ANALYSIS OF COMPOSITE BUILDING STRUCTURES UNDER HORIZONTAL DYNAMIC LOADS

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1. Introduction

In the actual construction practice, most buildings likely to be exposed to horizontal dynamic loads (wind gusts, earthquakes etc.) have vertical load-bearing structures of non-symmetrical floor plan, composite, or else, varying vertically. Structurally it means that the vertical load-bearing structures of the building are frameworks, columns, independent or connected bearing walls (panel frames) or combinations thereof. For example, an office block or a hotel may have a one- or two-storey ground floor hall of big floor area (reception, congress hall, restaurant, club rooms etc.) differing in floor plan from the upper storeys devoted to the main function of the building, with structures or floor plans that may also differ by storeys or storey groups.

A method will be presented below for analyzing the effect of horizontal dynamic loads on general and composite building structures.

2. Mathematical model

Since in general, vertical structural members are not doubly symmetrical, storeys can only perform "coupled" (simultaneous torsional and bending) vibrations in the horizontal plane. Accordingly, the mathematical model represents a system of as many masses as there are storeys in the building, where elastically connected masses consist of diaphragms rigid in the (horizontal) floor planes. Building masses including wall and accessory masses for each storey are assumed to be distributed in the storey floor plane, without any stipulation on the mass distribution. The forces being horizontal, masses are assumed to be mobile only in their planes. Elastic constraints connect the lowermost mass to the ground, and the superior ones to the underlying ones each. In conformity with the general plane motion, constraints exert a force and a couple. In addition, also the damping constraint proportional to the sidesway or rotation rate has been reckoned with. The model assumed for two adjacent (k-th and $k+l$-th) masses is seen in isometry in Fig. 1.

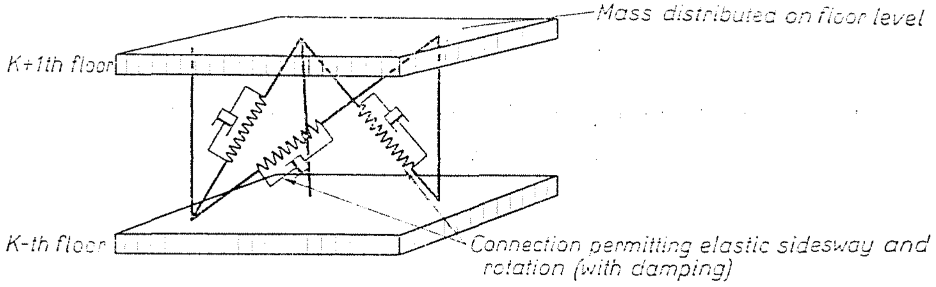

Fig. 1. Schematic model

3. Differential equation system of motion

Omitting details of computational assumptions and of relationships for determining the kinematic characteristics of the assumed model, we simply refer to [1].

For the previously described general case where the load-bearing structure of the building is different for each storey or storey group, neither the main directions of sidesway (u, v) and rotation centres 0 of storeys or storey groups will be identical. Therefore storey characteristics of motion are advisably referred to a co-ordinate system with arbitrary chosen axes (here x and y) and origin, stipulating that origins for each storey are aligned on a defined vertical and the corresponding co-ordinate axes are parallel in each storey. Thus, absolute sidesways of the origin at the i-th floor assumed to be rigid in plane are described by sidesways x_i and y_i and rotation φ_i (Fig. 2).

The system being linear elastic and of viscous damping, the matrix differential equation expressing the set of differential equations of motion is:

$$\mathbf{A}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{p}(t). \tag{1}$$

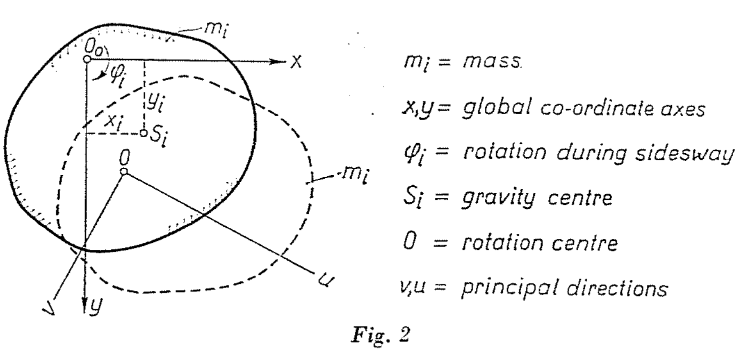

Fig. 2

In the actual case, a hypermatrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{M} & \mathbf{0} & -\mathbf{Y}_s\mathbf{M} \\ \mathbf{0} & \mathbf{M} & \mathbf{X}_s\mathbf{M} \\ -\mathbf{Y}_s\mathbf{M} & \mathbf{X}_s\mathbf{M} & \mathbf{J}_o \end{bmatrix}$$

with diagonal matrix elements containing masses concentrated in floor planes:

$$\mathbf{M} = \langle m_1, m_2, \ldots, m_n \rangle$$

mass inertia moments referred to the origin of co-ordinate system of masses concentrated in storey planes:

$$\mathbf{J}_o = \langle J_{o1}, J_{o2}, \ldots, J_{on} \rangle$$

gravity centre co-ordinates:

$$\mathbf{Y}_s = \langle y_{s1}, y_{s2}, \ldots, y_{sn} \rangle$$

$$\mathbf{X}_s = \langle x_{s1}, x_{s2}, \ldots, x_{sn} \rangle$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{xx} & \mathbf{K}_{xy} & \mathbf{K}_{xz} \\ \mathbf{K}_{yx} & \mathbf{K}_{yy} & \mathbf{K}_{yz} \\ \mathbf{K}_{zx} & \mathbf{K}_{zy} & \mathbf{K}_{zz} \end{bmatrix}.$$

Elements of hypermatrix **K** are so-called rigidity matrices containing forces and couples due to unit displacements. (To determine its elements, knowledge of rotation centres and principal directions is needed, see pp. 228—236 in [1].)

$\mathbf{d} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \boldsymbol{\varphi} \end{bmatrix}$ a hypervector of 3 n dimensions, its components are: displacement ordinatae $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ etc.

C contains damping matrices.

$$\mathbf{p}(t) = \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ \mathbf{m}_o \end{bmatrix} f(t)$$

are forces acting at each storey or better, force components and couples replacing them referred to the origin of the co-ordinate system.

4. Circular natural frequencies of the system

Circular natural frequencies and normal vibration modes of the system are obtained by solving the homogeneous part of Eq. (1). Omitting the effect of damping on the eigenfrequency, solution of the homogeneous part is:

$$(\mathbf{K} - \omega^2 \mathbf{A})\mathbf{d}_o = 0 . \tag{2}$$

Circular natural frequencies $(\omega_{o1}, \omega_{o2}, \ldots, \omega_{on})$ and normal vibration mode amplitudes $(\mathbf{d}_{o1}, \mathbf{d}_{o2}, \ldots, \mathbf{d}_{on})$ are given by eigenvalues and eigenvectors of the matrix equation. (Detailed numerical solutions see in [2].)

5. Analysis of the effect of dynamic excitation

Response functions of displacements due to dynamic (time-dependent) forces are given by solutions of Eq. (1). To simplify solution, the damping matrix will be assumed to be similar in construction to the rigidity matrix, accordingly, damping factors may be expressed as

$$c_j = \alpha_k k_j \qquad (j = 1, 2, \ldots, 3n).$$

Hence the damping matrix may be given as

$$\mathbf{C} = \alpha_k \mathbf{K}. \tag{3}$$

Introducing vector $\mathbf{z}^{(r)} = \frac{1}{c_r} \mathbf{d}_{or}$ and choosing the c_r value to meet condition

$$\mathbf{z}^{(r)*} \mathbf{A} \mathbf{z}^{(r)} = 1$$

vectors $\mathbf{z}^{(r)}$ can be combined into a matrix $\mathbf{Z}$. Using it after transformation $\mathbf{q} = \mathbf{Z}\mathbf{d}$, the system of differential equations (1) of $3n$ coupled terms is decomposed into $3n$ unconnected differential equations of the form:

$$\ddot{q}_i + (\alpha_k \omega_{oi}^2 + \alpha_m)\dot{q}_i + \omega_{oi}^2 q_i = P_{oi} f(\tau) . \tag{4}$$

Its solution being:

$$q_i = e^{-\alpha_i t}\left[q_{oi} \cos \beta_i t + \frac{1}{\beta_i} (\dot{q}_{oi} + \alpha_i q_{oi}) \sin \beta_i t\right] +$$

$$+ \frac{P_{oi}}{2\lambda_{i1} + c_i} \int_0^t e^{\lambda_{i1}(t-\tau)} f(\tau)\, d\tau + \frac{P_{oi}}{2\lambda_{i2} + c_i} \int_0^t e^{\lambda_{i2}(t-\tau)} f(\tau)\, d\tau \tag{5}$$

where

$$\lambda_{i1} = -\frac{c_i}{2} + \sqrt{\frac{c_i^2}{2} - \omega_{oi}^2}\,; \qquad \lambda_{i2} = -\frac{c_i}{2} - \sqrt{\frac{c_i^2}{2} - \omega_{oi}^2}$$

$$c_i = (\alpha_k \omega_{oi}^2); \quad \beta_i = \sqrt{\omega_{oi}^2 - \frac{c_i^2}{4}}\,; \quad \alpha_i = \frac{c_i}{2} .$$

6. Example

Natural frequencies and sidesways due to an excitation varying according to three consecutive sine half-waves of a building with vertical load-bearing structures as indicated in Fig. 3 have been determined as an example. The obtained circular natural frequencies and the normal vibration modes

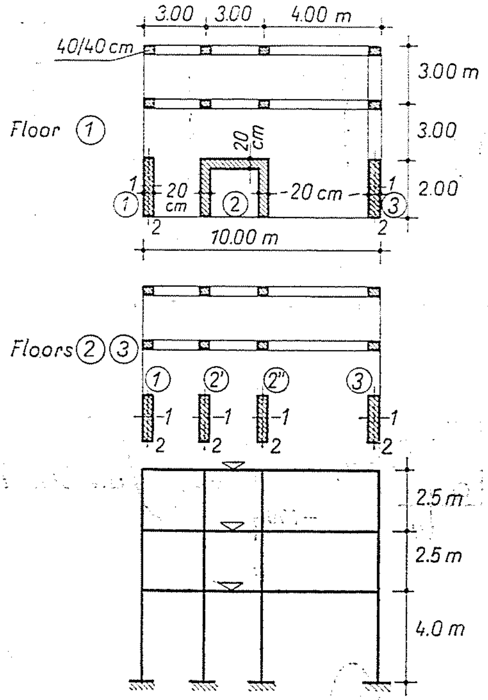

Fig. 3. Structural scheme of the building

are seen in Fig. 4, showing that circular natural frequencies are rather close. This is a hint to take coupled vibrations of buildings — where there is a likelihood — into consideration.

Analysis of the exciting force effect involved one force acting at the first storey as seen in Fig. 5. (Damped values were affected by subscript *d*.) Behaviour of systems without damping and with damping $c_i = 0,1\ c_{\mathrm{crit}}$ have been separately analyzed. From among results, response functions of sidesway in direction x are seen in Fig. 6. It is interesting to see that the damping effect is less during the first half-wave, to be considered in the analysis, since often it may be critical for the entire structure.

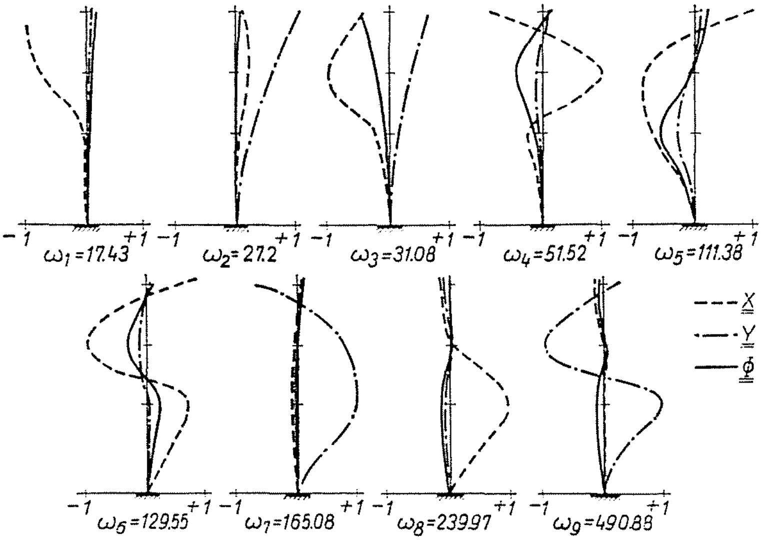

Fig. 4. **Circular eigenfrequencies and pertaining normal vibration modes**

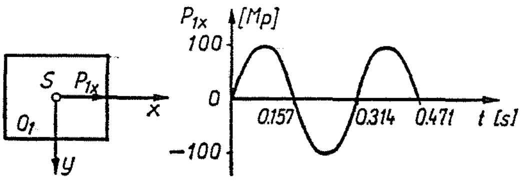

Fig. 5

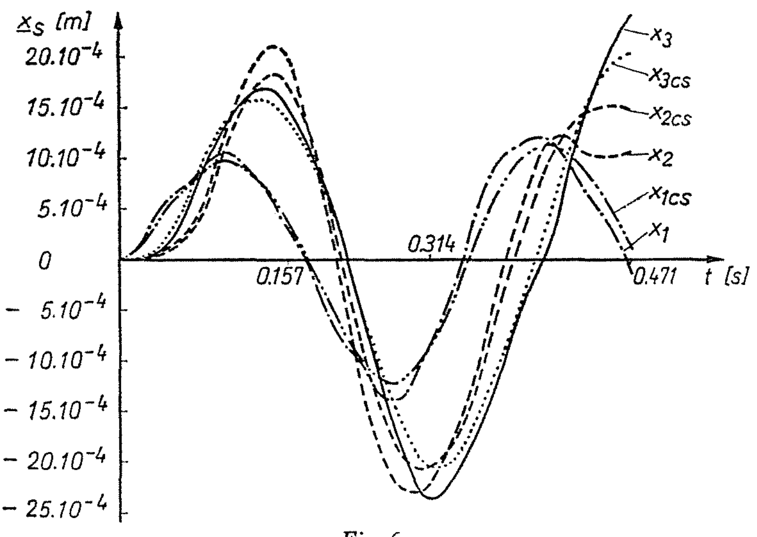

Fig. 6

Summary

A calculation method for determining horizontal dynamic effects in buildings of general load-bearing structure members may be different both in floor plane and in elevation, and also by dimensions. On hand of a developed numerical example, several phenomena of importance for the building will be pointed out that can be taken into consideration by the presented method.

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* In Hungarian.

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