CORRECTION OF TORSION BALANCE MEASUREMENTS USED FOR INTERPOLATING THE DEFLECTION OF THE VERTICAL

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Introduction

The following derivatives of potential W of the gravity field (gravity gradient values) can be measured by the torsion balance:

$$W_{zx} = \frac{\partial^2 W}{\partial z \partial x}, \qquad W_{zy} = \frac{\partial^2 W}{\partial z \partial y}$$

and

$$W_{\Delta} = \frac{\partial^2 W}{\partial y^2} - \frac{\partial^2 W}{\partial x^2}, \qquad W_{xy} = \frac{\partial^2 W}{\partial x \partial y}.$$

The two latter potential derivatives — quantities characteristic of curvature relations of the level surface — are applied for interpolation of deflection of the vertical [1, 2, 3].

A very simple relationship based on the potential theory can be written for the changes of $\Delta\xi_{ij}$ and $\Delta\eta_{ij}$ between arbitrary points i and j of the deflection components ξ and η as well as for curvature quantities W_{Δ} and W_{xy} measured by torsion balance:

$$\Delta\xi_{ij} \sin\alpha_{ji} - \Delta\eta_{ij} \cos\alpha_{ji} = \frac{n_{ij}}{4g} \{[(W_{\Delta} - U_{\Delta})_i + (W_{\Delta} - U_{\Delta})_j] \sin 2\alpha_{ji} + \\ + [(W_{xy} - U_{xy})_i + (W_{xy} - U_{xy})_j]\, 2\cos 2\alpha_{ji}\}, \tag{1}$$

where n_{ij} is the distance between points i and j, g the average value of gravity between them, U_{Δ} and U_{xy} are curvature gradients in the normal gravity field, whereas α_{ji} is the direction azimuth between the two points [3, 4, 5]. The computation being fundamentally an integration, practically possible only by approximation, in deriving (1) it had to be supposed that the change of potential derivatives between points i and j, measurable by the torsion balance, was linear — thus the equality sign in (1) is valid only for this case. This is a most rigorous requirement in balance measurements, tried to be obtained until now primarily by "smoothing out" gradient measurements through different corrections. The danger of this "smoothing" will be pointed out below.

Correction of torsion balance measurements

The results of balance measurements are influenced first of all by terrain relief and mass inhomogeneities. The effect of mass inhomogeneities cannot be corrected. The effect of the relief is usually calculated in two or three steps [3, 4]. There exists no uniform convention for computation limits — we are discussing corrections according to the following classification:

1) Effect of environment from 0 to 100 m; this is the terrain effect ($\delta W^{(1)}$).

2) Effect of environment from 100 to 5000 m; — the topographic effect ($\delta W^{(2)}$).

3) The effect from 5000 m to 5° — the cartographic effect ($\delta W^{(3)}$).

To determine the terrain effect, the altitude data of the immediate environment obtained by levelling are needed. In general the soil is planed around the measurement point in a 3 m circle, the levelling is carried out usually in 8 symmetric directions around the point at distances of 1.5, 2, 3, 5, 10, 20, 30, 40, 50 m. Levelling beyond 50 m has to be carried out only in case of important terrain unevennesses up to a distance of 100 m at a maximum. From the levelling results the terrain effect is established generally by means of a diagram or a table.

For calculation of the topographic and cartographic effects, the necessary altitude data are obtained from adequate RF maps. Knowing the altitude data, corrections are computed by the same method as the terrain effect.

To compare curvature data without and with corrections, distribution of values over the area of our experimental computation are shown in Figs 1 to 4. Part of the represented area is nearly plane, a small part of it is hilly. In this area of the "raw" data for W_Δ and W_{xy} of the balance measurements the so-called anomaly values of the curvature

$$\Delta W_\Delta = W_\Delta - U_\Delta \tag{2}$$

and

$$\Delta W_{xy} = W_{xy} - U_{xy} \tag{3}$$

were available for each station of which the isoline maps seen in Figs 1, 2 were plotted. Curvature values on the isolines are given in 10^{-9} cgs i.e. 1 E (1 Eötvös) units. In the figures the observation points are marked by small dots, the dark triangles are the initial and the end points of the interpolation

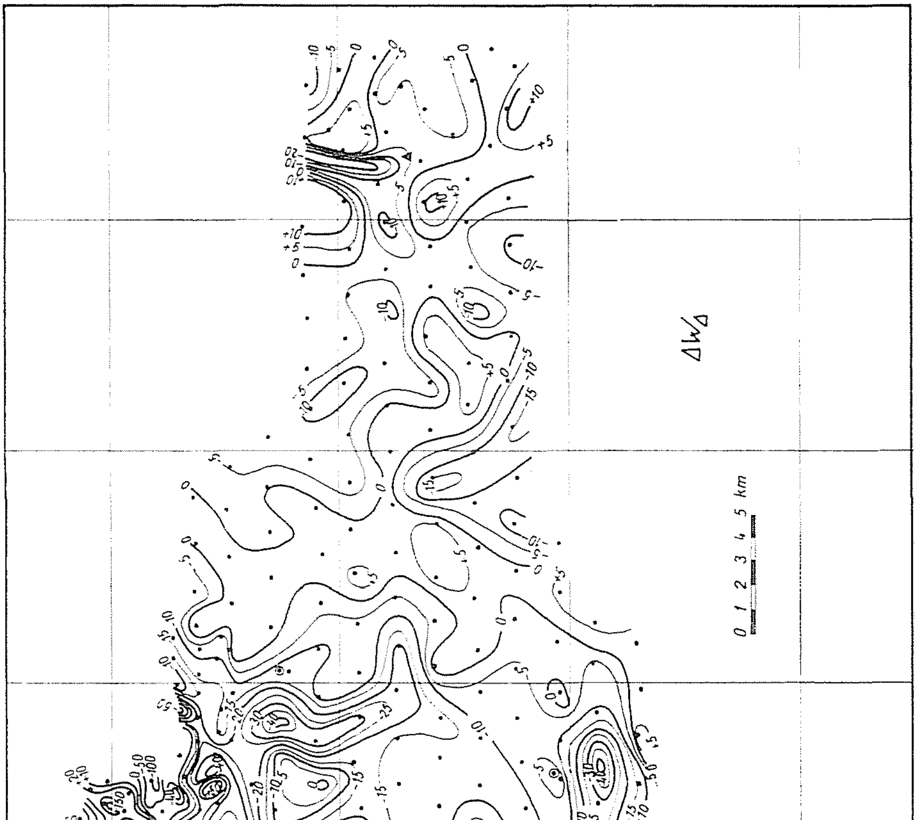

Fig. 1

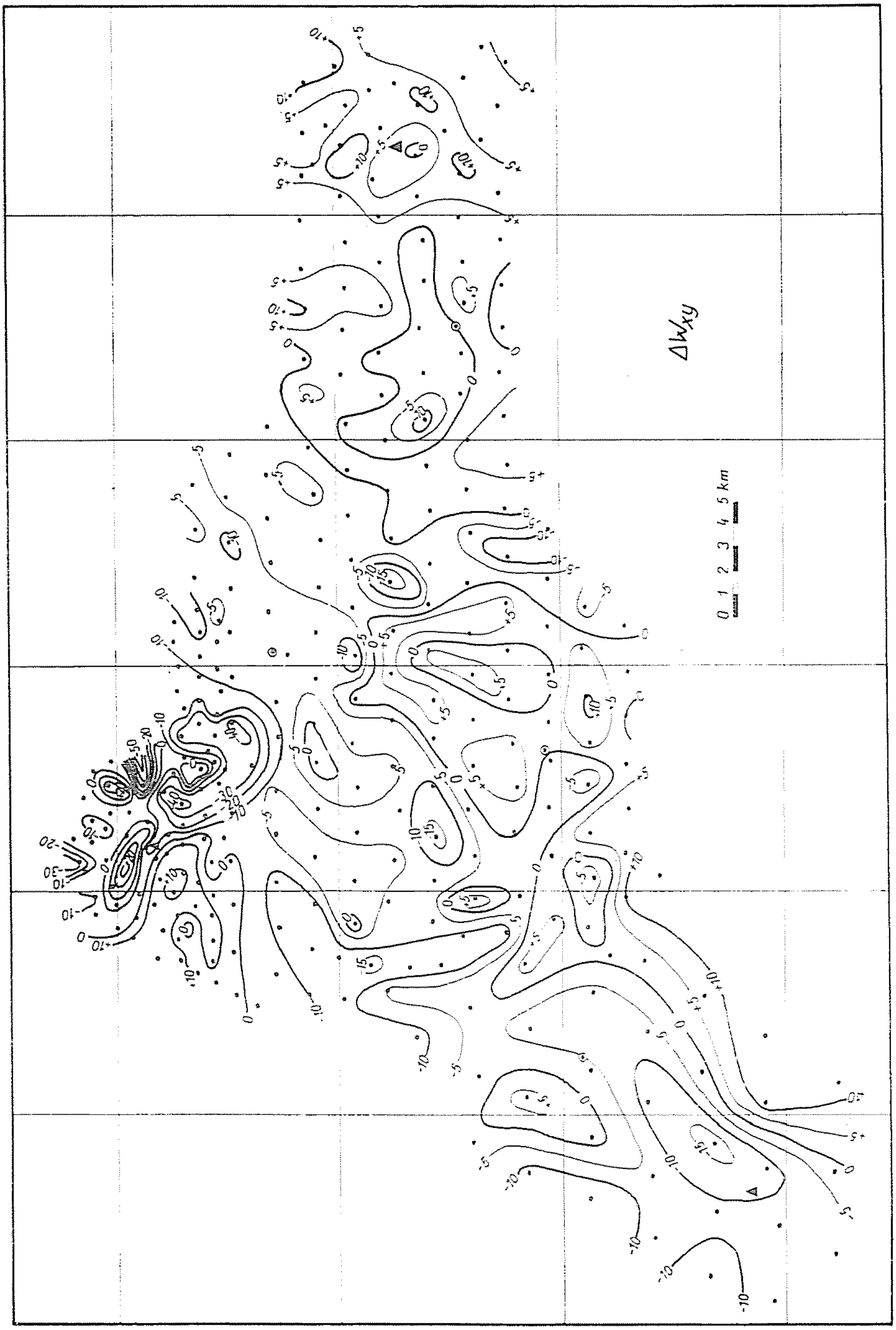

Fig. 2

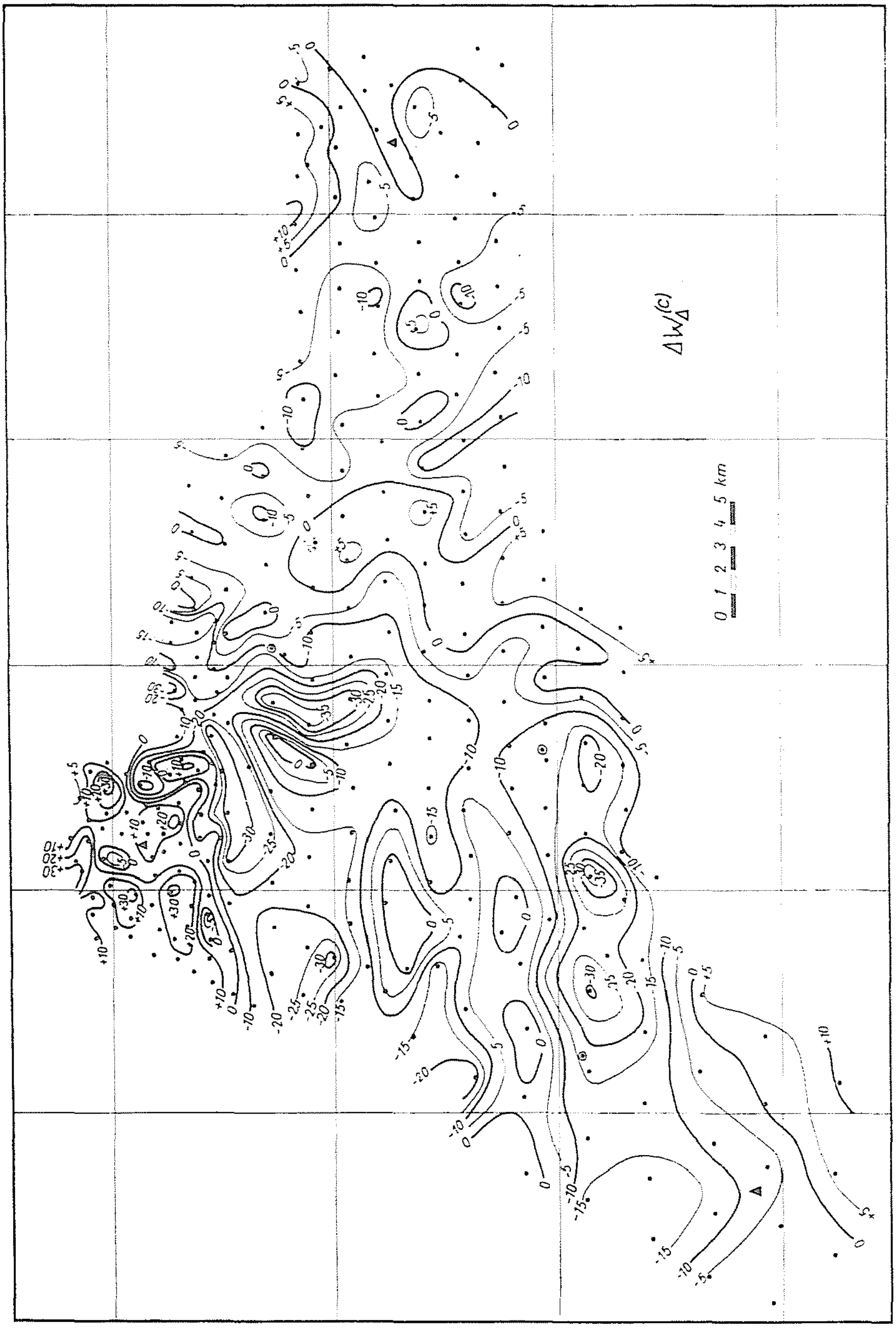

Fig. 3

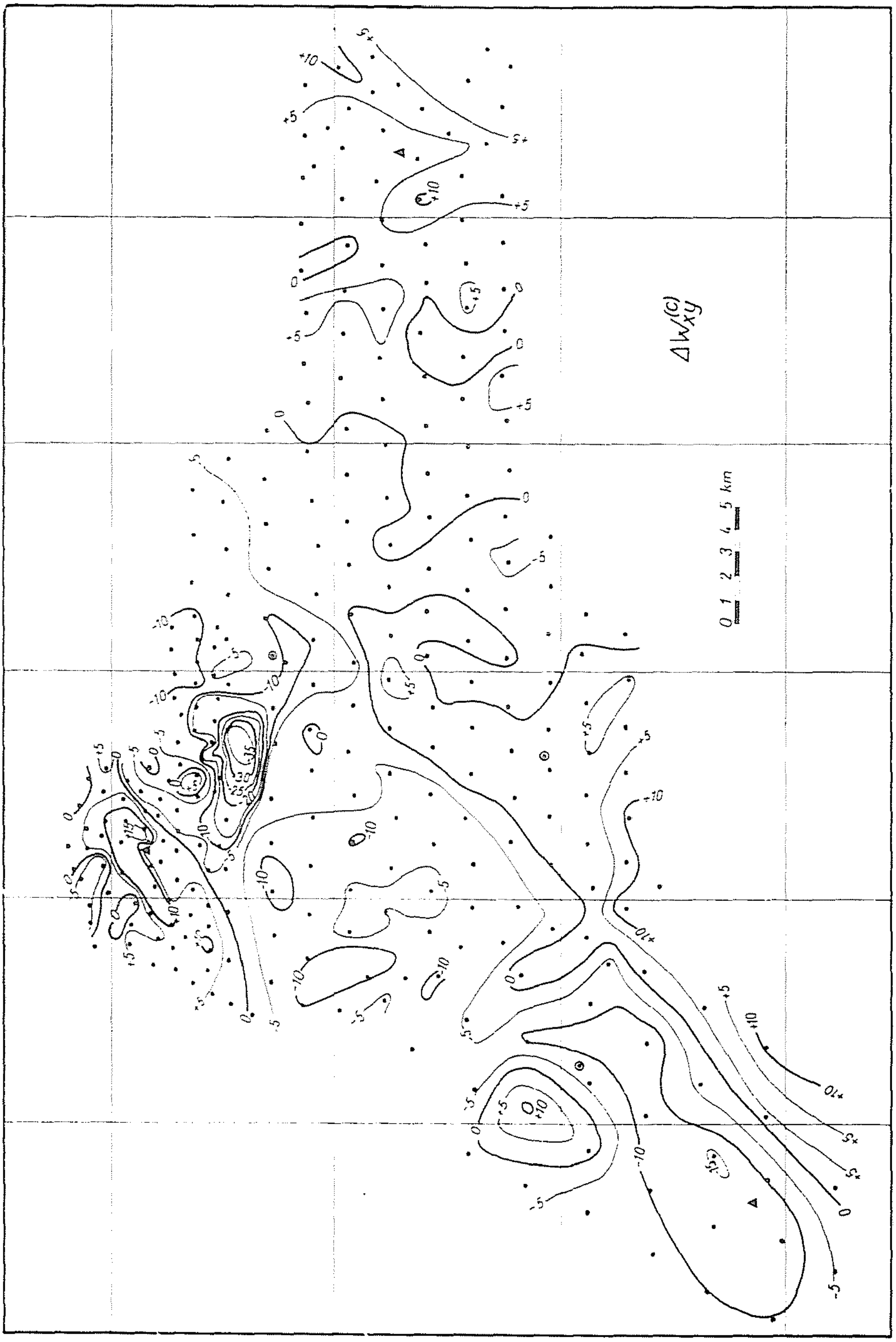

Fig. 4

lines. The observation stations are seen not to be set up with equal point density — observations were carried out on more "perturbed" territories with a higher point density.

From balance measurement data, corrected curvature data

$$\Delta W_{\Delta}^{(c)} = W_{\Delta} - \delta W_{\Delta}^{(1)} - \delta W_{\Delta}^{(2)} - \delta W_{\Delta}^{(3)} - U_{\Delta} \tag{4}$$

and

$$\Delta W_{xy}^{(c)} = W_{xy} - \delta W_{xy}^{(1)} - \delta W_{xy}^{(2)} - \delta W_{xy}^{(3)} - U_{xy} \tag{5}$$

for each observation station in the experimental area were established (i.e. subsurface irregularities of the curvature values), of which the isoline maps in Figs 3, 4 were plotted.

Comparison of Figs 1 and 3, and Figs 2 and 4, resp., shows correction to significantly alter the distribution pattern of curvature data needed for interpolating the deflection of the vertical, even in the almost plane areas in the bottom and in the middle parts of the figures. It is worth noting how much smoother the change of curvature values is, after carrying out the correction, taking into account the surface relief effect, however even so, many point pairs are found between which the curvature value change cannot be considered linear at all, thus between these point-pairs, in principle, relationship (1) would not be valid further on.

Finally, confidence of corrections is shortly discussed. Accuracy of the corrections is decidedly influenced by three factors:

1) Error of the measured altitude differences.

2) Error in the approximate density values used in the calculations.

3) Difference between the real and the theoretical, approximate terrain surface.

Taking into account the three sources of error according to [3] and [4], the mean error of the corrections is for both curvature values ± 4 E — against the curvature gradients' mean error measured by the torsion balance which is about ± 1 E. Care has to be taken because reliability of the corrections is much lower than reliability of measurements.

Experimental computations and the problem of corrections

Nearly all relevant publications [2, 3, 6, 7] started from the so-called subsurface irregularities calculated by relationships (4) and (5) because up to now, effect of the immediate environment of the observation points is considered to be involved in any case in the correction. Namely thereby the curvature gradients between points become smoother, permitting the change of the second potential derivatives between the points to be considered as linear in the integral approximative formula (1).

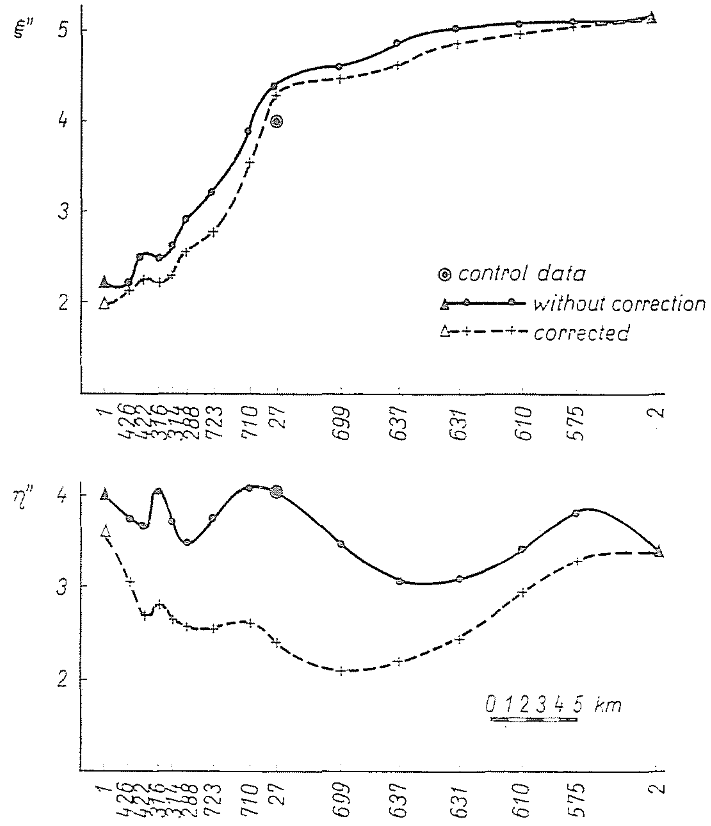

Fig. 5

In our opinion, application of corrections has to be revised because in the individual research territories just knowledge of the deflection of the vertical without correction is necessary. In this connection computations have been carried out in which it was investigated how the deflections of the vertical computed from topographic and subsurface irregularities according to (3), (4) and (5), resp., are related to each other [2].

Experimental computations were made by the so-called combined method [4, 8] in areas seen in Figs 1 to 4 [4, 5, 7, 9]. In Figs 1 to 4, interpolation chains were plotted between the astrogeodetic points, marked by black triangles in such a way that each chain contained also control points. For the same interpolation chains, the values of deflection components have been computed from (2) and (3) and from (4) and (5) applying curvature gradients without correction, and corrected for terrain effects, respectively. In the second instance, beside the torsion balance data, for the sake of uniformity, also the deflection components known in the initial and control points were provided with adequate terrain correction.

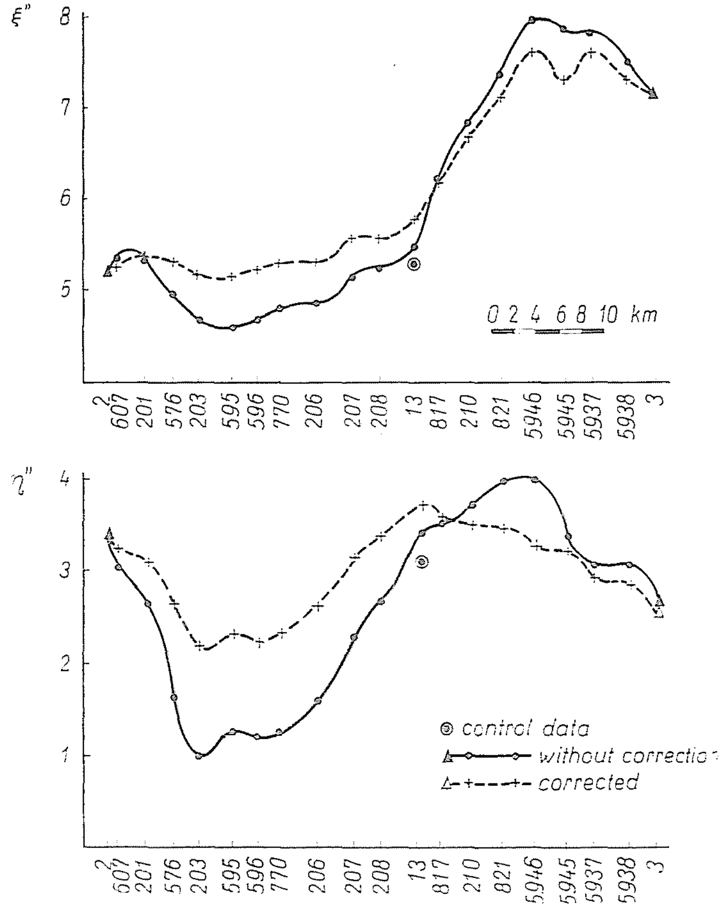

Fig. 6

For the sake of lucidity, the results have been plotted. Between the three astrogeodetic points, the components of deflection interpolated using the two different initial curvature gradients are shown in Figs 5 to 7. Abscissae of the diagrams show the differences of nodal point distances along the straight line connecting the two end points, and the ordinates the values of components ξ and η in seconds of arc. In the control points the known values for components ξ and η were marked by circled points. (Remark that in the control points the deflection components with and without correction coincide within the plotting accuracy.)

Investigating Figs 5 to 7 it is conspicuous that in the control points, curves computed on the basis of topographic anomalies without correction approximate better the control values of deflection components than do the curves computed from the subsurface anomalies (starting from measurements provided with corrections). Exact explication of this phenomenon needs further investi-

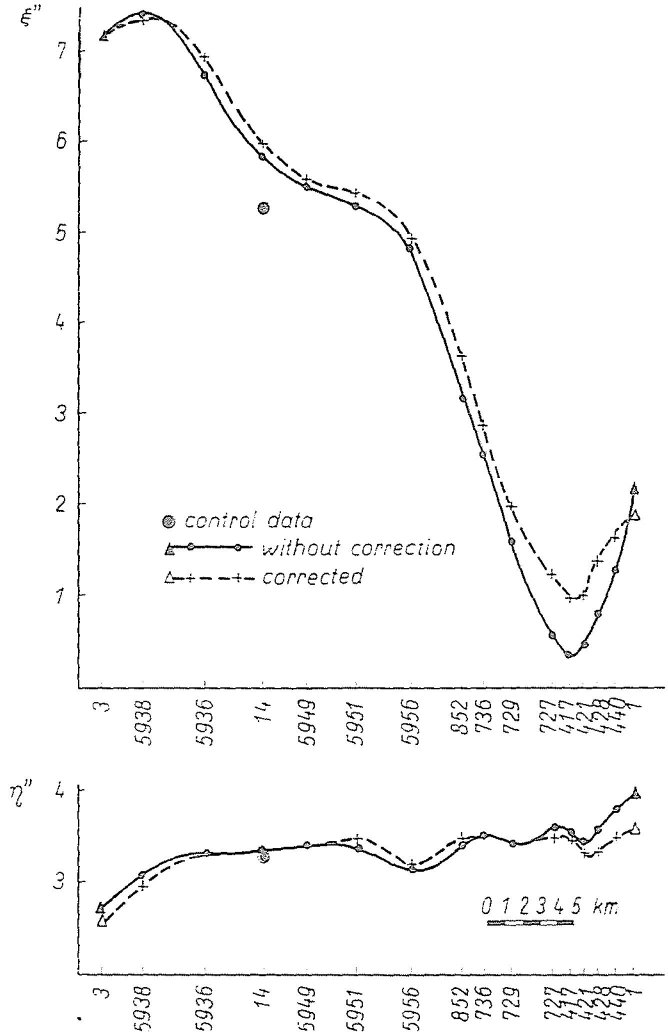

Fig. 7

gations; however, based on the results obtained, it may be stated that using curvature data without correction — at least for our test area — at least as accurate ξ and η values can be interpolated as from torsion balance measurements with correction for the terrain effect, although the experimental area can be considered neither completely level, nor "unperturbed". Therefore it can be stated that in not quite plane terrains, reliability of the interpolation method can be improved not only by terrain correction of the curvature gradients but e.g. by adequately choosing the point density of interpolation nets.

In our opinion, since deflections of the vertical are partly due to visible masses, partly to underground mass undulations—supposing that the reference ellipsoid is adequately situated — it is obvious that torsion balance curvature gradients have to be provided with the corrections earlier mentioned; depending on the intended use of deflection values computed from them.

If e.g. the deflections of the vertical are to be used for geophysical purposes, it is advisable to start from the subsurface anomalies of the curvature gradients, in our case from curvature gradients according to (4) and (5). In this case, of course, it has also to be taken into account that the terrain effect is reflected not only in the curvature gradients measured by torsion balance but also in the deflection component values determined at the astrogeodetic points. Therefore beside the torsion balance measurement data also the initial deflection components have to be provided with corresponding terrain correction. In Figs 5, 6, and 7 it is seen how much smoother the slope of graphs with correction is. This is obvious, as values ξ and η are exempt of the disturbing effect of the terrain surface and reflect primarily the effect of underground mass undulations. For purposes of geophysical structure research, just this is essential.

However, in using deflections of the vertical for geodetic purposes, it is advisable to start from the topographic values of curvature gradients according to (2) and (3), it being the only way to obtain the deflection values of the Earth surface corresponding to the observed relative astrogeodetic deflections. Namely, measurements carried out with geodetic instruments always refer to the level surface crossing the station point, as the vertical axis of the instruments, if correctly put up, is adjusted to the local vertical direction. In forming the local vertical direction, masses visible on the surface intervene as much as the subsurface mass inhomogeneities. Elimination of the terrain effect by correction falsifies the local vertical direction.

In certain cases the reduced subsurface values for deflections are needed for geodetic purposes, under such circumstances, however, relative deflection values of Earth surface astrogeodetic points have to be provided with adequate correction.

Conclusions

Reliability of results of interpolation deflection of the vertical, based on torsion balance measurements, is only satisfactory if change of corresponding potential derivatives between adjacent points of the interpolation net is linear — or at least approximately linear, — a fundamental supposition in deducing relationship (1). If this condition is inexistent in some part of the net, this interpolation method yields obviously useless results. According to practice up to now, the fulfillment of this fundamental condition was promoted by applying a correction on the curvature gradients measured by torsion balance, taking the terrain effect into consideration and adequately linearizing there the change of curvature gradients. According to our investigations, using corrections on the distribution, aspect of the curvature gradients changed very importantly and so did the interpolation deflection system, as the value of corrections in the investigated territory often equals or approximates the magnitude of the measurement results. In our opinion, application of these corrections has a serious physical consequence, therefore satisfaction of the mentioned fundamental condition ought to be attempted by choosing the interpolation net point density always according to the given conditions. In consequence, on more disturbed territories, where the change of curvature gradients is rapid and irregular, the distance of adjacent points of the interpolation net has to be chosen in a way that the change of corresponding potential derivatives between them is linear. According to our experiences it is useful and advantageous before beginning the computations to construct isoline maps from the measured curvature values similarly as in Figs 1 to 4 and to design therefrom the interpolation nets of optimum point density.

According to the above results the problem of correction is not concluded but further detailed investigations are necessary in this field.

Summary

Fundamental relationship of the interpolation of deflection of the vertical from torsion balance measurements is written, pointing out the basic condition of its applicability. The necessity of correcting the torsion balance measurement results by taking the effect of surface relief into consideration has a deeper physical background. Experimental calculation results show the accuracy of interpolation calculations to be improved by choosing an adequate geometry of the interpolation networks rather than by correcting the torsion balance curvature gradients.

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