MODELLING OF SMALL CATCHMENT RUNOFF CONDITIONS

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by

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Hydrological disclosure of small catchment areas and description of runoff conditions are increasingly urgent because of the growing demands on water management. Modelling of runoff conditions is one of the most important engineering hydrologic problems in small catchment areas, providing the basis for water management planning. A procedure applicable in this country for determining both floods due to precipitation (rain) and the entire runoff hydrograph will be presented. The model may be applied for mountainous and hilly catchment areas, up to 6000 sq.km. The procedure is based on a nonlinear regressive parameter calculation, whereas runoff conditions are described by the theory of stochastic processes. Accordingly, the hydrological systems theory, the hydrological statistics and the stochastic process theory have been made use of.

1. Systems theory bases for runoff calculation

The extreme intricacy of hydrological processes imposes partly to develop theoretical *models*, and partly, to consider hydrological cycles as a *system*. All these require the use of an important *mathematical* and *computing apparatus*. The systems theory provides *ample classifying and analysing possibilities* in hydrology. It is expedient to start from the operative form

$$y(t) = T[x(t)]; \qquad [x(t) \in X,\, T;\, t_0 \leqq t < +\infty]$$

where $x(t)$ is the input signal or input vector (time dependent) (e.g. precipitation), $y(t)$ the output or answer signal (e.g. runoff), T the system operator. Since determination of latter is impossible in practice, usually $x(t)$ and $y(t)$ are related, replacing $x(t)$ by the so-called standard input signals and determination of the output signals results in the description of the system. The properties of the system (linearity, steadiness, casuality, memory, stability) have a very important role in modelling.

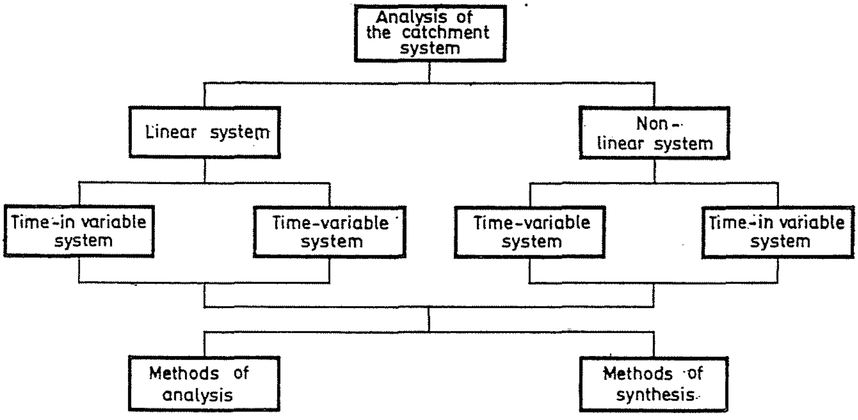

Fig. 1. Analysis possibilities of catchment systems

Classification and possibilities of analysis of catchment systems are shown in Fig. 1 [1, 8]. Based on these the linear models *(Nash, Dooge, Sato-, Mikkawa)*, the time-variant linear etc. models and the non-linear models *(Amorocho, Hino, Stanford, Dooge, Boughton*, the model described in this study, etc.) can be examined.

Among the system models the following, of an importance for Hungarian practice hence rather accomplished, is given a special stress.

2. Non-linear correlation model for determining runoff parameters

The model is based on a stochastic-parametrical system of analysis, and makes use of the graphic method of correlation analysis (coaxial relationships).

Some applications are known in the literature, e.g. [7], [10], [2], *Chekhova* (1967), *Sakharova* (1970) etc. This method has been used for predicting the entire flood diagram [4, 5, 6]. *The non-linear correlation model* [5] *is time-variant, seasonally time-variant* and provides a possibility to determine floods due to rain (from March to November). This method suits individual calculation of flood parameters of Hungarian catchment areas, and can be extrapolated to any mountainous and hilly catchment area up to 6000 sq.km. Extrapolation is based on the analogy between hydrology and geography, therefore when working out, the complete geography of the catchments had to be determined.

For determining the triangular flood hydrograph, three (other than statistical) independent parameters have to be calculated, such as:

V [cu.ms] — mass of the flood;
$\Delta Q_{\max}$ [cu.m/sec] — water discharge excess at flood peak;
t_a [h] — duration of the inundation.

Time relation between precipitation and flood is characterized by τ [h], value of time delay.

The arriving flood can be transformed into a continuous form [3].

Starting from the precipitation parameters (in mm) at observation points and rain of n days,

$$C = \sum_{i=1}^{n} \frac{1}{m} \sum_{j=1}^{m} C_{ij}$$

$$C_{\max} = \sum_{i=1}^{n} C_{i\,\max}$$

$$\Delta C = \sum_{i=1}^{n} C_{i\,\max} - \sum_{i=1}^{n} \frac{1}{m} \sum_{j=1}^{m} C_{ij} = C_{\max} - C$$

using further parameters, the flood characteristics may be calculated from the following functional relationship:

Insufficiency of runoff (shower retention):

$$Z = f(Mi_{20}, D, t_c, C) \qquad [\text{mm}]$$

(for calculating Z see e.g. Fig. 2). Accordingly, the flood mass is:

$$V = V_a + V_b = \int_{T_A}^{T_B} Q_\Delta(t)\mathrm{d}t = (C - Z)\, F \cdot 10^3 = L \cdot F \cdot 10^3 \qquad [\text{cu.m}]$$

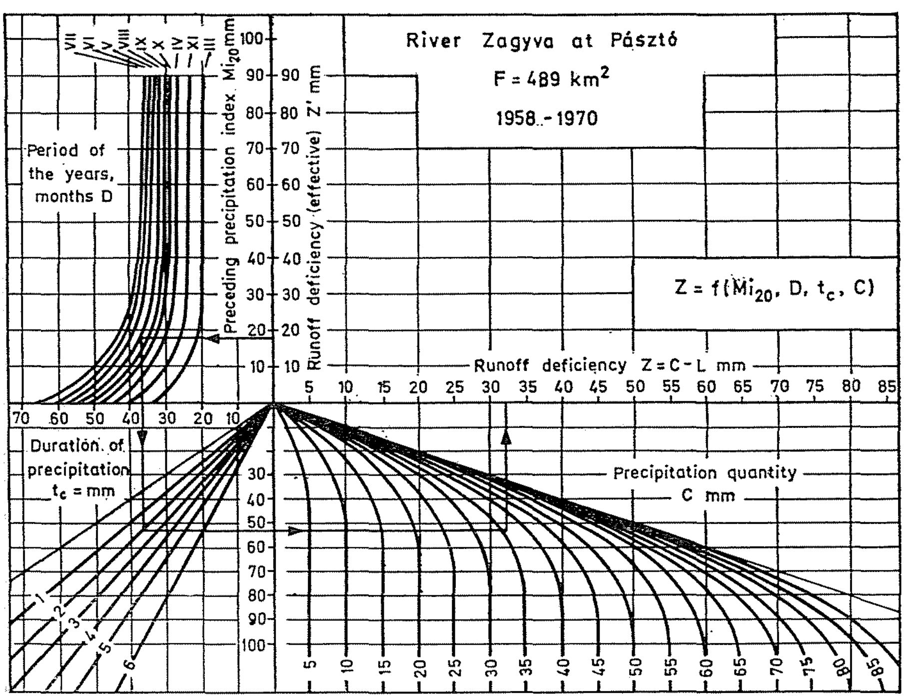

Fig. 2. Flood mass determination

(L in mm and the catchment area F in sq.km.)

The water discharge excess at peaking:

$$\Delta Q_{\max} = f(Q_{TA}, Mi_{20}, D, F_c, t_c, \Delta C, C) \qquad [\text{cu.m/s}]$$

and for checking:

$$\Delta Q_{\max} = f(C, t_c, L) \qquad [\text{cu.m/s}].$$

Relationships of the time parameters:

Flood duration:

$$t_a = f(C, t_c, Mi_{20}) \qquad [\text{h}].$$

Complete duration of the flood:

$$t_A = \frac{1}{1800} \cdot \frac{V\,[\text{cu} \cdot \text{m}]}{\Delta Q_{\max}\,[\text{cu} \cdot \text{m/s}]} \qquad [\text{h}].$$

Delay time:

$$\tau = f(C, t_c, Mi_{20}) \qquad [\text{h}].$$

Parameters computed in this way unambiguously define the approximative form of the flood hydrograph. It can be transformed to continuous by means of a nomogram [3, 5].

Some flood hydrograph examples modelled by this procedure are shown in Figs 3 and 4, for two catchment areas different by size and by character.

Relationships can be generalized by means of functions of the individual catchment and geographic parameter systems, in form of similar nomograms. Using the general relationships, the individual catchment categories are written by applying the empirical distribution function of the catchments:

$$\Phi[x^{(F)}] = P[\xi^{(F)} < x^{(F)}] = \sum_{x_k^{(F)} < x^{(F)}} P[\xi^{(F)} = x_k^{(F)}]$$

($x_k^{(F)}$ being the area of the k-th catchment, $k = 1, 2, \ldots, 37$).

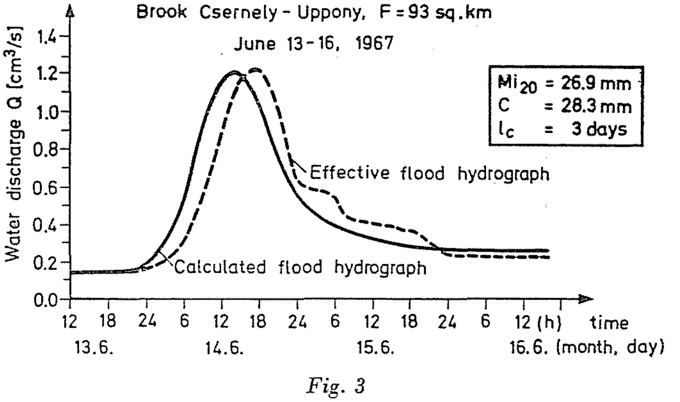

Fig. 3

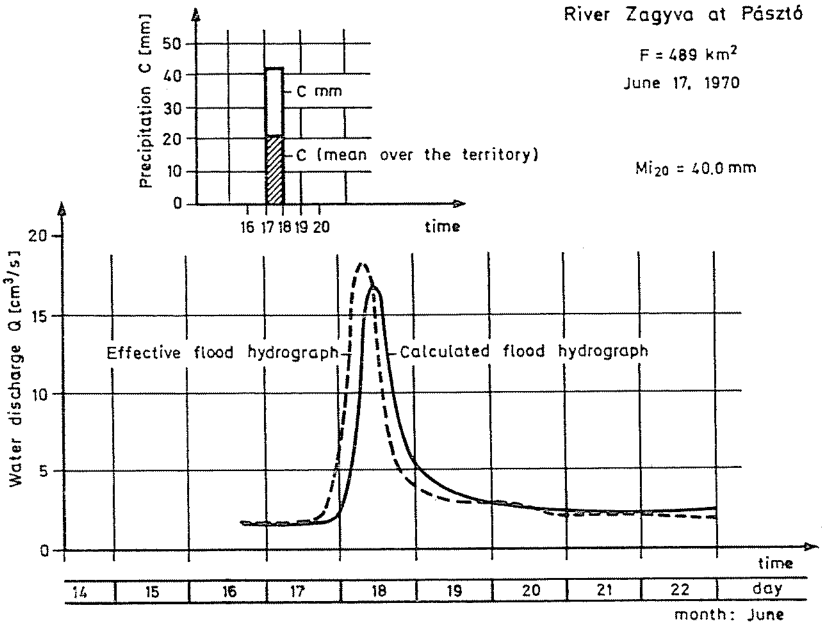

Fig. 4

3. Determination and analysis of water discharge hydrographs

The non-linear flood correlation model, complemented for basic loads (subsurface inflows) suits to approximate water discharge hydrographs. Basic loads are simplest determined by means of function $Q_a = f(\bar{C})$ (Q_a being the monthly average basic load in cu.m/s; $\bar{C}$ the monthly sum of precipitation in mm). Several problems have been solved by numerical correlation calculus of the multivariable linear model. For instance, the monthly average of water discharge basic load:

$$\bar{Q} = B_0 + B_1\bar{C} + B_2\bar{t} + B_3\bar{r}_p \qquad [\text{cu} \cdot \text{m/s}]$$

(where $\bar{t}$ is the monthly mean temperature in °C; $\bar{r}_p$ the monthly average vapour pressure in Pa).

The principial flow chart for the computer program of the approximate water discharge hydrograph comprises computation of both the flood and the basic load, as well as their superposition (Fig. 5).

Many practical problems may be solved by modelling the water discharge hydrograph according to the theory of *stochastic processes and the Bayes' analysis.* The water discharge hydrograph is handled as an independent, steady stochastic process with the differential equation system: [9]

$$p_n'(t) = -\lambda[p_n(t) - p_{n-1}(t)] \qquad (n = 0, 1, 2, \ldots)$$

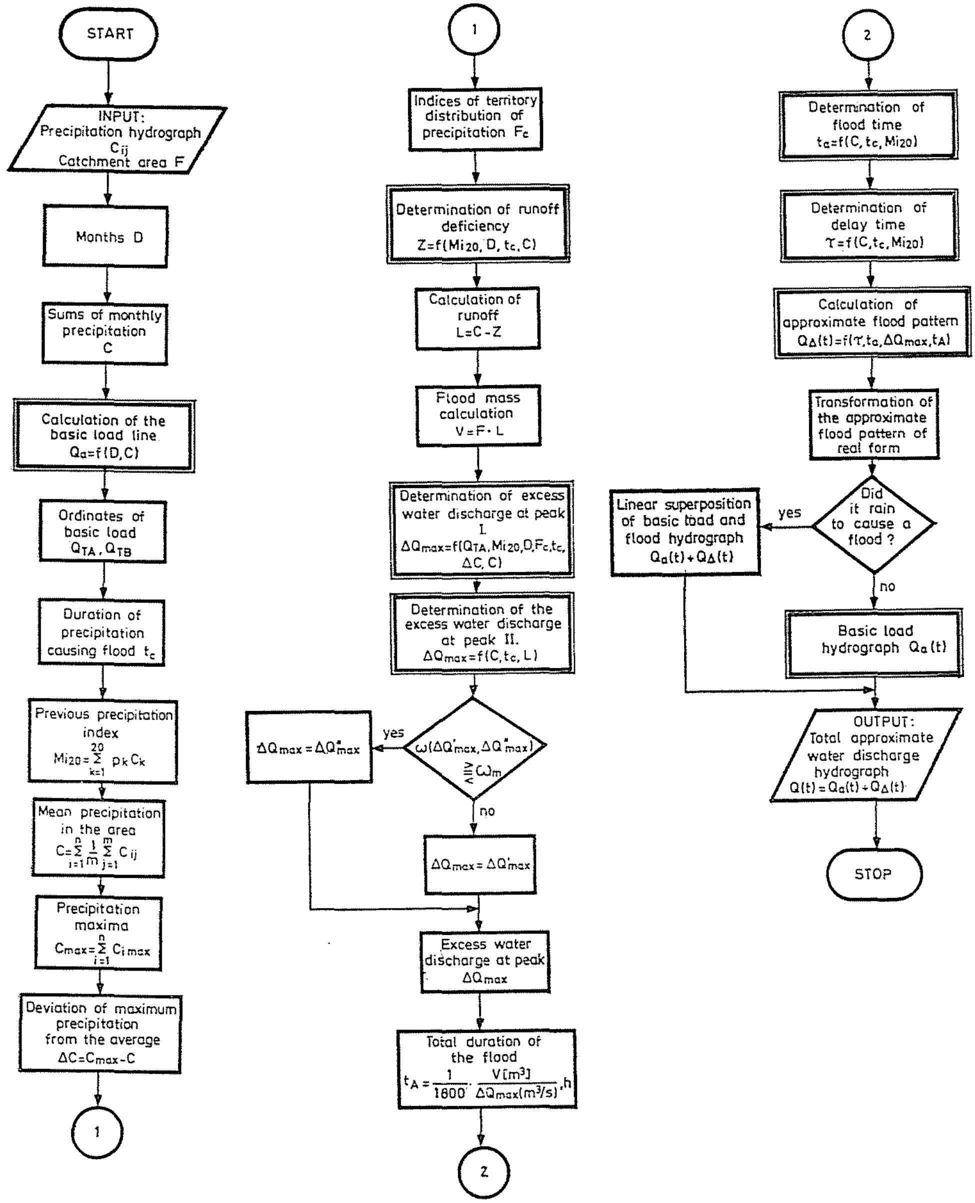

Fig. 5. Principial flow chart for the approximate water discharge hydrograph calculation

where n is the number and $p_n(t)$ the probability of flood "events". The solution is

$$p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \qquad (n = 0, 1, 2, \ldots,)$$

a *Poisson's* process with the parameter λt, of the expected value:

$$\bar{n} = \sum_{n=0}^{\infty} n \cdot p_n(t) = \lambda t$$

and the mean rate of events:

$$\lambda = \frac{\bar{n}}{t} .$$

Thus, the important conclusion can be drawn that the number of floods arriving in unit time or in a period follows the *Poisson's* distribution while the sequence times (ϑ) are of exponential distribution:

$$P(\vartheta < t) = 1 - e^{-\lambda t} = F(t)$$

and the average sequence time is:

$$\bar{\vartheta} = \int_0^{\infty} \vartheta f(\vartheta)\, \mathrm{d}\vartheta = \frac{1}{\lambda} .$$

Application of the stochastic process theory and the *Bayes'* analysis provided a possibility to realize the idea of using waters of small catchments in form of surface draw-off, merely inserting a low-capacity service reservoir.

Summary

Modelling of floods in small catchments due to precipitation (rain) and of the complete water discharge hydrograph is discussed in this study. The discharge hydrograph is modelled by superposing the basic load and the floods.

The basic load (subsurface total runoff) may be determined by a multivariable linear regressive model. For determining the floods a multivariable non-linear model was developed. The resulting discharge hydrograph may be written as an independent, *steady* stochastic process. Characteristic functions of the stochastical process can be determined, suitable for solving statistical forecasts and other engineering-hydrologic problems (e.g. storage calculation).

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