DESIGN OF RESERVOIR CAPACITY FOR VARIABLE WATER DEMAND

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By

I. V. NAGY

Department of Water Management, Institute of Water Management and Hydraulic Engineering, Technical University, Budapest

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For the greater part of the methods of dimensioning reservoirs, the water supply from the reservoir is assumed to be constant or to vary in a predetermined way. The statistic character of the water demand is taken into account at most by assuming normal distribution [1]. At the same time, it is evident that in dimensioning reservoirs for supplying irrigation water and lowering flood peaks, the water demand should be considered as a random variable because it depends on a wide range of hydrometeorological factors varying at random.

In the present paper both the inflow and the water demand are treated as random variables, and one of the methods of analytic (statistical) and graphical determination, permitting arbitrary selection of the critical dimensioning probability, is proposed.

1. Analytic (statistical) method

As a first approximation, in applying yearly averages for carryover reservoirs, the inflow into the reservoir and the water demand are assumed to be a series of independent events. It is evident that this approximation cannot be adopted to every water course. If the coefficients of autocorrelation significantly differ from zero, then the calculated storage capacity is always less than the actual one [2].

Further, as a first approximation, the distributions of inflow and water demand are assumed to be independent of each other. Neither this condition will always be fulfilled, and the object of the further investigation is to eliminate this restriction to a certain extent.

For simplifying the discussion, it is convenient to use dimensionless variables, therefore the following notation will be used:

$$k = \frac{W}{\overline{\overline{W}}}; \qquad \alpha = \frac{q}{\overline{\overline{W}}}; \qquad \beta = \frac{V}{\overline{\overline{W}}}$$

wherein:

k — so-called modular factor of inflow to the reservoir;
α — regulation factor of the reservoir;
β — volume factor of reservoir;
W — mean yearly inflow;
$\overline{W}$ — multiannual mean inflow;
q — yearly mean water yield;
V — volume of reservoir.

Consider the water shortage $z = \alpha - k$ as a random variable and let us find its distribution function. The problem may be solved by means of the distribution functions of random variables α and k, i.e., be from previous measurements:

$$F(x) = P(\alpha < x)\,; \qquad G(x) = P(k < x)\,,$$

thus

$$G^*(x) = P(k > x)\,,$$

noticing that, if $G(x)$ is a continuous distribution function, so

$$G^*(x) = 1 - G(x)\,.$$

Hence, the distribution function of the water shortage:

$$H(x) = P(z < x)\,.$$

By definition of the regulation factor, the possible values of α are in the interval $[0, 1]$ permitting the following division of the interval:

$$0 = a_0 < a_1 < a_2 \ldots < a_n = 1$$

or

$$I_1 = [0, a_1],\ I_j = [a_{j-1}, a_j]\,, \qquad j = 2, 3, \ldots n\,.$$

Let us apply the theorem of total probability:

$$P(z < x) = P(\alpha - k < x) = P(k > \alpha - x) =$$

$$= \sum_{j=1}^{n} P(k > \alpha - x/\alpha \in I_j)\, P(\alpha \in I_j) \tag{1}$$

or, in other form:

$$P(k > a_j - x/\alpha \in I_j) \leq P(k > \alpha - x/\alpha \in I_j) \leq P(k > a_{j-1} - x/\alpha \in I_j) \tag{2}$$

namely, if

$$\alpha \in I_j, \quad a_{j-1} < \alpha \quad \leq a_j, \text{ and thus}\,,$$

$$a_{j-1} - x < \alpha - x \leq a_j - x$$

so, from the relationship $k > a_j - x$ it follows that $k > \alpha - x$, and from the relationship $k > \alpha - x$ it follows too, that $k > a_{j-1} - x$.

Therefore,

$$P(k > a_j - x) \leq P(k > \alpha - x) \leq P(k > a_{j-1} - x)$$

and the same inequality is valid also to the conditional probabilities. It may readily be seen that, if $A \supset B$, and C is an optional event, then $AC \supset BC$ and $P(AC) > P(BC)$, thus:

$$P(A/C) = \frac{P(AC)}{P(C)} \geq \frac{P(BC)}{P(C)} = P(B/C) .$$

Consider now the last term of inequality (2). To that the following can be written:

$$P(k > a_{j-1}/\alpha \in I_j) = P(k > a_j - x/\alpha \in I_j) +$$
$$+ P(a_{j-1} - x < k \leq a_j - x/\alpha \in I_j) .$$

By the way, if events A and B are mutually exclusive, i.e. $P(A + B) = P(A) + P(B)$ and C is an optional event, then also products AC and BC are mutually exclusive and thus:

$$P(A + B/C) = \frac{P[(A + B)C]}{P(C)} = \frac{P(AC + BC)}{P(C)} =$$
$$= \frac{P(AC) + P(BC)}{P(C)} = P(A/C) + P(B/C) .$$

In our case, however, k and α are independent and therefore the conditional probability is not to be taken into account, i.e., omitting the conditionality:

$$P(k > a_{j-1} - x/\alpha \in I_j) = P(k > a_{j-1} - x)$$
$$P(a_{j-1} - x < k \leq a_j - x/\alpha \in I_j) = P(a_{j-1} - x < k \leq a_j - x)$$

therefore,

$$\sum_{j=1}^{n} P(k > a_j - x)P(\alpha \in I_j) \leq \sum_{j=1}^{n} P(k > \alpha - x/\alpha \in I_j)P(\alpha \in I_j) \leq$$

$$\leq \sum_{j=1}^{n} P(k > a_{j-1} - x) P(\alpha \in I_j) + \sum_{j=1}^{n} P(a_{j-1} - x < k \leq a_j - x) P(\alpha \in I_j)$$

where the last sum in the right-hand side will in the following be denoted by S. According to the above:

$$P(k > a_j - x) = G^*(a_j - x)$$

and

$$P(a_{j-1} - x < k \leq a_j - x) = G^*(a_{j-1} - x) - G^*(a_j - x) ,$$

hence

$$P(\alpha \in I_j) = F(a_j) - F(a_{j-1})$$

and

$$\sum_{j=1}^{n} G^*(a_j - x)[F(a_j) - F(a_{j-1})] \leq P(z < x) \leq$$

$$\leq \sum G^*(a_{j-1} - x)[F(a_j) - F(a_{j-1})] + S\,.$$

Refining the division $0 = a_0 < a_1 < \ldots < a_n = 1$ beyond all limits ($n \to \infty$, $\max(a_j - a_{j-1}) \to 0$), then $S \to 0$.

Here it is to be noted that if $G^*(x)$ is a monotonous function then to an optional ε value a δ value may be found, where

$$|G^*(x_1) - G^*(x_2)| < \varepsilon \qquad \text{if} \qquad |x_1 - x_2| < \delta.$$

Therefore, if the subdivisions of the interval α are finer than δ, then to every $j = 1, 2, \ldots, n$ it may be written

$$[G(a_{j-1} - x) - G(a_j - x)] < \varepsilon$$

and because it is a positive value, no generation of the absolute value is needed. Accordingly:

$$S = \sum [G^*(a_{j-1} - x) - G(a_j - x)\ [F(a_j) - F(a_{j-1})] \leq$$

$$\leq \varepsilon \sum_{j=1}^{n} F(a_j) - F(a_{j-1}) = \varepsilon\,.$$

The attention should be drawn to the fact that the sum at th^ right and left-hand sides of the above expression have limiting values which are independent of the subdivision of the range, hence, considering the lower and upper approximate sums of *Stieltjes'* integral:

$$[= \int G^*(y - x)\, \mathrm{d}F(y)]$$

results in the *distribution function of the water shortage*:

$$H(x) = \int G^*(y - x)\, dF(y) = \int G^*(y - x)\, f(y)\, dy$$

wherein $f(y) = F(y)$, the density function of α.

2. Graphostatistic method

Since, according to the initial assumption, $z = \alpha - k$, $k = \alpha - z$, $k_j = a_j - x_0$, therefore, if x_0 is fixed, $P(z < x_0)$ may be approximated by the lower or upper sum:

$$P(z < x_0) \approx \sum_{i=1}^{n} G^*(a_j - x_0)[F(a_j) - F(a_{j-1})]$$

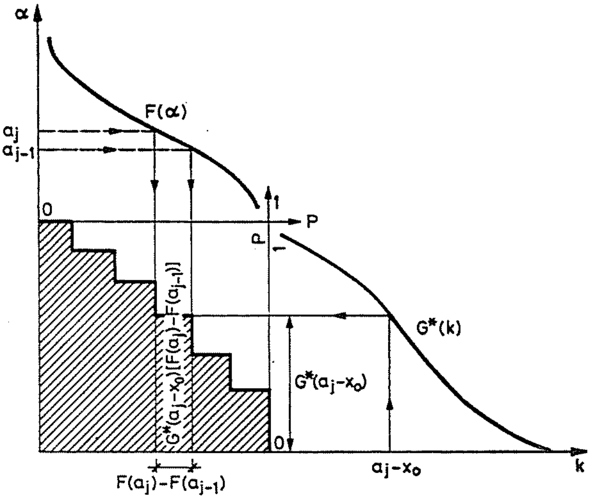

Fig. 1

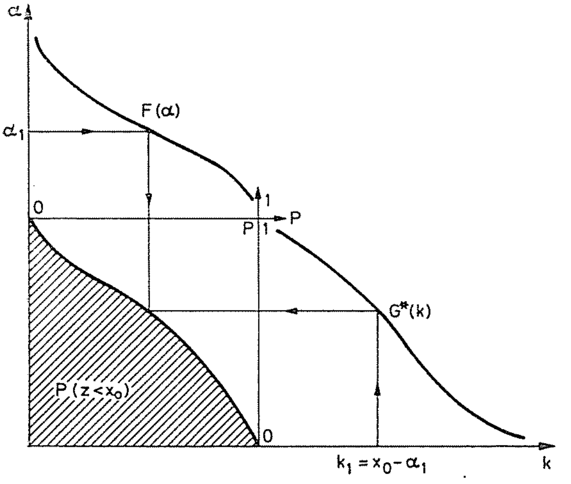

Fig. 2

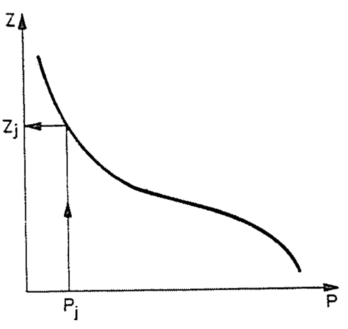

Fig. 3

and this sum is equal to the shaded area under the step-like function in Fig. 1. Refining the subdivision in conformity with POTAPOV's suggestion [3], in the limiting case the step-like function may be replaced by a continuous curve, and the area under the curve is equal to the probability $P(z < x_0)$ (Fig. 2).

By fixing the values $z_1, z_2, \ldots, z_n$ and adopting the values $\alpha_1, \alpha_2, \ldots, \alpha_n$, the values $k = \alpha - z$ may be calculated to yield different probabilities $P(z_i < x_0)$. Calculating the probabilities associated with given z_i values (determined graphically by planimetry), the empirical distribution function $H(z)$ of the water shortage z can be constructed (Fig. 3).

The needed storage capacity may be determined from the condition $\beta \geq z_i$.

Summary

The distribution function of the water shortage may unambiguously be derived from the distribution functions of the inflow and the water demand provided the inflow into the reservoir (stream flow) and the water demand are independent of each other. The (empirical) distribution function of the water shortage may easily be constructed graphoanalytically and therefore also the storage capacity of given probability may be determined. The solution presented may be applied also to other instances of the *general water balance equation.* A further task is to take the dependence between the inflow and the water demand into consideration.

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Prof. Dr. Imre V. NAGY, H-1521, Budapest