

ANALYSIS OF PANEL BUILDINGS BY THE USE OF RIGID PANEL MODELS

By

S. KALISZKY--K. WOLF

Department of Civil Engineering Mechanics, Technical University, Budapest

Received: November 27, 1978

1. Introduction

For the analysis of prefabricated panel buildings different models and methods are used. In the *frame model* the horizontal and the vertical wall stripes between the openings are considered as beams and columns of a frame [1], while in the *continuous model* developed by ROSMAN the vertical wall stripes are supposed to work as cantilevers and the horizontal lintels, providing interaction between the cantilevers, are replaced by continuous, elastic membranes [2, 3]. The use of these two models leads to the solution of a system of linear equations and differential equations, respectively. For the analysis of panel structures the *finite element method* is also widely applied. Here, following the arrangement of the openings, the prefabricated panels are usually subdivided into further rectangular elements [4, 7]. In this method the system of linear equations to be solved is very large and therefore the analysis leads to long numerical calculations.

In this paper a new model of analysis of panel buildings will be suggested, where each prefabricated panel is considered a single element, further called *plate* or *panel*. The plates are assumed to be perfectly *rigid* in their own plane and perfectly flexible perpendicular to it, and to be interconnected at the corners and at the middle points of the edges by springs acting in tension or compression and in shear, respectively. Accordingly, the plates bear forces only in their middle plane and, concerning plane problems, each plate has three degrees of freedom.

In fact, the panels are never perfectly rigid, therefore their elastic behaviour has to be simulated by the proper choice of the springs. Besides, by the use of the springs the effect of the deformation of the panel joints can also be characterized in a simple manner. The model described above will be called *rigid panel model*.

Although by using the rigid panel model a whole building can be investigated, in this paper only the analysis of a wall will be presented. The model

of this plane problem is illustrated in Fig. 1. Here the springs at the corners work in tension or in compression, while throughout the springs placed at the middle points of the edges transmit shear forces. The springs are supposed to be infinitesimal, therefore the sizes of the rigid plates are equal to those of the prefabricated panels.

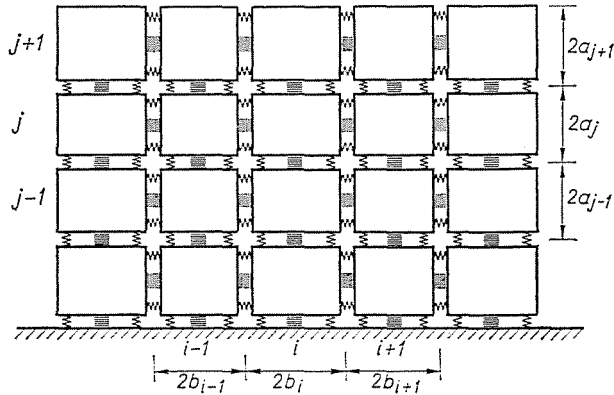


Fig. 1

Further, first the basic equations of a panel will be derived and then the analysis of a whole wall will be described. Briefly also the determination of the spring coefficients will be treated and finally two simple numerical examples will illustrate the application of the model and the comparison of its results with those of the finite element method.

2. Analysis of a panel

Figure 2/a illustrates the model of the panel ij of the wall under consideration. Here the panel itself and its edges are represented by a rigid plate connected by 12 elastic springs and by 4 *line elements*. The latter are supposed to be perfectly rigid and their use is expedient from the point of view of the connection of the neighbouring plates. In order to take elastic deformations of the panels more accurately into account, the springs placed along the same edge of a plate are assumed to interact with each other, i.e., their coefficients to be dependent.

The forces acting on a plate are illustrated in Fig. 2/b. Denoting the external forces reduced to the middle point of the plate and the spring forces acting along the edge k with the vectors

$$\mathbf{q}_{ij} = \begin{bmatrix} P_{ijx} \\ P_{ijy} \\ M_{ij} \end{bmatrix} \quad \mathbf{s}_{ijk} = \begin{bmatrix} N'_{ijk} \\ N''_{ijk} \\ T_{ijk} \end{bmatrix} \quad (k = 1, 2, 3, 4) \quad (1)$$

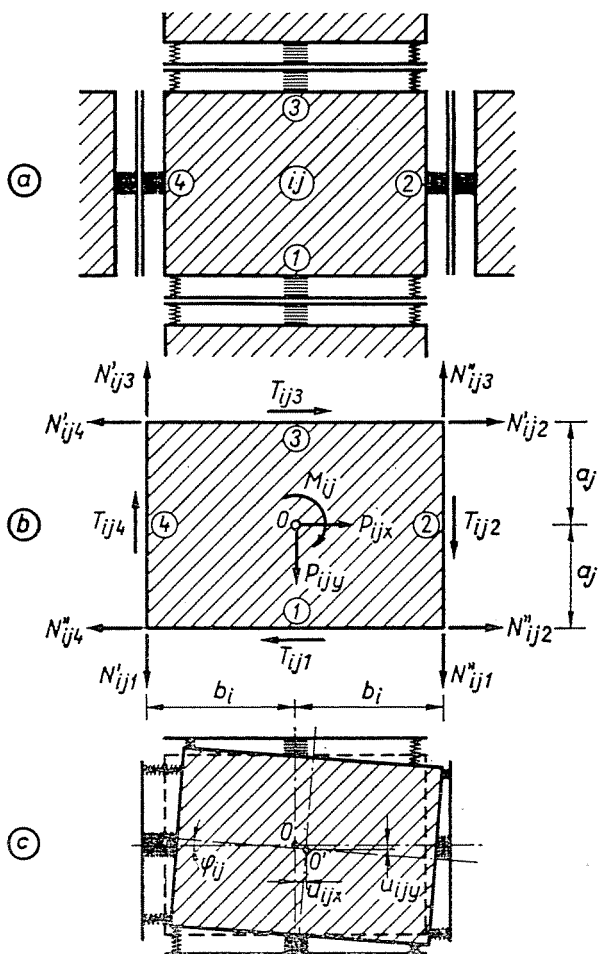


Fig. 2

the equilibrium of the plate can be expressed by the matrix equation:

$$\begin{bmatrix} \mathbf{G}_{ij1}^* & \mathbf{G}_{ij2}^* & \mathbf{G}_{ij3}^* & \mathbf{G}_{ij4}^* \end{bmatrix} \begin{bmatrix} s_{ij1} \\ s_{ij2} \\ s_{ij3} \\ s_{ij4} \end{bmatrix} + \begin{bmatrix} P_{ijx} \\ P_{ijy} \\ M_{ij} \end{bmatrix} = \mathbf{0} \tag{2}$$

or

$$\mathbf{G}_{ij}^* s_{ij} + \mathbf{q}_{ij} = \mathbf{0}. \tag{3}$$

The matrices in these equations are:

$$\mathbf{G}_{ij1}^* = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 0 \\ -b_i & b_i & a_j \end{bmatrix} \quad \mathbf{G}_{ij2}^* = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ a_j & -a_j & b_i \end{bmatrix}$$

(4)

$$\mathbf{G}_{ij3}^* = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -1 & 0 \\ b_i & -b_i & a_j \end{bmatrix} \quad \mathbf{G}_{ij4}^* = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & -1 \\ -a_j & a_j & b_i \end{bmatrix}.$$

Let us denote the displacements of the middle point of the plate, the deformations of the springs and the initial discontinuities due to deficiencies in the joints along the edge k by the vectors

$$\mathbf{u}_{ij} = \begin{bmatrix} u_{ijx} \\ u_{ijy} \\ \Phi_{ij} \end{bmatrix} \quad \mathbf{e}_{ijk} = \begin{bmatrix} \Delta'_{ijk} \\ \Delta''_{ijk} \\ \Gamma_{ijk} \end{bmatrix} \quad \mathbf{t}_{ijk} = \begin{bmatrix} \omega'_{ijk} \\ \omega''_{ijk} \\ \psi_{ijk} \end{bmatrix}.$$

(k = 1, 2, 3, 4)

Then the *compatibility* of the motion of the plate and of the neighbouring line elements can be expressed by the equations (Fig. 2/c):

$$\begin{bmatrix} \mathbf{G}_{ij1} \\ \mathbf{G}_{ij2} \\ \mathbf{G}_{ij3} \\ \mathbf{G}_{ij4} \end{bmatrix} \begin{bmatrix} u_{ijx} \\ u_{ijy} \\ \Phi_{ij} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{ij1} \\ \mathbf{e}_{ij2} \\ \mathbf{e}_{ij3} \\ \mathbf{e}_{ij4} \end{bmatrix} + \begin{bmatrix} \mathbf{t}_{ij1} \\ \mathbf{t}_{ij2} \\ \mathbf{t}_{ij3} \\ \mathbf{t}_{ij4} \end{bmatrix} = \mathbf{0} \quad (6)$$

or

$$\mathbf{G}_{ij} \mathbf{u}_{ij} + \mathbf{e}_{ij} + \mathbf{t}_{ij} = \mathbf{0}. \quad (7)$$

Further, both the panels and the springs will be assumed to consist of a linear elastic material. Then the forces and the deformations of the springs along edge k have the following linear relationship:

$$\mathbf{F}_{ijk} \mathbf{s}_{ijk} = \mathbf{e}_{ijk}. \quad (8)$$

Here \mathbf{F}_{ijk} denotes the flexibility matrix of the springs of the edge k . Since these springs were assumed to interact, \mathbf{F}_{ijk} is not a diagonal matrix. The determination of its elements will be illustrated in Chapter 4.

The equilibrium, the compatibility and the material equations (3), (7) and (8) of the panel ij can be comprised in a single matrix equation [5]:

$$\begin{bmatrix} \mathbf{0} & \mathbf{G}_{ij1}^* & \mathbf{G}_{ij2}^* & \mathbf{G}_{ij3}^* & \mathbf{G}_{ij4}^* \\ \mathbf{G}_{ij1} & \mathbf{F}_{ij1} & & & \\ \mathbf{G}_{ij2} & & \mathbf{F}_{ij2} & & \\ \mathbf{G}_{ij3} & & & \mathbf{F}_{ij3} & \\ \mathbf{G}_{ij4} & & & & \mathbf{F}_{ij4} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{ij} \\ \mathbf{s}_{ij1} \\ \mathbf{s}_{ij2} \\ \mathbf{s}_{ij3} \\ \mathbf{s}_{ij4} \end{bmatrix} + \begin{bmatrix} \mathbf{q}_{ij} \\ \mathbf{t}_{ij1} \\ \mathbf{t}_{ij2} \\ \mathbf{t}_{ij3} \\ \mathbf{t}_{ij4} \end{bmatrix} = \mathbf{0} \quad (9)$$

or

$$\begin{bmatrix} \mathbf{0} & \mathbf{G}_{ij}^* \\ \mathbf{G}_{ij} & \mathbf{F}_{ij} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{ij} \\ \mathbf{s}_{ij} \end{bmatrix} + \begin{bmatrix} \mathbf{q}_{ij} \\ \mathbf{t}_{ij} \end{bmatrix} = \mathbf{0} \quad (10)$$

Here \mathbf{F}_{ij} denotes the *flexibility matrix* of the panel ij .

Assuming for the sake of simplicity that $\mathbf{t}_{ij} = \mathbf{0}$ and expressing \mathbf{s}_{ij} from (10):

$$\mathbf{s}_{ij} = -\mathbf{F}_{ij}^{-1} \mathbf{G}_{ij} \mathbf{u}_{ij} \quad (11)$$

we obtain:

$$\mathbf{K}_{ij} \mathbf{u}_{ij} = \mathbf{q}_{ij}. \quad (12)$$

Here

$$\mathbf{K}_{ij} = \mathbf{G}_{ij}^* \mathbf{F}_{ij}^{-1} \mathbf{G}_{ij} \quad (13)$$

is the *stiffness matrix* of the panel ij . Using Eq. (9) this matrix can be written in the following summation form:

$$\mathbf{K}_{ij} = \sum_{k=1}^4 \mathbf{K}_{ijk}. \quad (14)$$

Here \mathbf{K}_{ijk} denotes the stiffness matrix of the edge k :

$$\mathbf{K}_{ijk} = \mathbf{G}_{ijk}^* \mathbf{F}_{ijk}^{-1} \mathbf{G}_{ijk}. \quad (15)$$

When analysing a single panel, the displacements \mathbf{u}_{ij} and the spring forces \mathbf{s}_{ij} can be determined from Eqs (12) and (11), respectively.

3. Analysis of a wall

The model of a wall of a panel building can be constructed by fitting to each other the line elements of the models representing the single panels (see Fig. 2/a). Then the displacements of the connected rigid line elements must be identical, and since the line elements are unloaded, the corresponding forces of the neighbouring springs are also equal.

3.1. Determination of the displacements

In order to determine the displacements of the plates we shall construct the stiffness matrix of the whole wall. Let us consider the l -th panel of the wall and two neighbouring panels denoted by n and m (Fig. 3). In our further

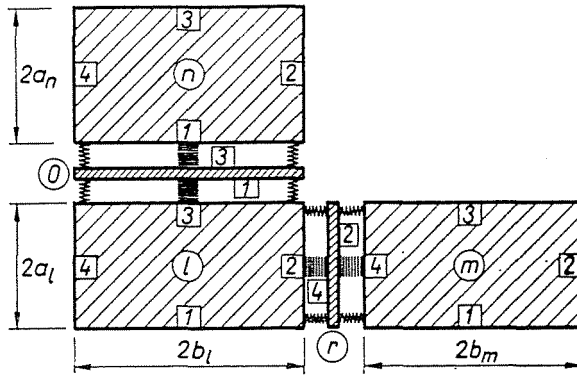


Fig. 3

investigations the line elements will be also considered as rigid plates with zero size in one direction. Accordingly, the results of Chapter 2 are valid for these elements, too, the only difference is, that in the matrices \mathbf{G} and \mathbf{G}^* in case of the line elements o and r , $a = 0$ and $b = 0$ should be substituted. The line elements are unloaded ($\mathbf{q}_0 = \mathbf{q}_r = \mathbf{0}$) and their displacements will be denoted by \mathbf{u}_0 and \mathbf{u}_r .

Firstly let us consider the plates l , o and n . Because of the interaction between the displacements of the plates l , o and o , n , respectively, the equilibrium equations (12) of the plates will be coupled, so we get the form:

$$\begin{bmatrix} \mathbf{K}_l & \mathbf{K}_{lo}^{31} & & \\ & \mathbf{K}_o^1 + \mathbf{K}_o^3 & \mathbf{K}_{on}^{31} & \\ & & & \\ & \mathbf{K}_{no}^{13} & \mathbf{K}_n & \end{bmatrix} \begin{bmatrix} \mathbf{u}_l \\ \mathbf{u}_o \\ \mathbf{u}_n \end{bmatrix} = \begin{bmatrix} \mathbf{q}_l \\ \mathbf{0} \\ \mathbf{q}_n \end{bmatrix} \quad (16)$$

Here \mathbf{K}_l and \mathbf{K}_n denote the stiffness matrices of the plates l and n (see Eq. (14)), while the others can be determined by the analogy of matrix (15) derived for the edges in the following way:

$$\left. \begin{aligned} \mathbf{K}_{lo}^{31} &= \mathbf{G}_{l3}^* \mathbf{F}_{l3}^{-1} \mathbf{G}_{o1} \\ \mathbf{K}_{ol}^{13} &= \mathbf{G}_{o1}^* \mathbf{F}_{l3}^{-1} \mathbf{G}_{l3} \\ \mathbf{K}_{on}^{31} &= \mathbf{G}_{o3}^* \mathbf{F}_{n1}^{-1} \mathbf{G}_{n1} \\ \mathbf{K}_{no}^{13} &= \mathbf{G}_{n1}^* \mathbf{F}_{n1}^{-1} \mathbf{G}_{o3} \\ \mathbf{K}_o^1 &= \mathbf{G}_{o1}^* \mathbf{F}_{l3}^{-1} \mathbf{G}_{o1} \\ \mathbf{K}_o^3 &= \mathbf{G}_{o3}^* \mathbf{F}_{n1}^{-1} \mathbf{G}_{o3} \end{aligned} \right\} \quad (17)$$

In Eq. (16) the sum $\mathbf{K}_o^1 + \mathbf{K}_o^3$ can be considered as the stiffness matrix of the plate o and accordingly the following notation will be used:

$$\mathbf{K}_o = \mathbf{K}_o^1 + \mathbf{K}_o^3. \quad (18)$$

From the second row of Eq. (15) \mathbf{u}_o can be expressed:

$$\mathbf{u}_o = -\mathbf{K}_o^{-1} (\mathbf{K}_{ol}^{13} \mathbf{u}_l + \mathbf{K}_{on}^{31} \mathbf{u}_n). \quad (19)$$

Substituting this result into the first row of Eq. (16) we obtain a direct relationship between the external forces and displacements of panels l and n :

$$\left[\begin{array}{c|c} \mathbf{K}_l - \mathbf{K}_{lo}^{31} \mathbf{K}_o^{-1} \mathbf{K}_{ol}^{13} & -\mathbf{K}_{lo}^{31} \mathbf{K}_o^{-1} \mathbf{K}_{on}^{31} \\ \hline -\mathbf{K}_{no}^{13} \mathbf{K}_o^{-1} \mathbf{K}_{ol}^{13} & \mathbf{K}_n - \mathbf{K}_{no}^{13} \mathbf{K}_o^{-1} \mathbf{K}_{on}^{31} \end{array} \right] \begin{bmatrix} \mathbf{u}_l \\ \mathbf{u}_n \end{bmatrix} = \begin{bmatrix} \mathbf{q}_l \\ \mathbf{q}_n \end{bmatrix}. \quad (20)$$

Using analogous notations as in (17), similar relationships can be derived for the external forces and the displacements of the plates l and m :

$$\left[\begin{array}{c|c} \mathbf{K}_l - \mathbf{K}_{lr}^{24} \mathbf{K}_r^{-1} \mathbf{K}_{rl}^{42} & -\mathbf{K}_{lr}^{24} \mathbf{K}_r^{-1} \mathbf{K}_{rm}^{24} \\ \hline -\mathbf{K}_{mr}^{42} \mathbf{K}_r^{-1} \mathbf{K}_{rl}^{42} & \mathbf{K}_m - \mathbf{K}_{mr}^{42} \mathbf{K}_r^{-1} \mathbf{K}_{rm}^{24} \end{array} \right] \begin{bmatrix} \mathbf{u}_l \\ \mathbf{u}_m \end{bmatrix} = \begin{bmatrix} \mathbf{q}_l \\ \mathbf{q}_m \end{bmatrix}. \quad (21)$$

The interaction of the panels in the analysis of a whole wall is seen from Eqs (20) and (21) to modify the stiffness matrix of a panel. Nevertheless, even now the block \mathbf{K}_{ll} of the stiffness matrix of the whole wall can be calculated as the sum of the stiffness matrices of the four edges of the l -th panel:

$$\mathbf{K}_{ll} = \sum_{k=1}^4 \mathbf{K}_{llk}. \quad (22)$$

Instead of Eq. (15), however, matrices \mathbf{K}_{llk} for e.g. the edges $k = 3$ and $k = 2$ are obtained from the formulae:

$$\left. \begin{aligned} \mathbf{K}_{ll3} &= \mathbf{G}_{l3}^* \mathbf{F}_{l3}^{-1} \mathbf{G}_{l3} - \mathbf{K}_{lo}^{31} \mathbf{K}_o^{-1} \mathbf{K}_{ol}^{13} \\ \mathbf{K}_{ll2} &= \mathbf{G}_{l2}^* \mathbf{F}_{l2}^{-1} \mathbf{G}_{l2} - \mathbf{K}_{lr}^{24} \mathbf{K}_r^{-1} \mathbf{K}_{rl}^{42} \end{aligned} \right\} \quad (23)$$

The blocks \mathbf{K}_{lm} and \mathbf{K}_{ln} are given by the equations:

$$\left. \begin{aligned} \mathbf{K}_{ln} &= \mathbf{K}_{nl}^* = -\mathbf{K}_{lo}^{31} \mathbf{K}_o^{-1} \mathbf{K}_{cn}^{31} \\ \mathbf{K}_{lm} &= \mathbf{K}_{ml}^* = -\mathbf{K}_{lr}^{24} \mathbf{K}_r^{-1} \mathbf{K}_{rm}^{24} \end{aligned} \right\} \quad (24)$$

Using formulae (22) and (24) the stiffness matrix \mathbf{K} of the whole wall under consideration can be constructed and then from equation

$$\mathbf{K}\mathbf{u} = \mathbf{q} \quad (25)$$

the displacements \mathbf{u} of the plates can be determined.

3.2. Determination of the spring forces

In the analysis of a whole wall, the displacements of the line elements require also the spring forces (11) of a single panel to be modified. In order to derive the new formula, let us consider again the panels illustrated in Fig. 3. Applying formula (11) for the spring forces \mathbf{s}_{lo} and \mathbf{s}_{on} arising between the plates l, o and o, n , respectively, we obtain the relationships

$$\left. \begin{aligned} \mathbf{s}_{lo} &= -(\mathbf{F}_{l3}^{-1} \mathbf{G}_{l3} \mathbf{u}_l + \mathbf{F}_{l3}^{-1} \mathbf{G}_{ol} \mathbf{u}_o) \\ \mathbf{s}_{on} &= -(\mathbf{F}_{n1}^{-1} \mathbf{G}_{o3} \mathbf{u}_o + \mathbf{F}_{n1}^{-1} \mathbf{G}_{n1} \mathbf{u}_n) \end{aligned} \right\} \quad (25)$$

Since the element o is unloaded, therefore $\mathbf{s}_{lo} = \mathbf{s}_{on}$ and these forces can be considered as the forces \mathbf{s}_{ln} arising between the plates l and n :

$$\mathbf{s}_{ln} = \mathbf{s}_{lo} = \mathbf{s}_{on}. \quad (27)$$

Then \mathbf{u}_o can be calculated from Eqs (26):

$$\mathbf{u}_o = (\mathbf{F}_{l3}^{-1} \mathbf{G}_{ol} - \mathbf{F}_{n1}^{-1} \mathbf{G}_{o3})^{-1} \cdot (-\mathbf{F}_{l3}^{-1} \mathbf{G}_{l3} \mathbf{u}_l + \mathbf{F}_{n1}^{-1} \mathbf{G}_{n1} \mathbf{u}_n). \quad (28)$$

Since the displacements \mathbf{u}_l and \mathbf{u}_n are already known, \mathbf{s}_{ln} can be determined from any of Eqs (26). The spring forces acting on the other three edges of the panel l can be calculated in a similar manner.

4. Determination of the spring coefficients

Since the springs interconnecting the rigid elements simulate the elastic behaviour of the panels and the joint, also their coefficients should be determined in two steps.

The panels are solid or have openings (windows or doors). Their elastic behaviour may be analysed e.g. by the finite element method [4, 7] and then, by using the results of this analysis and introducing some approximations, the spring coefficients can be determined.

For example the edges of the panels may be assumed to remain straight and the springs of an edge comprise only the elastic deformations of the half panel at the edge in question. Then a unit vertical displacement has to be introduced e.g. at point a of the panel illustrated in Fig. 4 and using the finite element method the stiffness coefficients k'_{aa} , k'_{ab} , k'_{ac} of the springs a , b , c of the edge 3 can be determined. Repeating the same procedure for the unit displacements at points b and c we obtain all the elements of the stiffness matrix of edge 3, and the inverse of this matrix will be the flexibility matrix F_{ij3} of the edge under consideration. In this manner the flexibility matrices of all practically encountered panel types can be determined in advance. These results can be tabulated or plotted in diagrams or stored in a computer. Then the analysis of a panel building involves relatively simple calculations.

Developing an approximate method, there are, of course, many other possibilities for the determination of the spring coefficients. The investigation of this problem will be the subject of further research.

The second part of the spring coefficients refers to the elastic behaviour of the joints (connection between steel bars and cement grouting). Reliable information in this question is expected, first of all from tests and measurements (see e.g. [6]). Knowing for example the stiffness coefficient k''_{aa} representing the elastic behaviour of the joint at point a of a panel (see Fig. 4.), then the *total* stiffness coefficient k_{aa} of the spring in question is determined by the relationship

$$\frac{1}{k_{aa}} = \frac{1}{k'_{aa}} + \frac{1}{k''_{aa}}.$$

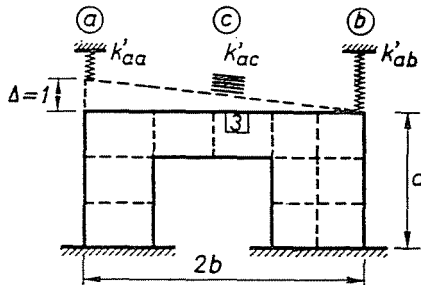


Fig. 4

5. Examples

In Fig. 5 a wall constructed of 12 identical solid panels is seen. The wall is built in at its bottom edge and its upper part is subjected to a uniformly distributed horizontal load. The elastic constants of the material are $E = 3 \cdot 10^6 \text{ Mp/m}^2$, $\nu = 0.15$ and the thickness of the wall is 15 cm. For the sake of simplicity the deformations of the joints will be neglected.

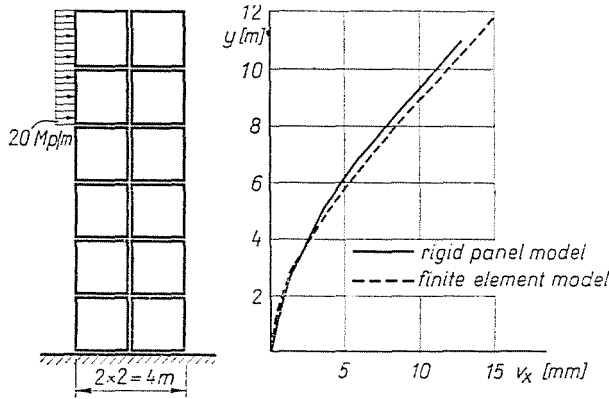


Fig. 5

The problem has been analysed by using the rigid panel model and by the finite element method. In the latter case each panel was subdivided into $2 \times 2 = 4$ further elements. The horizontal displacements v_x of the wall obtained by the two different methods are given in the diagram of Fig. 5.

The wall illustrated in Fig. 6 is constructed of 15 identical solid panels. The bottom edge is built in and the top edge is partially loaded. The wall thickness and the elastic constants are the same as in the former example. Analysing the wall by the rigid panel model and by the finite element method

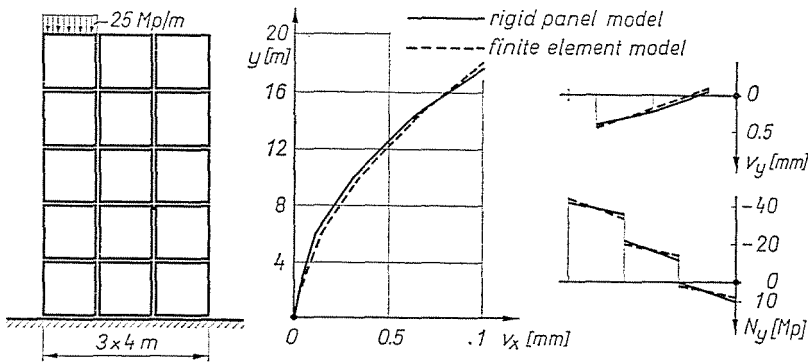


Fig. 6

(in the latter case each panel was subdivided into $4 \times 4 = 16$ further elements) the horizontal displacements v_x , the vertical displacements v_y in the section $y = 8^m$ and the vertical normal forces N_y in the section $y = 0$ have been calculated. The results are illustrated in Fig. 6.

Considering these two examples, the results obtained by the use of the rigid panel model can be stated to but slightly differ from those of the finite element method. The maximum difference is less than 5%. This seems to prove the reliability of the suggested new model. Nevertheless, general statements can only be given after performing a greater number of numerical investigations.

6. Conclusions

The model suggested fits very well the actual structural system of panel buildings, therefore it seems to be suitable for theoretical investigations and practical applications. The advantage of the model is to permit the effect of the joints to be taken into consideration and to deliver directly the internal forces arising in the joints. Besides, the analysis needs relatively short computer time, much less than e.g. the finite element method. The condition of the economical application of the model is, however, to know the spring coefficients of panels with different sizes and openings. This problem needs further investigations. The suggested method can be extended to the elastic analysis of a whole building and to the limit analysis of panel structures, too. By the use of the model, also the interaction between the structure and the subgrade can be easily investigated. The research into these problems is in progress.

Summary

A new model is presented for the elastic analysis of prefabricated panel buildings. In this model the panels work as rigid plates interconnected along their edges by elastic springs acting in tension or compression and in shear. By the appropriate choice of their coefficients the springs are made to represent the elastic behaviour of both the panels and the joints.

The application of the model leads to relatively simple calculation and according to the numerical examples the accuracy of its results seems to be satisfactory for practical use.

References

1. MACLEOD, I. A.: Analysis of Shear Wall Buildings by the Frame Method. Proc. Instn. Civ. Engrs. 1973. 55. (593—503).
2. KALISZKY, S., GYÖRGYI, J., LOVAS, A.: The Internal Forces of Panel Buildings Due to Vertical Displacements. (In Hungarian) Magyar Építőipar 1978 (8).
3. ROSMAN, R.: Statik und Dynamik der Scheibensysteme des Hochbaues. Springer Verlag Berlin 1968.
4. ZIENKIEWICZ, O. C.: The Finite Element Method in Engineering Science. McGraw Hill, New York 1971.

5. SZABÓ J.-ROLLER, B.: Anwendung der Matrizenrechnung auf Stabwerke. Akadémia Kiadó, Budapest 1978.
6. BJARNE CHR. JENSEN: Some Applications of Plastic Analysis to Plain and Reinforced Concrete. Institute of Building Design Report 123. Technical University of Denmark. Lyngby 1977.
7. BOJTÁR, I.: Analysis of Spatial Plate Structures. Periodica Polytechnica. Civ. Eng. Vol. 23. (1979) 2.

Prof. Dr. Sándor KALISZKY }
Assistant Károly WOLF } H-1521, Budapest