointing out problems of graphic interpretation, normally by isolines.

Isoline movement maps (velocity maps) are constructed from observed vertical displacements of bench marks in a given area after a time, related to their initial position.

Repeated determinations of bench mark elevations permit to represent Earth surface displacements during a period between any two measurements by isolines.

Obviously, in practice, the problem is much more difficult: namely several factors affect the reliability of the isoline map constructed from observed vertical displacements of bench marks related to the effective vertical displacements of the tested area. (For details see an earlier discussion [1].)

In reality, the outlined ideal process of vertical movement investigation is impaired by the gradual destruction of bench marks.

Therefore in fact, no protracted investigation of movements can rely throughout on the same bench marks. Any insistance on it would result in an increasingly sketchy, ever less detailed movement map, and at last it would be clear that data for the remaining scarce identical bench marks are quite insufficient for a real movement map.

Previously, these problems have been referred to ([1]), stating, however, that inherent possibilities of graphic representation permit to investigate movements on bench marks that are not all original, if those perished in the meanwhile are replaced before each masurement. Namely most of the involved bench marks have already been involved in previous determinations, of a number sufficient to construct isoline velocity maps from their vertical displacements. (This conception has the enormous advantage to permit after a long while all bench marks to be exchanged though maintaining continuity of movement investigation, at the same density of control points, and for an arbitrary period.)

In order to present movements during a longer period including several consecutive intervals in a single isoline map, contents of isoline maps for several (consecutive) intervals have to be summarized [1], essentially a problem of summing graphic information (velocity maps): in plain words, superposing isolines.

This problem may be solved by making use of research results by HOVÁNYI, L. in our special field. HOVÁNYI [2, 3, 4] developed the mining geometry problem of determining and representing resultant surfaces by isolines (total layer thicknesses, total mineral wealth, geostatic pressure etc.).

A simple summation problem, like that of ours, is easy to solve by either of two means [4]:

a) Finding intersections of two curve families traced to the same scale and superposed, then writing the sum of values of the two intersecting curves to the intersection; plotting a third family of curves from these values delivering an isoline map, essentially a sum of both former ones.

b) Covering both curve families by a square net of a density needed for the designed accuracy, then writing to each node of both square nets its numerical value obtained by interpolating between adjacent isolines. A third square net will be traced, with nodes marked with sums of node values of both former square nets, used to construct a family of curves (isoline map), sum of both former curve families. (Obviously, in tracing the square nets, identity of nodal points has to be cared for.)

Among the described methods, for our aims (summation of velocity maps), method b) seems to be the more convenient one, namely:

— Richness in details (hence summation accuracy) of the resultant curve family obtained by method a) depends on the number and distribution of intersections; in adverse cases the number of intersections may be as low as to suffice only for a very sketchy (hence inaccurate) velocity map. (For instance, exactly in our case, the likely similarity of movements by nature in consecutive periods may result *a priori* in curve families scarcely intersecting each other.)

— Method b) — more laboursome than the former — yields summation of arbitrary richness of details (accuracy). Besides, any number of curve families — not only two — can be summarized. Also, the isoline maps to be summed need not be of the same scale (or of the resultant isoline map); only the relative density of square nets has to be constant for each isoline map.

These graphic summation methods are represented in Fig. 1. By the way, summation of movement curve families will only have been recourse to if

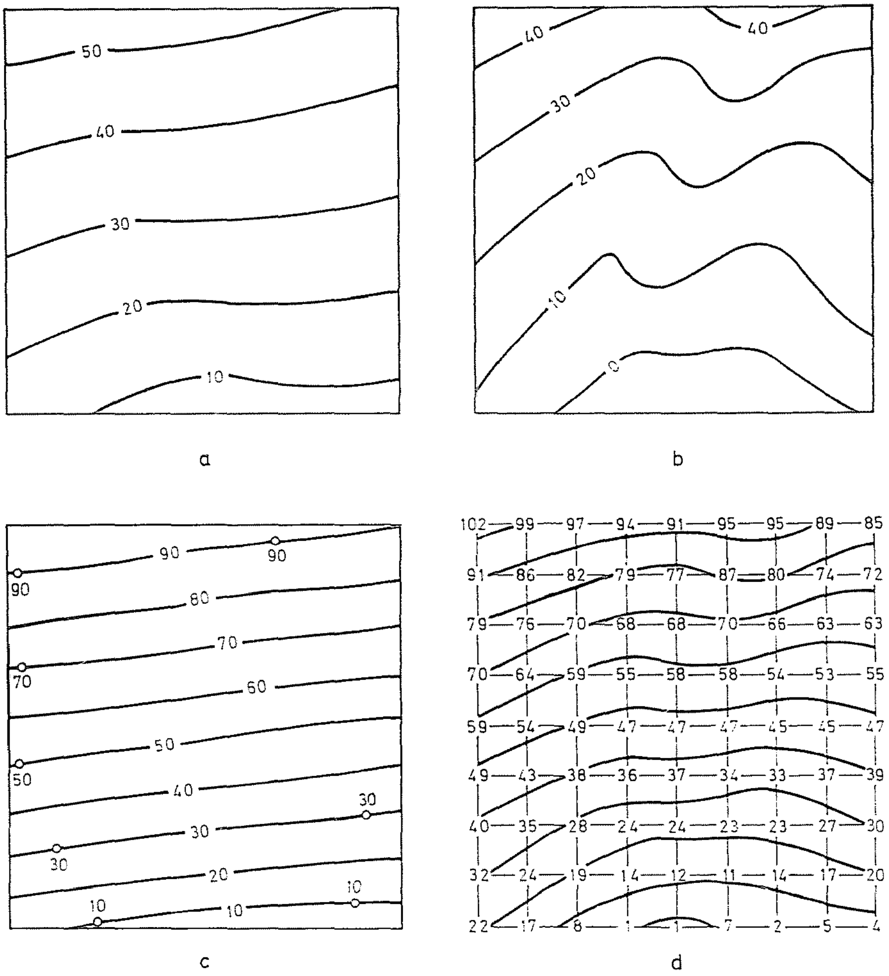

Fig. 1. a) Family of curves 1; b) Family of curves 2; c) Resultant family of curves constructed from intersections; d) Resultant family of curves constructed by means of square net

curve families (isoline maps) of the same area for subsequent intervals are applied to produce isoline velocity map of the area for a longer (or full) time of investigation.

In some cases, however, the intervals are not continuous but either discontinuous or fully or partly overlapping, and of different durations, or the terrains affected are more or less different. (A case where isoline movement maps constructed from various past measurements are to be united into more detailed or more extended velocity maps.)

Let us consider the case seen in Fig. 2. Bench marks of the tested area indicated by black dots have first been measured e.g. in 1930, then 18 years

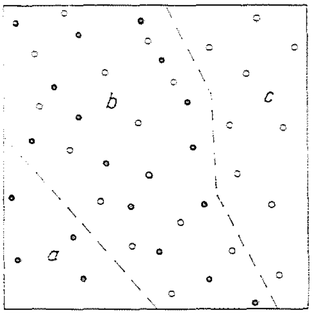

Fig. 2

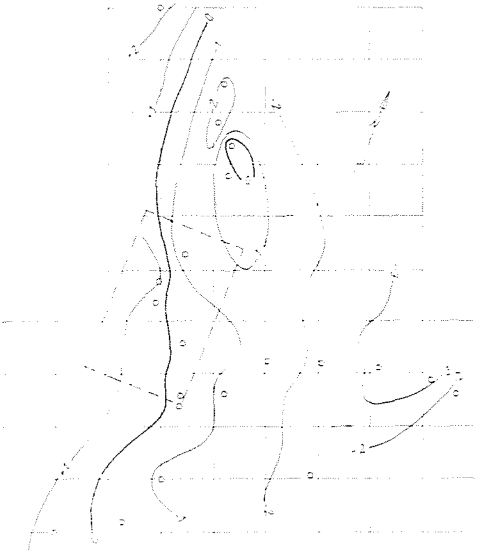

Fig. 3

later, in 1948; bench marks in empty circles date first from 1940, then, after 22 years, from 1962. Intervals are seen partly not to be equal, and partly, to overlap. Also, velocity maps for the areas *a* and *c* are seen to be only constructible from one or the other measurement pairs, possible from both for area *b*.

Theoretical steps of constructing a homogeneous, continuous velocity map for the entire area involve:

1. First, elimination of heterogeneity due to various lengths of intervals by either:

— transforming the content (isolines) of one map into the interval of the other; or

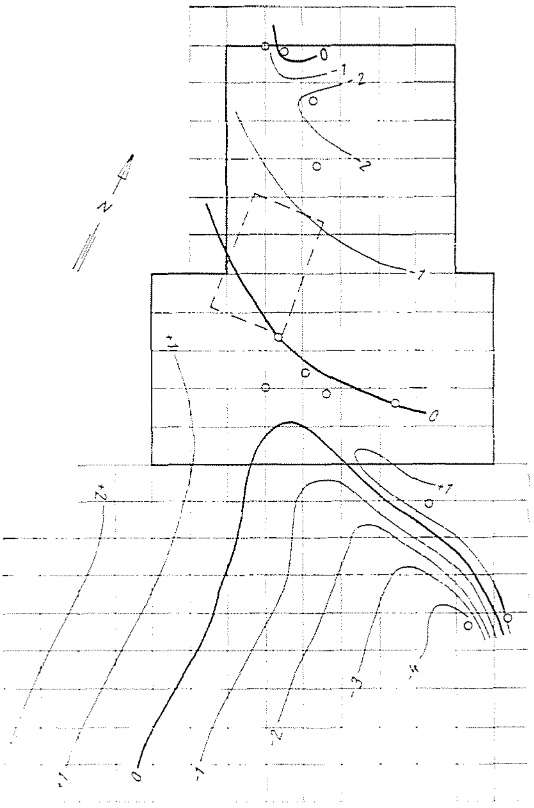

Fig. 4

- transforming the content (isolines) of both into a unit interval (e.g. 1 year or 10 years).

Technically, the transformation may be either by

- transforming the numerical values of discrete points originally applied for constructing the isolines and reconstructing with them the (rounded-up) isolines; or by
- revaluating the isolines themselves; reducing e.g. the 5, 10, 15 . . . mm isolines for 25 years to 10 years does not affect their form but their value is reduced to 2, 4, 6 . . . mm. (In that event, isolines of fractional value are possible, though unusual, not disturbing mathematical operations of the next step.)

Obviously, these theoretical and technical methods of transformation also apply for the homogenization of several, rather than two, isoline maps. Let us keep considering the example in Fig. 2.

2. Simple arithmetic mean of the two — already homogeneous — curve families for area *b* in Fig. 2 is formed. (Being acquainted with the way of

Fig. 5

summarizing the isoline maps, formation of the arithmetic means of maps will be no problem even if details are not specified here. Also here the square net method is expedient, obviously constructing isolines of integer values.)

3. Isoline mean map for area *b* has to be joined to isoline maps for areas *a* and *c*.

The final outcome will be the velocity map of the area for the period of 1930 to 1962, in dimensions of mm/year, mm/decade or in some other, convenient dimension. (Remark that partly, this resultant velocity map is continuous over the entire area, and partly, it is richer in details for area *b* than the original velocity maps.)

Of course, the problem illustrated in Fig. 2 is only one alternative among the several possibilities; both simpler and complexer problems may arise.

If, for instance, the entire tested area is covered (represented) by isoline maps to be averaged, (such as an area type *b* in Fig. 2), steps 1 and 2 could be contracted. Namely, averaging would involve at the same time homogenization of isoline maps. (Keeping with the previous example: the numerical values of isolines of 5, 10, 15, . . . mm constructed for 25 years but to be reduced to 10 years would be multiplied by a factor 1/2,5, while the numerical values of 10-year isolines would enter intact the averaging.)

Also weighting may be applied in the presented graphic procedure if justified by quality differences between measurements (measurement pairs), stressing in particular the correctness of weighting correlations.

As a final, concrete example, let us outline — based on research report [5] — the process of constructing the resultant map of movements for the region of town *Székesfehérvár* from isoline maps constructed from past measurements.

Three previous measurements have been found for the region, of an accuracy, and of a number of bench marks identical between them, permitting at all to construct isoline maps. These measurements included:

1. Town levelling by *István Hazay* and *Béla Szentiványi* in 1949.
2. 1961—1966 measurement on lines of the third national levelling net belonging to the concerned region.
3. 1971—1972 measurements initiated by and under the guidance of *Vilmos Vincze*, performed by the *Department of Surveying, Technical College of Surveying, University of Forestry and Timber Industry* [6].

21 points of the 1971/72 measurements (net) were already encountered in the 1949 town net, five of which, and other eight ones were included in the 1961—66 measurements.

These relations (common points) permitted to construct two, independent isoline maps. Both were constructed in mm/decade dimension. Fig. 3 shows movement map traced from common points of the 1949 and the 1971/72 town nets, that constructed from common points of the 1961/66 national net and the 1971/72 town net is seen in Fig. 4. Identical points are marked by empty circles. Area represented in Fig. 3 has been outlined in thick line in Fig. 4. The inner city is marked by a rectangle in dash line.

The two movement maps of identical dimension have been used to construct the resultant movement map seen in Fig. 5 by the square net method and unweighted averaging. Obviously, also the presented movements are of mm/decade dimension. Precision (accuracy) of averaging appears from the density of the square net used for the construction, indicated only by its nodes in Fig. 5, to avoid crammedness of the figure. For the sake of orientation, in addition to the inner city, also suburban lakes are indicated.

Figs 3, 4 and 5 will not be appreciated here from the aspect of the represented movements; they are only intended to explain the method of uniting the velocity maps.

Summary

Vertical displacements of parts of the earth surface may be represented on isoline maps, to be constructed from displacements of identical points in consecutive geodesic measurements (precise levellings). After a while, however, original bench marks in an area gradually perish.

A method will be presented for constructing isoline maps of movements over longer periods, by uniting isoline maps for shorter intervals (irrespective of perished identical bench marks). The same method is suggested for cases where isoline maps are available for one and the same area or for adjacent area parts constructed from different groups of identical bench marks, to be united into more detailed isoline maps or those representing movements of larger, continuous regions. As an illustration, the process of constructing a united isoline map for the region of town Székesfehérvár is described.

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