TESTS WITH SIMPLE ELASTIC-PLASTIC FRAMES

By

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1. Introduction

1.1. *The research project*

To promote the adoption of plastic design of steel structures, specifications were issued in several countries, containing special design rules and procedures for plastic analysis and design.

The most simple and popular approach in this field being the so-called rigid-plastic (first-order, simple-plastic) analysis — operating with rigid-plastic material model — one of the main purposes of these specifications is to define the restrictions on the use of this method.

The rigid-plastic limit load is known to be connected to the formation of a yield mechanism due to the developing plastic hinges. So — in using rigid-plastic analysis — all phenomena to interfere with this mode of failure are to be excluded. In case of planar frames they belong to two categories:

(i) Effects of change in geometry in the plane of the structure leading to secondary bending moments reducing ultimate load and influencing the mode of failure. They can be overcome by choosing an appropriate in-plane stiffness.

(ii) Local buckling of plates (webs and flanges) and lateral buckling of beams and beam-columns, leading again to lower limit load by diminishing the ultimate moment of the members or their deformation capacity needed for the development of the predicted yield mechanism. They can be excluded by appropriate choice of width-to-thickness ratio of the plates and sufficiently close spaced lateral supports.

1.2. *Factors influencing the investigations*

The research project carried out in the Laboratory of the Department of Steel Structures, Technical University, Budapest (Fig. 1) was connected on the one hand to the preparation of a new version of the Hungarian specification for plastic design, on the other hand to the adoption of a standard system of one-storey frames (CONDER-IPARTERV) based on a licence bought from the British firm CONDER and produced in series according to the plans of

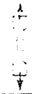
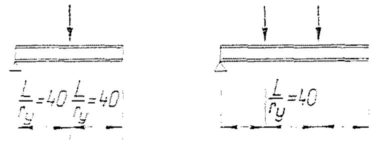
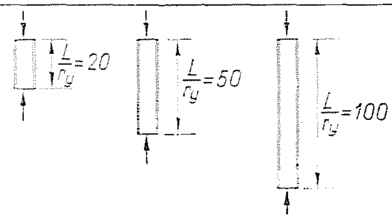
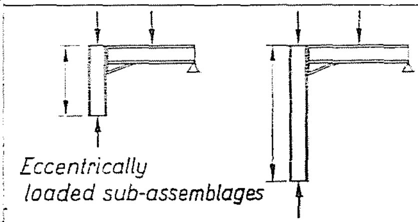
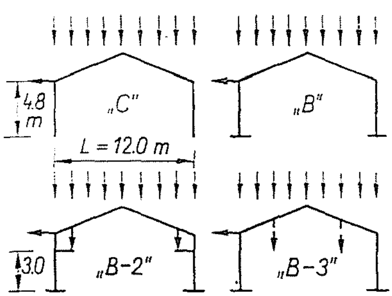
	Form of specimens	Number	Scope of investigation
Tensile specimens		cc 200	Material characteristics
Beam specimens		8	Ultimate moment and behaviour of plastic hinges
Axially compressed specimens		24	Effect of residual stresses; column buckling
Subassemblages under eccentric load		8	Behaviour in case of different forms and connections (welded, bolted) of frame corners
Frames		14	Effect of different systems of lateral supports in case of pinned and clamped columns; various slopes of rafters (10%, 20%, 30%) and cranes

Fig. 1. Experimental research project

the design office IPARTERV* and manufactured by the KGyV** Steel Works. The welded plate girders of this system have lower torsional rigidity than the rolled sections of the original structure, obvious from comparing the factors

$$T = \frac{G K A}{W_x^2}$$

* Industrial Building Design Co.
** Metallurgic Construction Enterprise.

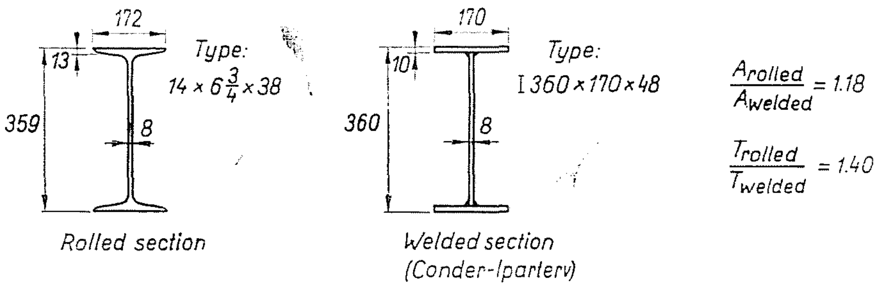
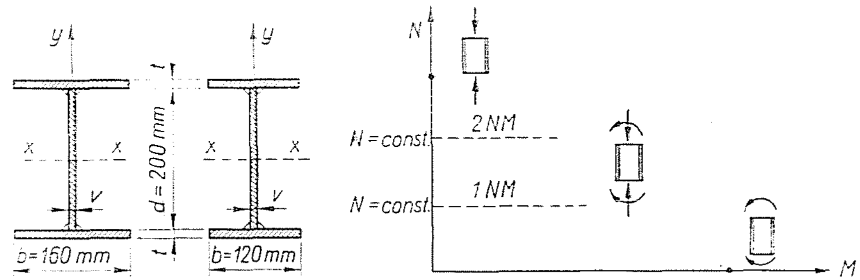


Fig. 2.



$\frac{d}{v}$	40				50				67				80			
$\frac{b}{t}$	16	20	26.6	32	16	20	26.6	32	16	20	26.5	32	16	20	26.6	32

Fig. 3

(GK being the torsional rigidity, A the cross-sectional area and W_x the elastic section-modulus) as seen in Fig. 2. *This leading to increased sensitivity to lateral-torsional buckling, the main purpose of the investigations was to check the influence of the lateral-torsional buckling of beam-columns on the failure load and to establish appropriate and economic rules for their lateral bracing requirements* (Chapter 4). Additionally the limits of negligibility of the effect of change in geometry were dealt with by checking the validity of the so-called modified *Rankine-Merchant* formula (Chapter 5) in case of the mentioned frames.

Tests concerning the appropriate and optimum choice of width-to-thickness ratio of plate elements were conducted under a separate project (Fig. 3).

Paper deals with the results of the first four tests on full-size one-story, one-bay frames as described in Chapter 3.

Other tests in the research projects according to Figs 1 and 3 will be dealt with in details in a subsequent publication.

Fig. 4. Test on frame *C*-3/1

2. Experimental methods and techniques

2.1. *Description of full-size experimental structures*

In the first series three different structures were tested, with pin-based columns; their span being 12.0 meters, the height of the column 4.8 meters (Fig. 4).

Test frame *C*-1 had a rafter with slope of 10% (5.7°), welded column sections I 400-180-58 and welded rafter sections I 270-135-31 (Fig. 5).

Test frame *C*-2 had a rafter with a slope of 20% (11.3°), welded column sections I 400-180-58 and welded rafter sections I 270-135-31 (Fig. 6).

Test frames *C*-3/1 and *C*-3/2 had rafters with a slope of 30% (16.7°), welded column sections I 360-170-48 and welded rafter sections I 300-150-37 (Fig. 7).

Rafter-to-column and mid-span connections consisted of high-strength prestressed bolts (Fig. 8).

2.2. *Loading of test frames*

Test frame *C*-3/1 was loaded by vertical loads; *C*-1, *C*-2 and *C*-3/2 by combined (vertical and horizontal) loads (Fig. 9).

Vertical loads were applied at points where purlins were planned to join the rafter; so column tops received 50 percent of the load acting at other points.

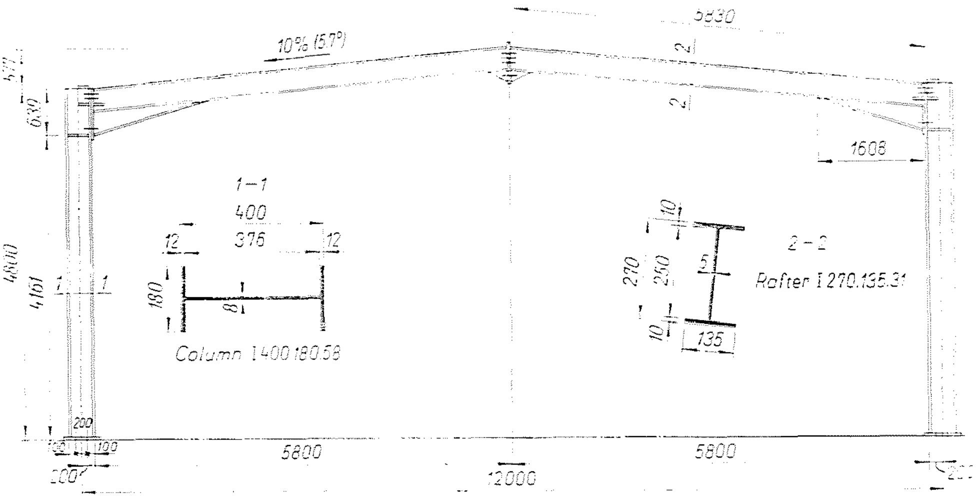


Fig. 5. Test frame *C*-1

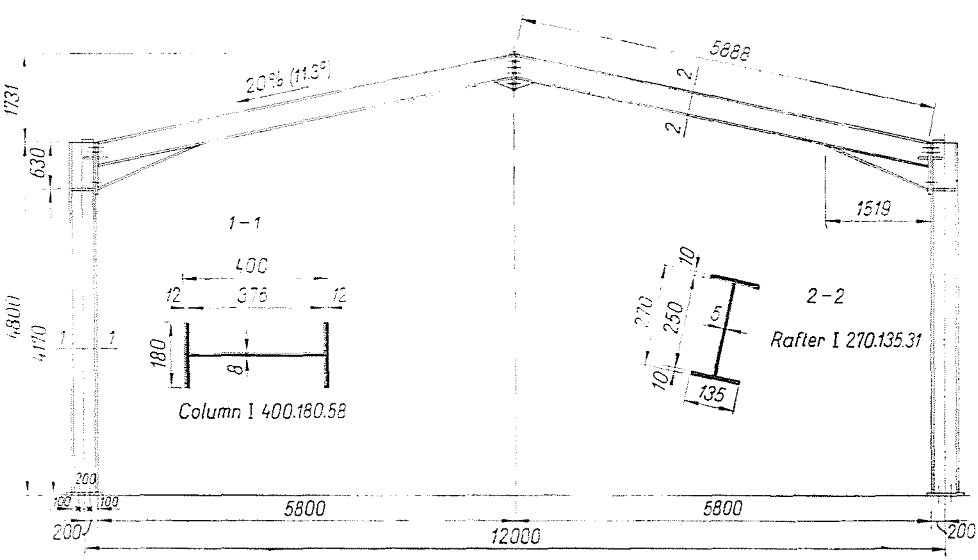


Fig. 6. Test frame *C*-2

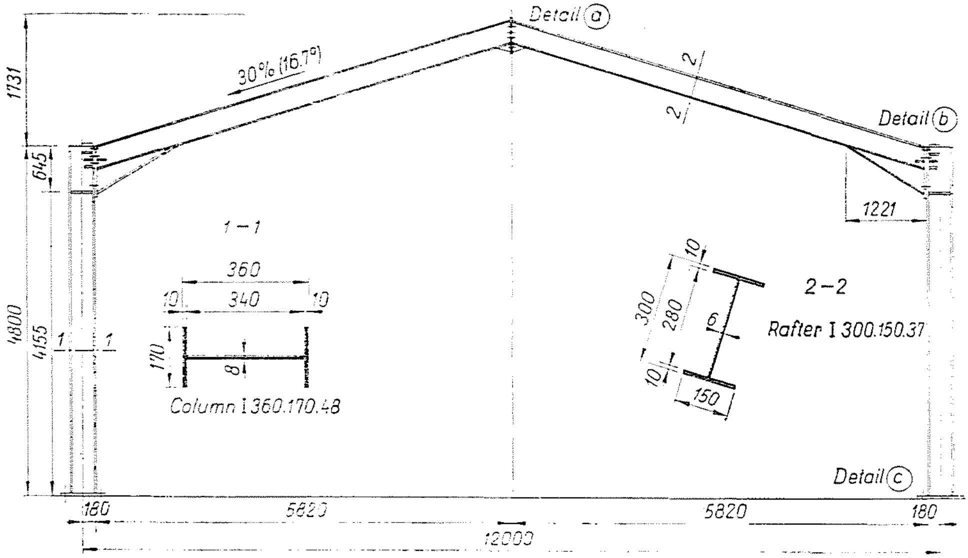


Fig. 7. Test frame *C*-3

Horizontal load was applied at the top of the left-side column, of a magnitude chosen to give the same effect as a distributed horizontal loading (wind load).

2.21. *Vertical loading system*

Vertical loads at the joining points of purlins were applied to the upper flange of rafter, so web and bottom flange were not restricted laterally. The appropriate ratio of vertical loads was achieved by a load-distributing girder system (Fig. 10), built up of simply supported beams. Loading was exerted by two hydraulic jacks of 400 kN capacity. To make horizontal displacement (sidesway) unrestricted, jacks were fastened not directly to the floor-slab (Fig. 11/a), but through a so-called gravity load simulator (Fig. 11/b) [2]. This latter consisted of three elements: two bars, and a rigid triangle. The two bars had pin-joints at both ends, resulting in a one-degree-of-freedom mechanism. Hydraulic jacks joined the rigid triangle. This mechanism produced a vertical load acting at the intersection of the two bar axes. Characteristics of the simulator are given in Fig. 11/c.

The simulator — designed and manufactured in the Laboratory of the Department of Steel Structures — is seen in Figs 12 and 13.

2.22. *Horizontal loading*

Horizontal loading was approximated by a slightly inclined tension rod joining an equipment fixed to the floor-slab. Horizontal loads were exerted by

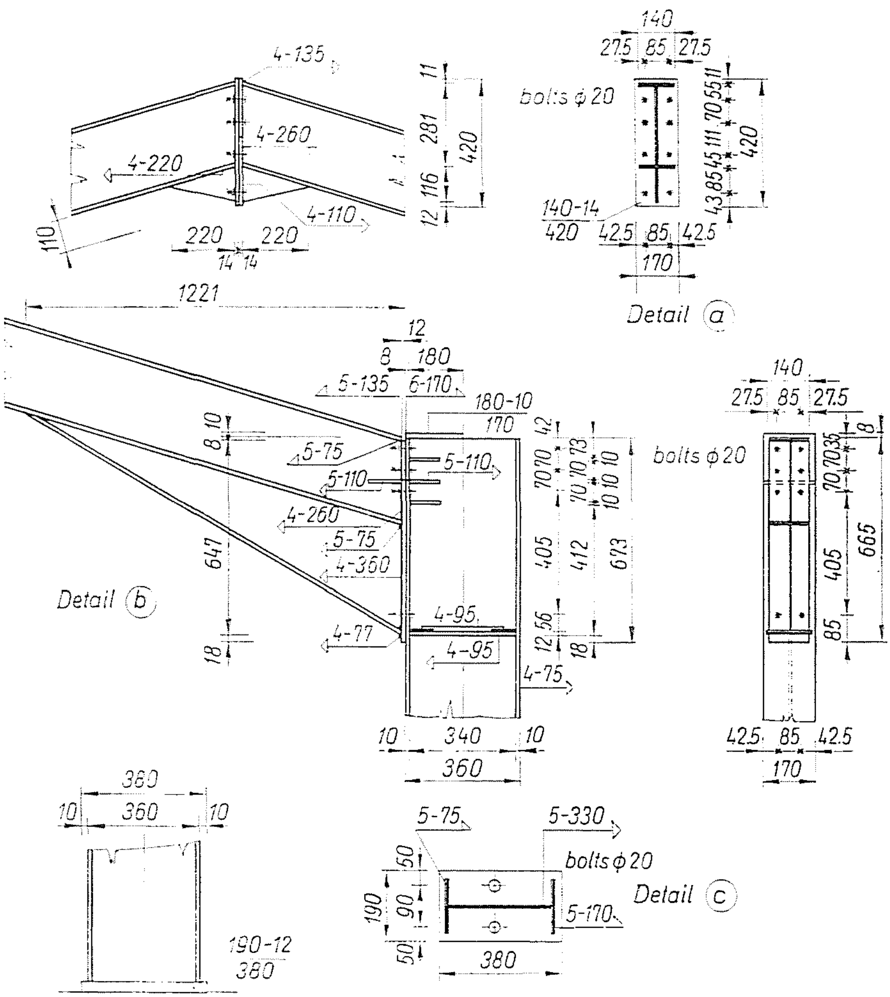


Fig. 8. Details of frame corner

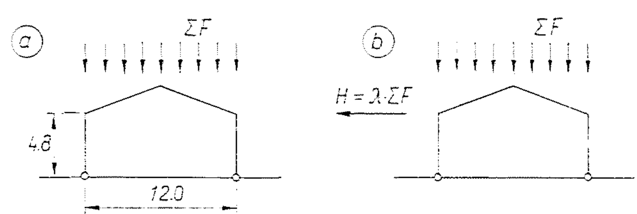


Fig. 9. a) Vertical load; b) Vertical and horizontal load

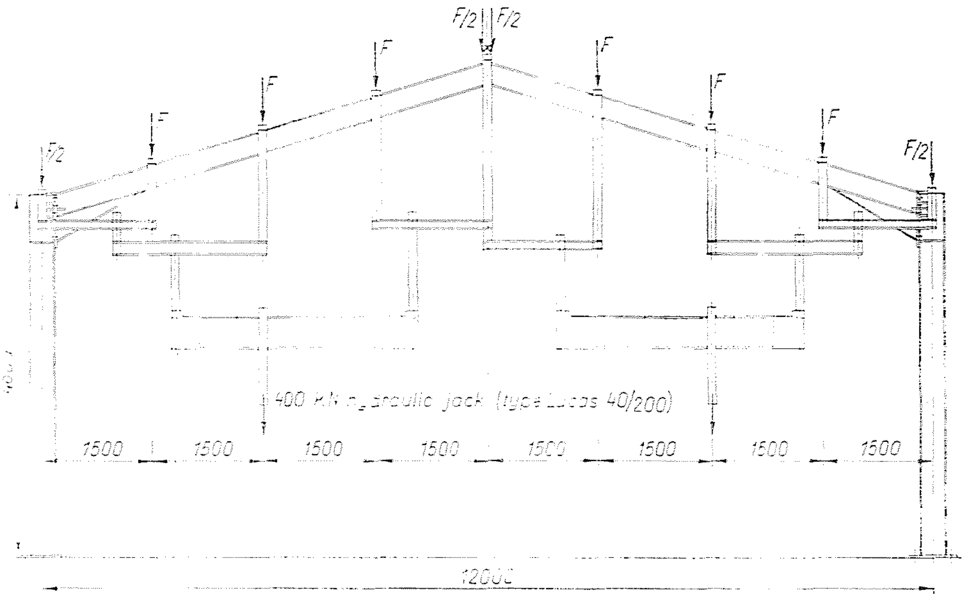


Fig. 10. Load distributing system

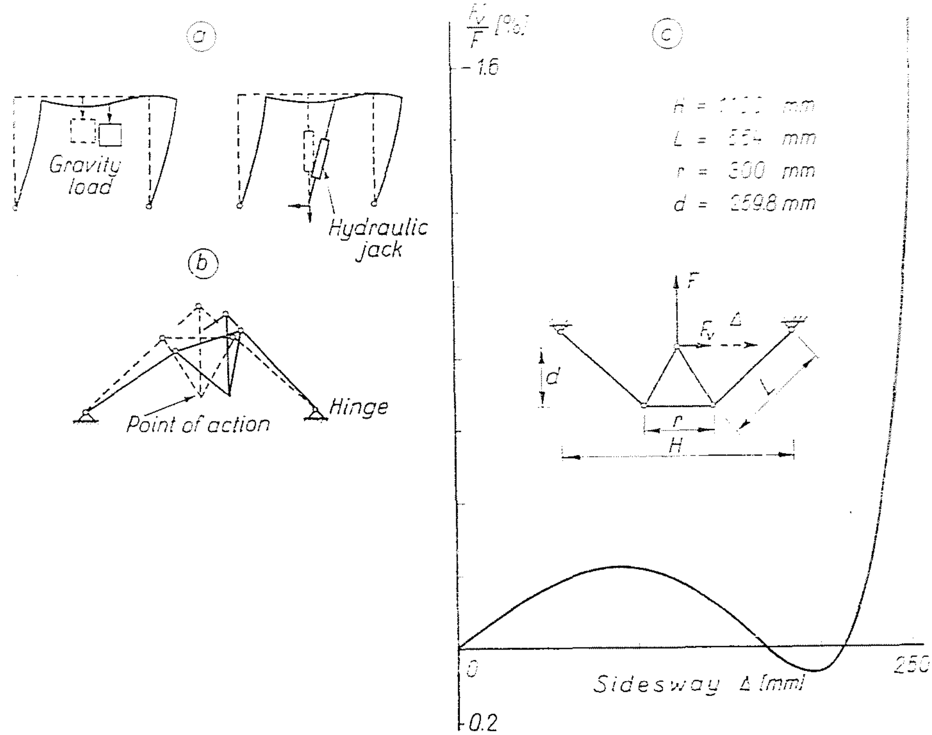


***Fig. 11.* Vertical loading system**

— — — Central position; ——— Swayed position

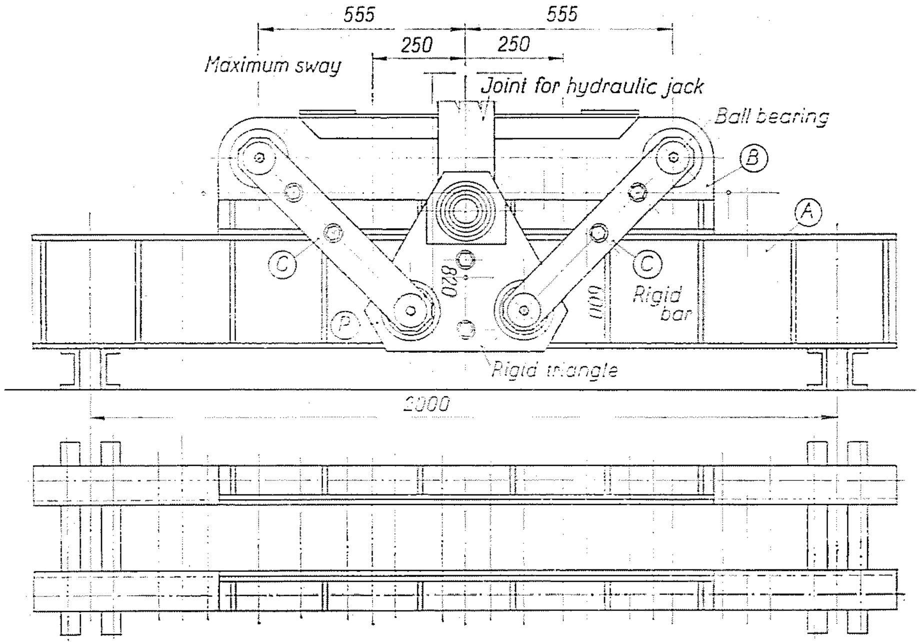


Fig. 12. Gravity load simulator

a hydraulic jack driven by the same oil-pressure circuit as the vertical jacks thus achieving a constant ratio of horizontal to vertical loads. Details are seen in Fig. 14.

2.3. *Lateral supports*

The original structure consists of frames and a "perpendicular" system of purlins, side-rails and wind-bracing. The effect of this latter system was simulated by a "back-ground" construction (Fig. 15) located at three meters behind the test frames, and interconnecting rods at the location of purlins and side-rails giving lateral support to the frames.

The main purpose of these tests being (as stated in Chapter 1) to find appropriate measures to exclude premature lateral-torsional buckling, different types of supports were applied (Fig. 16):

— Type a): a single rod with pin-joints giving lateral restraint to one flange only (Fig. 17);

— Type b): former type completed by a diagonal tie-back supporting the other flange;

— Type c): two rods with pin-joints giving lateral support to both flanges (Fig. 18);

— Type d): perpendicular girder with rigid joint to one flange, giving lateral and partial rotational restraint;

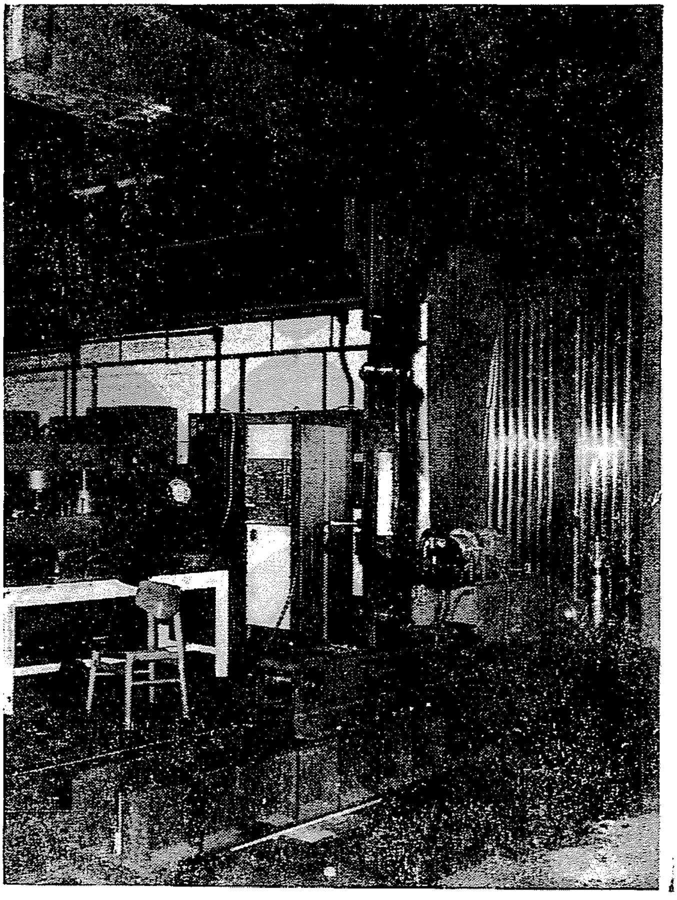


Fig. 13. Photo of vertical load simulator

— Type e): former type completed by a diagonal tie-back supporting the other flange;

— Type f): type d) completed by a parallel rod supporting the other flange.

Stiffness of girders in types d), e) and f) was chosen to simulate that of the purlins and side-rails, respectively.

2.4. *Measuring techniques*

During testing, in addition to the basic characteristics of structural behaviour (vertical and horizontal loads; vertical and horizontal displacements at different points of the structure), other data (rotation at column bases, forces in prestressed bolts, forces in supporting rods etc.) were recorded as well.

Displacements were measured by photogrammetric techniques as well: at different loading stages pairs of pictures of the structure were taken for the stereophotogrammetric evaluation of displacements.

Present paper reports on the values of loads and vertical deflections only in the mid-span.

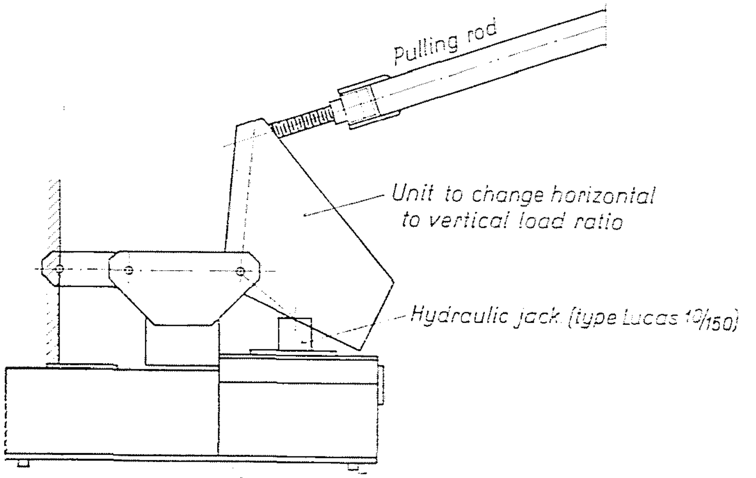


Fig. 14/a Horizontal loading equipment

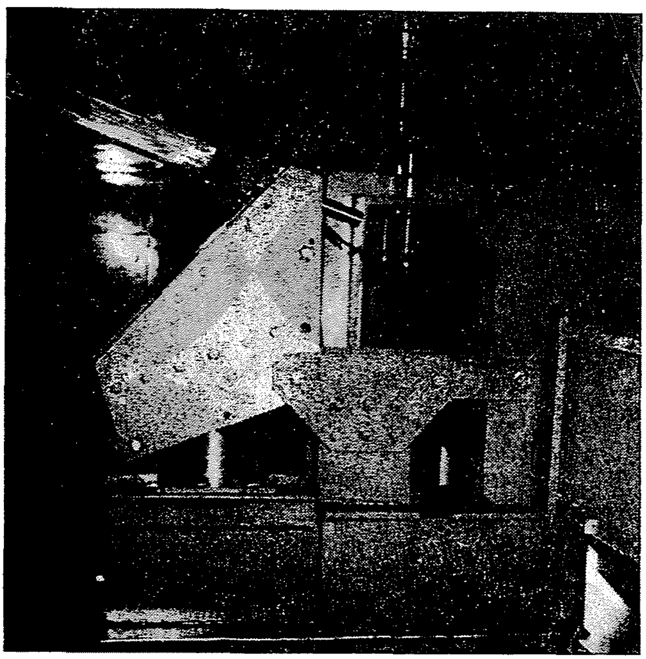


Fig. 14/b

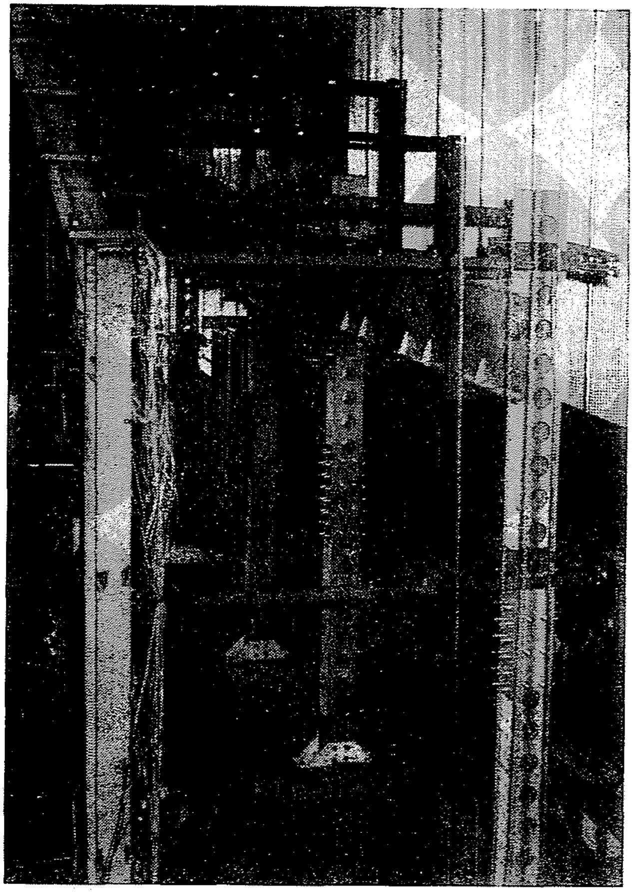


Fig. 15. Lateral supports

3. Short description of tests

3.1. *Test of frame C*-3/1 (August 16, 1977)

Lateral supports type a) were applied (joining the upper flange of rafter and outer flange of columns) at the location of planned purlins and side-rails.

Load was applied in steps, which followed each other after a 10 minutes' interval to make development of plastic deformation possible prior to taking the readings. Load-displacement diagram (vertical load vs. mid-span deflection) is demonstrated in Fig. 19. At load level No. 9, substantial lateral-rotational displacement of the inner flange of the left-side column was observed; similar, but less marked movements took place with the other column as well. Failure was due to the lateral-torsional buckling of the column, as seen in Fig. 20.

3.2. *Test of frame C-2* (January 11, 1978)

Lateral supports around the frame corners were of type c), thus giving complete lateral and torsional restraint at points indicated in Fig. 21.

Load-displacement diagram is seen in Fig. 22, demonstrating the development of a complete yield mechanism and showing a "yield-plateau" after reaching the limit load.

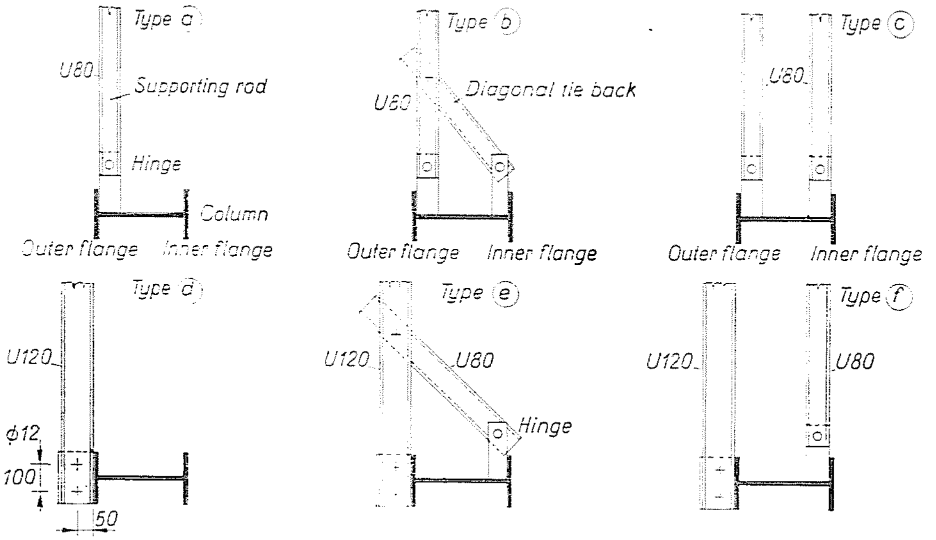


Fig. 16. Types of lateral supports

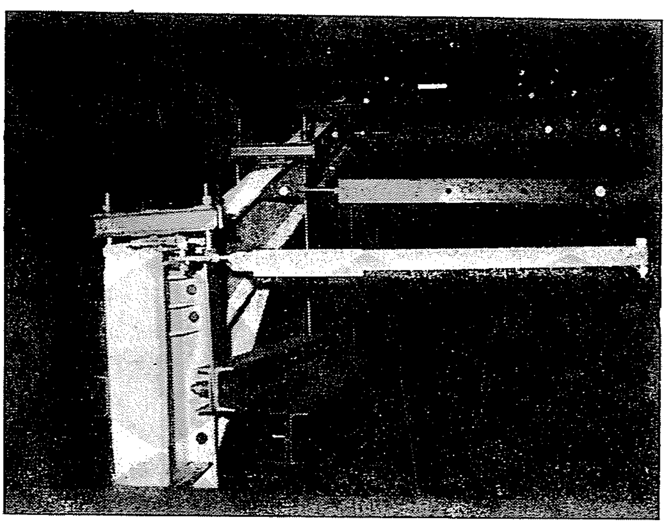


Fig. 17. Lateral support type a)

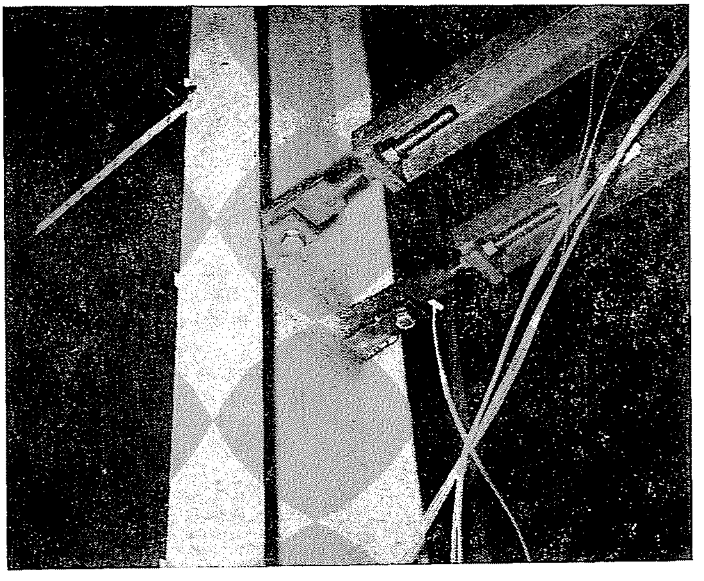


Fig. 18. Lateral support type c)

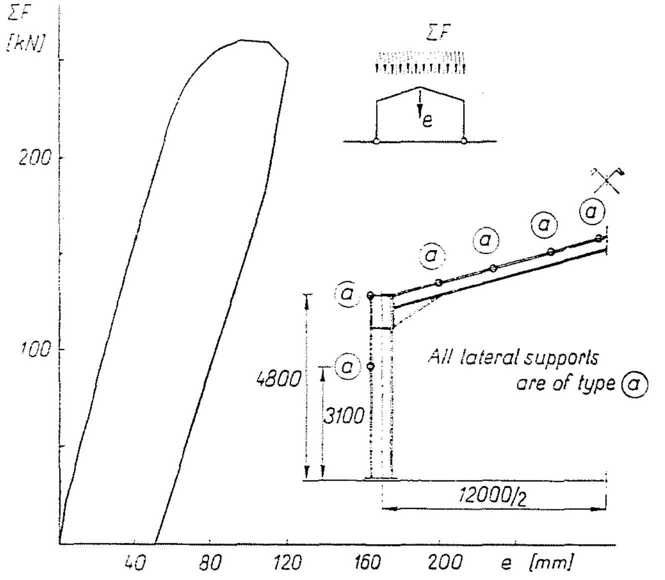


***Fig. 19.* Load-deflection diagram for test frame *C*-3/1**

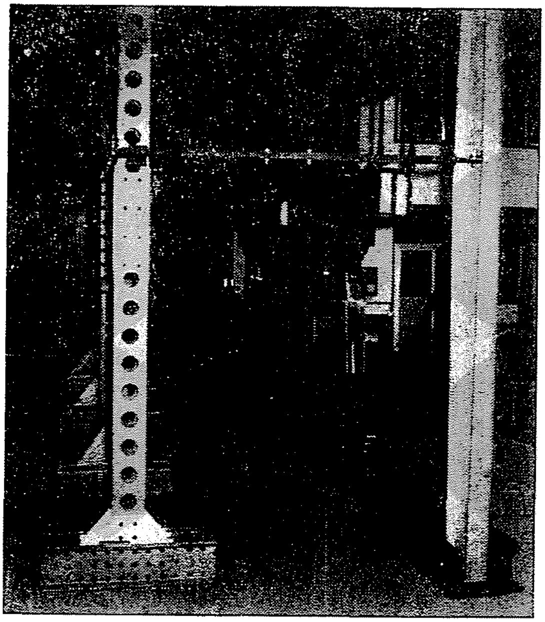


Fig. 20. Lateral torsional buckling of column of test frame C-3/1

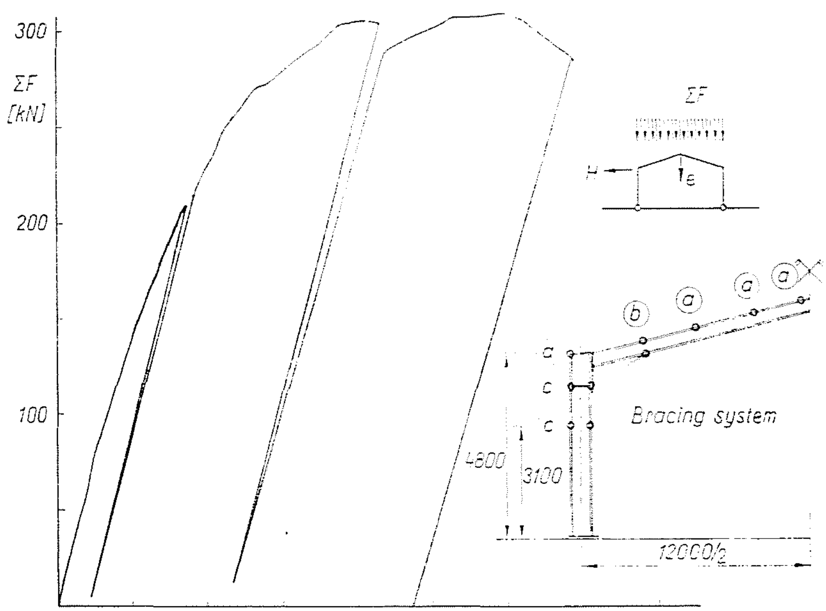


Fig. 21. Load-deflection diagram for test frame C-2

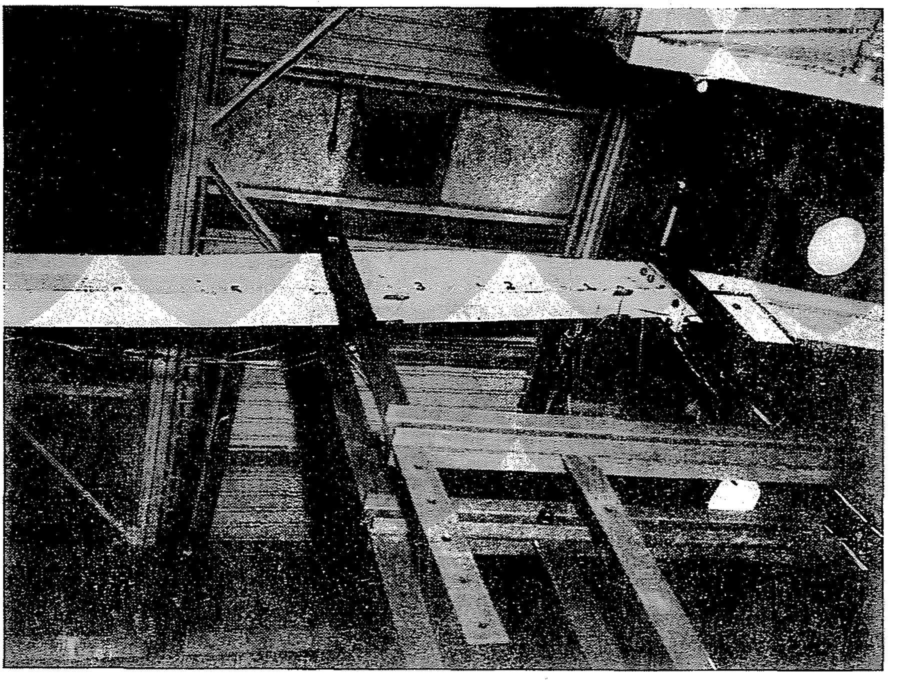


Fig. 22. Lateral buckling of rafter of test frame *C*-2

Final failure (drop in loads) was due to lateral buckling of the rafter around the mid-span, as seen in Fig. 22.

3.3. *Test with frame C*-1 (March 7, 1978)

Based on favourable results with test frame *C*-2, lateral supports were reduced. Below the haunch support type c) (giving full lateral and torsional restraint) was applied to the column; at a height of 3.1 meters a second support type d) was applied to the outer flange; the same was applied to the rafter at the end of the haunch.

Load-deflection diagram is given in Fig. 23. At the peak load, the diagram started to abruptly decrease, indicating loss of stability. Failure was due to lateral buckling of column and rafter.

3.4. *Test of frame C*-3/2 (March 15, 1978)

Effect of different types of lateral supports was investigated consecutively.

— System I. Support type d) was applied below the haunch, at a height of 3.1 meters to the outer flange of the column and at the end of the haunch to the upper flange of the rafter.

— System II. Support below the haunch was changed to type e) by adopting a diagonal tie-back. Rest of the supports corresponded to system I.
— System III. Support below the haunch was restored to type d) by removing the diagonal tie-back. The support at 3.1 meters was changed from type d) to type e). Supports of the rafter were unchanged.
— System IV. All three supports were of type e).
— System V. Support below the haunch was changed from type e) to type f), thus giving full lateral and torsional restraint at this cross section.

Load-deflection diagram is seen in Fig. 24. System of lateral supports was changed to a more effective one as soon as substantial lateral displacement

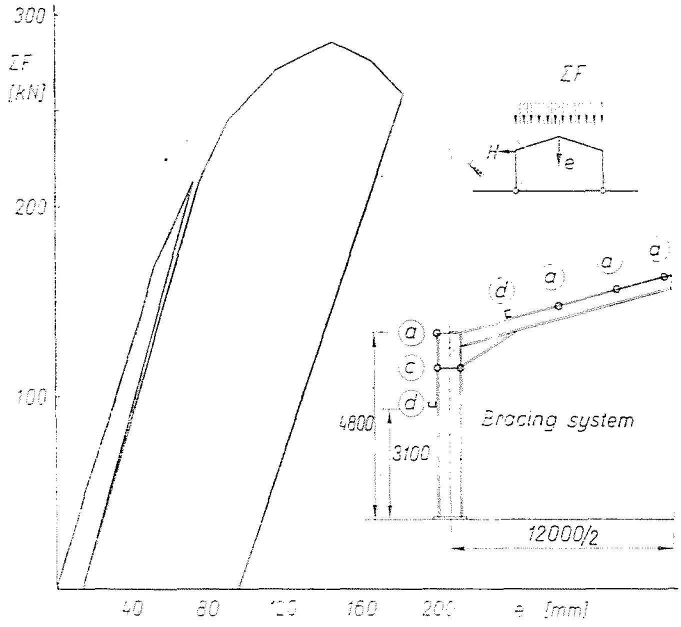


Fig. 23. Load-deflection diagram for test frame *C*-1

of the elements was observed. Tolerated value of displacement was 1/1000 of the length of elements. Larger displacement was regarded as the onset of buckling: loads were removed and the system of lateral supports was changed.

Failure was due to plate buckling in the plastic hinge below the haunch in the column (to develop the earliest) and lateral buckling around the midspan. Load-deflection diagram proves the formation of the predicted yield mechanism.

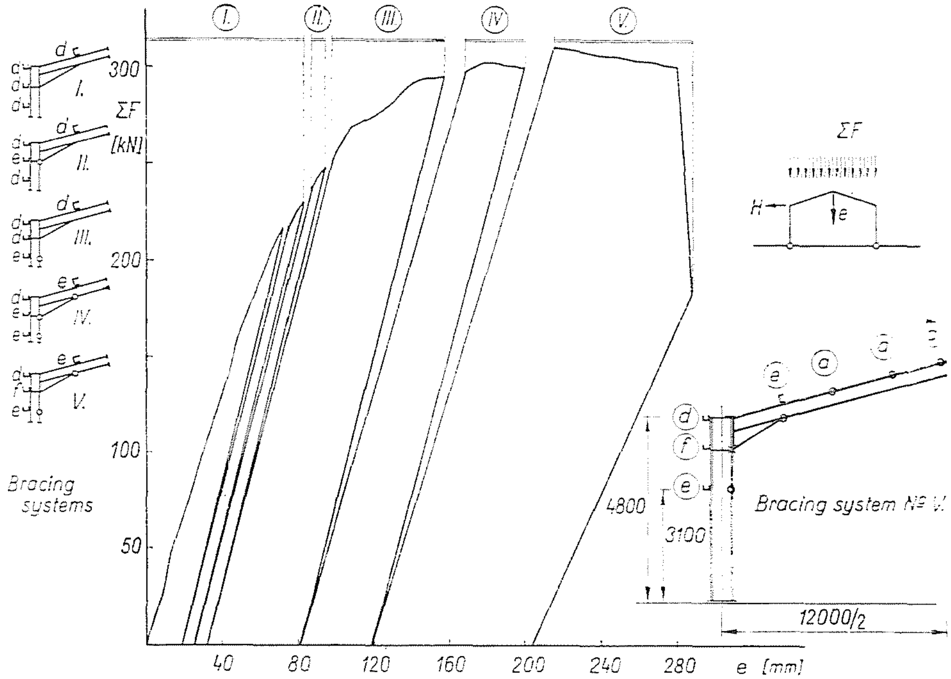


Fig. 24. Load-deflection diagram for test frame *C*-3/2

Failed structure is seen in Fig. 25, reproducing a photogrammetric picture. Local buckling in the cross section of the first plastic hinge is seen in Fig. 26.

4. Stability requirements. Experimental results

4.1. *General remarks*

Stability requirements in plastic design are well known to be more severe than those in elastic analysis. Not only instability phenomena causing reduction in ultimate moment of cross sections are to be excluded, but sufficient rotation capacity in the cross sections of supposed plastic hinges without decrease in bending moments is required, permitting also the assumed redistribution of mements and the formation of the yield mechanism.

The problem — involving lateral buckling of beams and beam-columns and plate-buckling of webs and flanges as the most important instability phenomena — may be demonstrated by Fig. 27, with ultimate moment M_p and slenderness ratio λ_i, characteristic of the case of instability, as co-ordinates. In case of a sufficiently low slenderness ratio ($\lambda_i < \lambda_i'$) critical moment may be as high as the full plastic moment M_p. Further decrease in slenderness does not add much to the ultimate moment ($\lambda_i'' \leq \lambda_i \leq \lambda_i'$), but results in increasing rotation capacity φ without drop of bending moment. Knowing the required amount of rotation, the adequate λ_i value can be determined in principle by the φ vs. λ_i diagram in the horizontal co-ordinate plane.

So stability analysis is twofold and includes the check of both the load capacity and the deformation capacity.

4.2. *Bracing requirements for continuous beams*

The relatively complicated procedure mentioned above was first developed by mostly American authors for the case of continuous beams under uniform moment and moment gradient. (Details and references see in [3] to [5].) Research resulted in simple design formulae, adopted by the specifications of different countries, among them by Recommendations of the E.C.C.S. (European Convention for Constructional Steelwork). Design rules prescribe on one

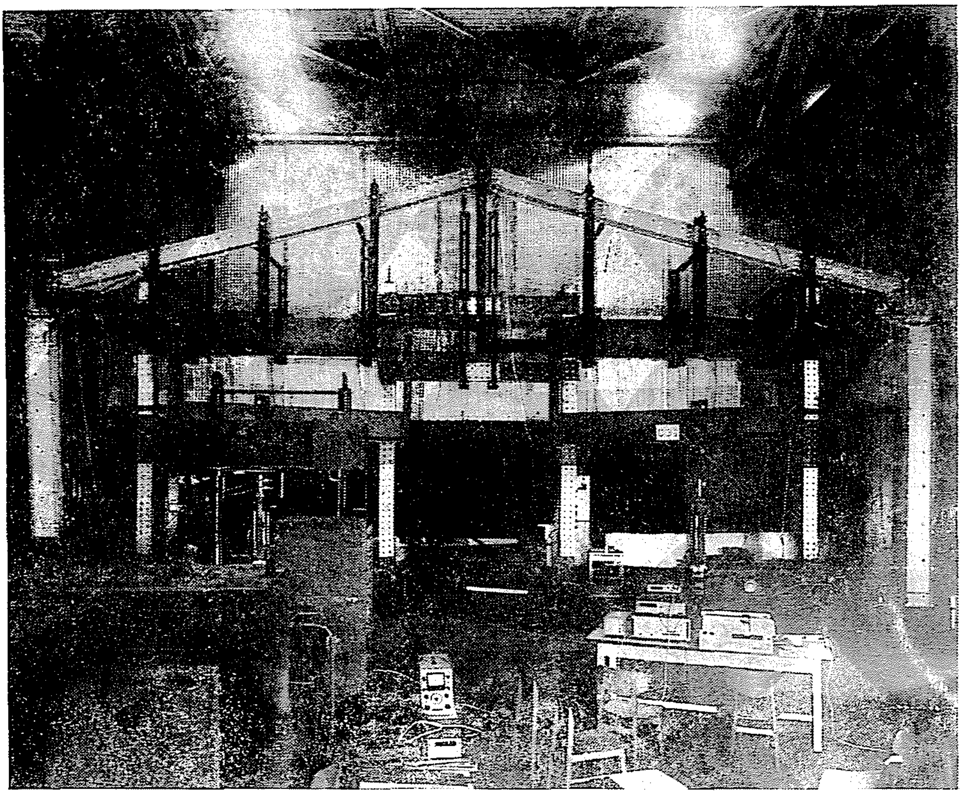


Fig. 25. Photogrammetric picture of test frame C-3/3

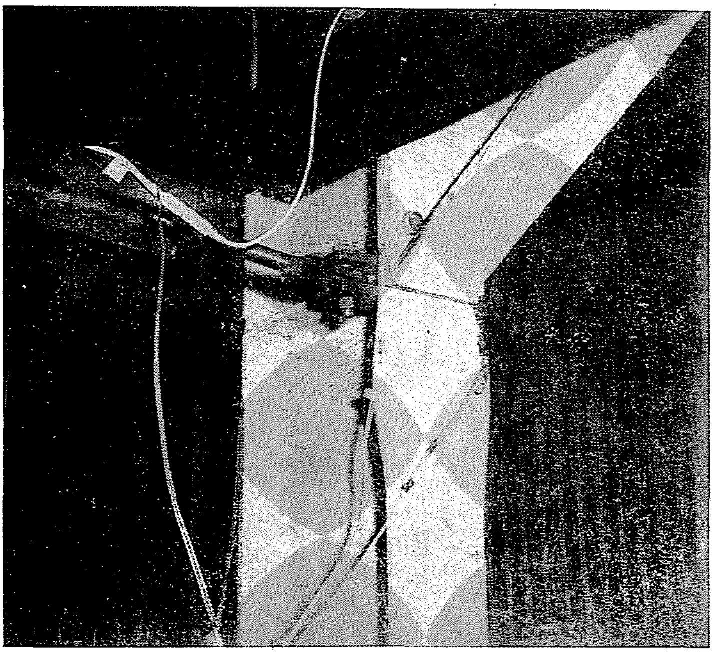


Fig. 26. Local buckling in the column cross section of test frame *C*-3

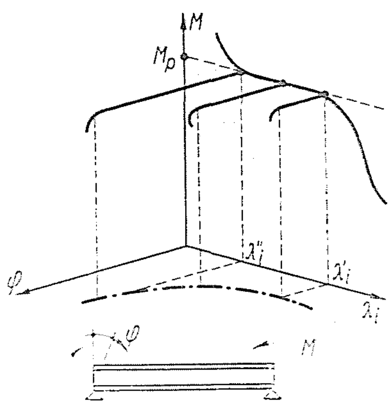


Fig. 27

hand the minimum thickness sufficient to avoid plate buckling, on the other hand bracing requirements (maximum spacing of lateral supports) in the vicinity of plastic hinges, excluding premature lateral buckling. So lateral supports are to be applied at the cross section of future plastic hinges spaced at $t = \gamma \cdot r_y$, where r_y denotes the radius of gyration around the weak axis of the cross section, factor γ depending on the shape of the moment diagram. In case

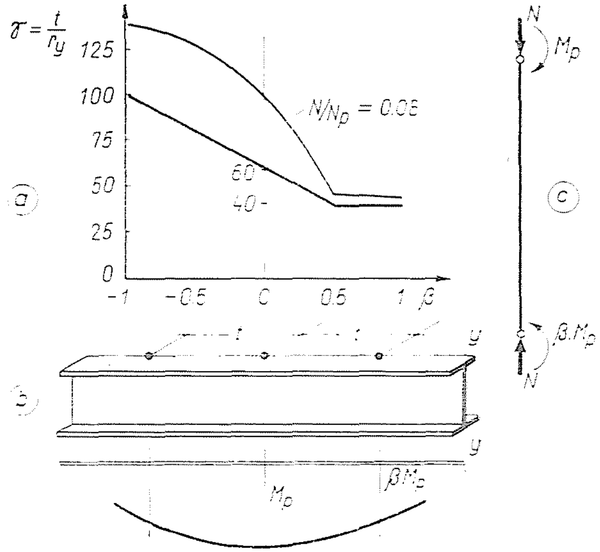


Fig. 28

of uniform or near-uniform bending moments ($\beta \geq 0.5$ in Fig. 28/b) for steel grade *St* 37 $\gamma = 40$; in case of rapid decrease of bending moments γ may increase depending on the β value according to the linear relationship traced in heavy line in Fig. 28/a. For different steel grades γ is to be multiplied by $\sqrt{\frac{\sigma_{Y,37}}{\sigma_Y}}$, $\sigma_{Y,37}$ and σ_Y denoting the yield stresses of the steel grade *St* 37 and of the beam material, respectively. These values meet both quoted requirements as proved experimentally as well ([3] to [6]).

It should be emphasized that lateral supports are effective if prevent both lateral displacement and rotation of the cross section. Supports (purlins, side-rails) joining the compression flanges (and connected to wind-bracing or similar structures) satisfy this condition (though it is desirable to give a certain torsional restaint by appropriate connection as well). Design rules also prescribe the required stiffness of bracing members [5].

Above requirements can be met in case of continuous girders and rafters of frames without major difficulty, as purlins or girders supporting the roof shell or floor-decks are usually rather closely spaced and — at least in the region of positive bending moments — join the compression flanges. In the region of negative moments the same can be achieved by simple structural means (e.g. diagonal tie-backs).

4.3. *Bracing requirements for beam-columns*

A complexer task is to give similar requirements for beam-columns. In the case of simple, one-storey frames (similar to those tested by us), lateral restraint is accomplished by side-rails, being generally flexible, located at major spacings and (in case of pin-based frames) joining the outer (tension) flange of the columns, often inefficient against rotation. Secondary elements, supporting the compression flange (as diagonal tie-backs) mean often structural difficulties or are unaesthetic and thus disliked by designers.

4.31. *Requirements in the E.C.C.S. Recommendations*

Different approaches may be found in the literature. Explanations [4] to the E.C.C.S. Recommendations give more severe requirements for beam-columns than those for beams dealt with above. This may be partly evident as in case of high axial forces lateral-torsional buckling approximates flexural buckling around the weak axis. So then — if axial forces N equal squash-load $N_p = A \cdot \sigma_Y$ (A being the cross-sectional area) in case of steel grade *St* 37 — the unsupported length must not exceed the value $t \cong 20\ r_y$, rather than $t = 40\ r_y$ given for pure bending.

4.32. *Proposals of British authors*

On the other hand other — mostly British — authors [7], [8], [9], [10], [11], [12], [13], with the structural difficulties mentioned above in mind, are giving more liberal requirements, first of all in cases with moment gradients $\beta = 0$, $\beta = -1$ encountered with pin-based or fix-based columns. Following basic ideas have been adopted [11]:

— Due to strain-hardening, actual failure load exceeds ultimate load computed by assuming elastic—perfectly plastic material.
— Thus, bracing requirements causing a 5 to 10 percent drop in computed ultimate moment can be tolerated.
— Rotation capacity may be regarded as sufficient if the M vs. φ diagram is "flat-topped", preferably if $\varphi_t \geq 3\varphi_e$ (with notations in Fig. 29), where φ_e and φ_t are measured at a reduced level, i.e. at 95 percent of the full-plastic moment M_p.

If between two supports giving full lateral and torsional restraint further lateral restraining elements (side-rails connected to the tension flange only) are applied, their beneficial effect may be taken into consideration [14], [15]. In this case two lateral-torsional buckling modes may develop (Fig. 30):

(i) between the full restraints around a restrained axis (defined by the intermediate supports),

(ii) between intermediate supports, such that the cross sections at these supports do not rotate, allowing thus to assume full torsional restraint there as well.

In case of unequal end moments (M_{max} and βM_{max} in Fig. 30) the concept of equivalent moment M_e is applied. The critical values of end moments M_{cr} and $\beta \cdot M_{cr}$ can be computed by elastic analysis, together with the value

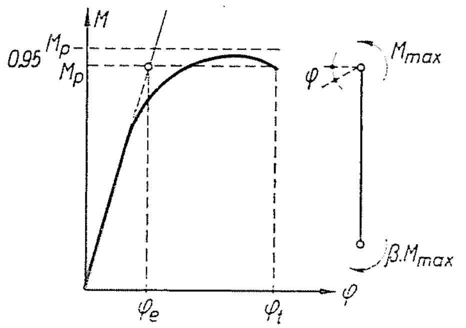


Fig. 29

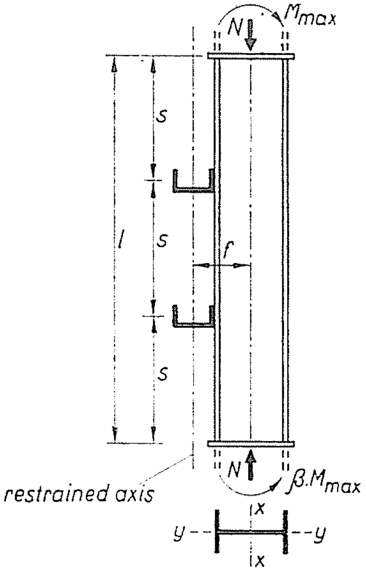


Fig. 30

$M_{e,cr}$; this latter being the critical moment of the same beam-column under uniform moment causing lateral-torsional buckling. At a sufficient accuracy [16], [17] $\mu = M_{e,cr}/M_{cr}$ can be regarded — independent of end conditions — to depend only on the ratio β and its value may be approximated by several formulae [3], [4]; most simply by

$$\mu = 0.6 + 0.4\beta; \qquad \text{if} \qquad -0.5 \leq \beta \leq 1;$$

$$\mu = 0.4; \qquad \text{if} \qquad -1 \leq \beta \leq -0.5.$$

In case of lateral-torsional buckling around a restrained axis a similar procedure can be applied [9], [15], the μ value being a function of factor α containing compression force N, cross sectional properties (as torsional rigidity GK, flexural rigidity EJ_y, radii of gyration r_x and r_y, depth D and the distance f between restrained and column axes:

$$\alpha = \frac{G \cdot K - N(r_x^2 + r_y^2 + f^2)}{\frac{\pi^2 EJ_y}{l^2}\left(\frac{D^2}{4} + f^2\right)} \cdot D^2 .$$

In this case the μ value can be approximated — based partly on data in [15], partly on an approximate analysis — by the formula

$$\mu = A(\alpha) + B(\alpha) \cdot \beta + C(\alpha) \cdot \beta^2$$

where:

$$A = 0.5 + 0.028(\alpha+1); \quad \text{if} \quad -1 \leq \alpha < 5;$$

$$A = 1 - \frac{1}{2\sqrt[4]{\alpha}} \quad \text{if} \quad 5 \leq \alpha;$$

$$B = 0.5 - 0.125\sqrt{\alpha + 1}\ ; \quad \text{if} \quad -1 \leq \alpha < 0;$$

$$B = \frac{1}{2\sqrt[3]{\alpha + 24}} \quad \text{if} \quad 0 \leq \alpha;$$

$$C = 1 - A - B.$$

The concept of equivalent moment (valid only in the elastic range) may be extrapolated for the plastic ranges. There are two possibilities:

(i) substituting the non-uniform bending moment diagram by a uniform bending moment $M_e = \mu M_{max}$ or

(ii) choosing the substituting uniform bending moment as $M_{\max}$ and reducing the actual length l of the column to an effective length $\nu \cdot l$.

In the case of column buckling (Fig. 31) $\nu = \sqrt{\mu}$, and in the elastic range both methods give the same result. In the inelastic range, however, the second method is much more conservative, applying a greater "plastic reduction" due to the higher buckling stress. In cases of column buckling usually the second method is adopted [18]; in the case of lateral buckling, literature prefers the first-mentioned, less conservative way [3] [4].

Based on the research results quoted above, graphs were prepared [7], [9] for bracing requirements, giving less severe results first of all in the case of high torsional rigidity and double-curvature bending ($\beta \leq 0$), and in the case of applying intermediate supports (side-rails) under uniform moment as well, than those in 4.2 and 4.31. For the columns of frames in our tests (having low torsional rigidity) and for the case $N/N_p = 0.08$ ($N_p = A \cdot \sigma_Y$) the values gained from these graphs are shown in thin line in Fig. 28/a. Theoretical research based on several hypotheses and approximations (as the choice of initial curvature and twist, neglecting plastic reduction in shear modulus G,

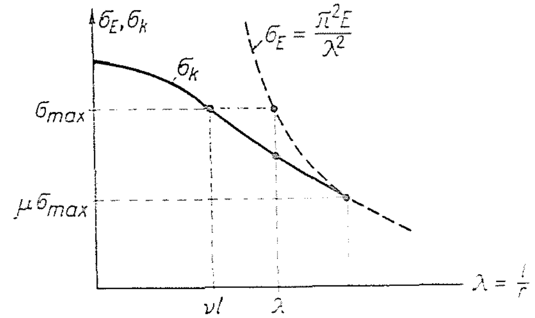


Fig. 31

etc.) required to be confirmed by experiments with beam-columns [11] which gave favourable answer at least when accepting the quoted reduced requirements for bracing.

4.4. *Test results*

4.41. *Effect of lateral buckling of beam-columns*

It seemed advisable to extend the experiments from isolated beam-columns to complete structures. Thus — from the fourteen test-frames — tests with four pin-based and four fix-based structures were conducted so as to furnish information about bracing requirements. A short summary about the tests with four pin-based frames (C-1, C-2, C-3/1, C-3/2) is given as follows.

Test frame C-3/1 — as seen in Fig. 32 — had lateral supports only at points where — from other structural reasons — purlins and side-rails were applied. (In the cross section of plastic hinges there was no lateral support.) Bracing had a pinned connection to the tension flange around the frame corner and to the compression flange around the mid-span. According to the load-deflection diagram in Fig. 32 (where e is the vertical displacement of the mid-span cross section) premature failure occurred at 85 percent of the computed simple plastic limit load, due to insufficient bracing causing lateral-torsional buckling of the columns. Twist visible for the naked eye was observed at loads as low as 70 percent of the limit load.

Lateral supports in the test with frames C-3/2 and C-2 were located — at least in the final phase — to meet the severe requirements in 4.31. Lateral

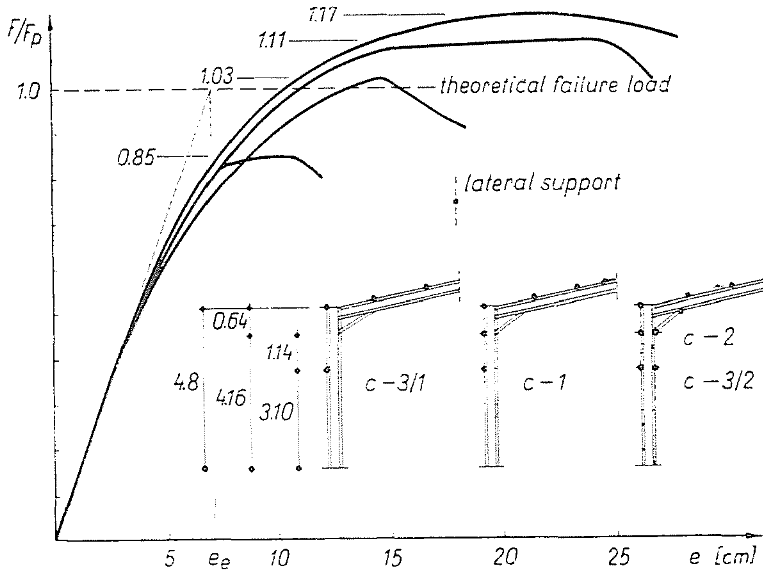


Fig. 32

support giving complete restraint against lateral displacement and rotation was applied in the cross section of plastic hinges in the columns (just below the haunches); similar restraint was given at a distance $t = 30\ r_y$; the remaining part of the column being — according to the elastic analysis — safe enough against lateral buckling. Similar principles were applied with the rafters. Failure loads of the two frames were 17 percent and 11 percent higher, resp., than the computed simple plastic limit load due to the effect of strain hardening and presumably to the non-perfect pin-base. Evidently failure was caused by unrestricted yielding (formation of yield mechanism) and decrease in load carrying capacity occurred only after plate buckling around the plastic hinge (Fig. 26). The near-horizontal branch of the diagram was long enough; the descending branch cut the horizontal line $F/F_p = 1$ (F_p being the computed limit load) at a distance $e > 4e_e$ (where e_e is the displacement of a structure supposed to behave elastically up to the limit load defined in Fig. 32).

Finally, test frame C-1 had complete restraint at the cross section of plastic hinges; other supports were connected to the tension flange only around the frame corner and in the columns. The distance measured between full restraint and column base was $t \sim 100\ r_y$, which satisfied conditions in 4.32 (see thin line in Fig. 28/a, $\beta = 0$).

Failure due to lateral-torsional buckling of column and rafter occurred at 103 per cent of computed limit load. This experiment proved that, on the one hand, requirements suggested by British authors were sufficient to reach the needed limit load, on the other hand some disadvantages were observed:

(i) column twisting visible to the naked eye started very soon (at loads as low as 50 percent of the limit load, thus at working loads),

(ii) load-deflection diagram was not "flat-topped"; decrease in load carrying capacity was soon observed which rendered the structure more sensitive to geometrical imperfections than that with more conservative bracing.

The effect of these phenomena had to be cleared by the results of the remaining part of experiments, which will be reported on in a subsequent publication.

4.42. *Effects of change in geometry*

Experiments furnished information about the correctness of the so-called modified *Rankine-Merchant* formula [4]:

$$F_f = \frac{F_p}{0.9 + \dfrac{F_p}{F_{cr}}},$$

an approximate relationship between simple plastic limit load F_p, elastic buckling load P_{cr} and the second-order failure load F_f (taking effects of change in geometry into account). Formula contains the assertion that for $F_p/F_{cr} \leq 1/10$, effects of change in geometry may be neglected.

Values in our experiments have been compiled in Table 1 (load in kN). While computing F_{cr} the broken rafter was replaced by a horizontal beam with the same span, and axial forces only in columns were regarded.

Table 1

	F_p	F_{cr}	F_p/F_{cr}	F_{exp}
C-1	276	2008	0.138	285
C-2	293	2008	0.146	325
C-3/1	306	2391	0.128	261
C-3/2	266	2391	0.111	310

Though in all cases the ratio F_p/F_{cr} exceeded 1/10, experimental failure load F_{exp} was greater than the computed (first order) limit load — except for the frame C-3/1, which failed prematurely in lateral buckling.

5. Draft of Hungarian Specifications for Plastic Design

Based partly on experiments (quoted above and to be reported on in a subsequent publication), partly on the literature, design rules concerning lateral-torsional buckling of beam-columns are summarized as follows.

In case of lateral-torsional buckling of a beam-column with uniform, bisymmetric I cross section between two complete restraints, the elastic critical moment and the axial load (M_{cr} and N_{cr}, respectively) are defined by the eigenvalue problem of the system of differential equations

$$EJ_{\omega}\theta'' - (GK - Nr_p^2)\,\theta - Mu = 0$$

$$EJ_y u'' - M\theta + Nu = 0,$$

with boundary conditions $\theta = \theta'' = u = u'' = 0$ at both ends, where EJ_y and GK denote flexural rigidity (due to the weak axis) and torsional rigidity, respectively, J_{ω} the warping modulus $J_{\omega} = \frac{D^2}{4} J_y$, D being the distance between the centers of the flanges, r_p the polar radius of gyration, u and θ the lateral displacement and rotation of cross sections, respectively.

The well-known solution of the problem [18] leads to the interaction formula

$$\left(\frac{M_{cr}}{M_E}\right)^2 - \left(1 - \frac{N_{cr}}{N_E}\right)\left(1 - \frac{N_{cr}}{N_{\omega}}\right) = 0,$$

where

$N_E = \frac{\pi^2 E J_y}{l^2}$, Euler-load of a centrally compressed column due to the weak axis buckling;

$N_{\omega} = N_E \frac{c^2}{r_p^2}$, critical load of a centrally compressed column due to torsional buckling,

$M_E = N_E \cdot c$, elastic critical moment of the column under uniform moment:

$$c = \sqrt{\frac{GK}{N_E} + \frac{J_{\omega}}{J_y}} = \sqrt{\frac{G \cdot K}{N_E} + \frac{D^2}{4}} \quad .$$

The character of the interaction formula is demonstrated in Fig. 33. Reduction due to plasticity can be taken into account by introducing the critical stress σ_{cr} of the extreme compression fibre:

$$\sigma_{cr} = \frac{N_{cr}}{A} + \frac{M_{cr}}{W_x} = \frac{N_{cr}}{A}\left(1 + \frac{d}{m}\right),$$

(A denoting the cross-sectional area, W_x the elastic section modulus, $m = W_x/A$ and $d = M_{cr}/N_{cr}$) and regarding it as the critical buckling stress of a fictitious

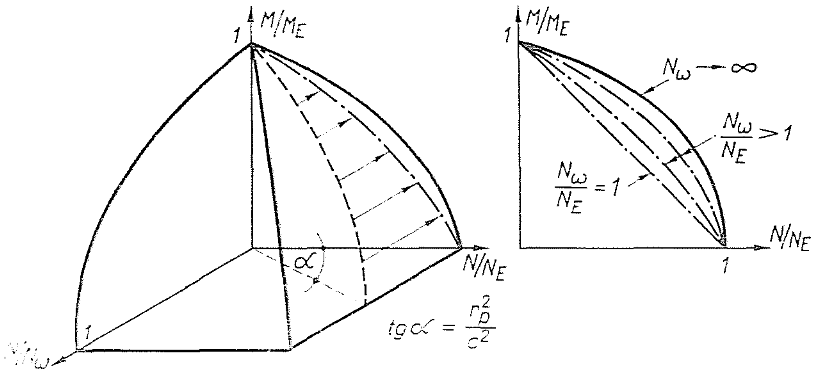


Fig. 33

column with slenderness ratio λ_i:

$$\sigma_{cr} = \frac{N_{cr}}{A}\left(1 + \frac{d}{m}\right) = \frac{\pi^2 E}{\lambda_i^2} .$$

This leads (with approximation $m \sim D/2$) to

$$\lambda_i = \lambda_y \sqrt{\frac{s_1 + \sqrt{1 + s_2^2}}{e\left(1 + \dfrac{m}{d}\right)}} ,$$

where

$$\lambda_y = \frac{l}{r_y}; \qquad H = \frac{G\,K\,A}{\pi^2\,E\,W_x^2} \cong 0{,}04\,\frac{K}{Am^2} ;$$

$$e = \sqrt{1 + H\lambda_y^2}; \quad s_{1,2} = \frac{m^2 e \pm r_p^2}{2m.d.e} .$$

Thus the problem is reduced to that of the buckling of an axially compressed column, where slenderness ratio λ_i defines the buckling coefficient $\varphi(\lambda_i) = \sigma_K/\sigma_Y$ [19] (σ_K being the limiting buckling stress) including the effect of both plastic reduction and initial curvature and twist (regarding the latter statement see [20]).

Using interaction formula for limiting values of simultaneous bending moment and axial force (Fig. 34):

$$\frac{N}{N_p} + \frac{M}{1{,}1\,M_p} \leq 1$$

$$\frac{M}{M_p} \leq 1$$

(M_p and N_p being the full-plastic bending moment and squash load, respectively), effect of lateral-torsional buckling may be expressed by reducing the right-hand side by $\varphi(\lambda_i) = \sigma_K/\sigma_Y$:

$$\frac{N}{N_p} + \frac{M}{1{,}1\,M_p} \leq \varphi(\lambda_i)\,.$$

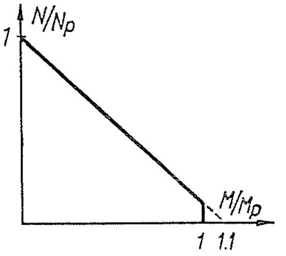


Fig. 34

Taking the effects of change in geometry by the factor

$$\psi = \frac{1}{1 - \dfrac{N}{N_{E,x}}}$$

into account ($N_{E,x}$ being the Euler-load belonging to buckling in the plane of bending moments) and — in case of moment gradient — using the concept of equivalent moment (4.32) results in the design formula:

$$\frac{N}{N_p} + \frac{\psi\mu\, M_{\max}}{1{,}1\,M_p} \leq \varphi(\lambda_i)\,.$$

Similar procedure can be applied in case of lateral-torsional buckling around a restrained axis (Fig. 30) at a distance f from the column axis. Assuming simply supported beam-column with I-section again, M_{cr} and N_{cr} are defined by the eigenvalue problem of the differential equation

$$(EJ_\omega + EJ_y f^2)\theta'' - [GK - Nr_f^2 + 2\,Mf]\,\theta = 0,$$

with end conditions $\theta = \theta'' = 0$; where $r_f^2 = r_p^2 + f^2$. This leads to the interaction formula

$$\frac{M_{cr}}{\widetilde{M}_E} + \frac{N_{cr}}{\widetilde{N}_\omega} = 1,$$

where:

$$-\widetilde{N}_{\Theta} = N_E \frac{\dfrac{J_{\Theta}}{J_y} + \dfrac{GK}{N_E} + f^2}{r_f^2},$$

and

$$-\widetilde{M}_E = \widetilde{N}_{\Theta} \cdot \frac{r_f^2}{2f}.$$

Repeating former procedure to include plastic reduction and effect of initial imperfections,

$$\lambda_i = \sqrt{\frac{2}{1+\dfrac{m}{d}} \cdot \frac{\dfrac{r_f^2}{D \cdot d} + \dfrac{2f}{D}}{1+\left(\dfrac{2f}{D}\right)^2 + H\lambda_y^2}}$$

where notations are as defined previously. In design formula

$$\frac{N}{N_p} + \frac{\psi\mu\, M_{\max}}{1{,}1\, M_p} \leq \varphi(\lambda_i)$$

factor μ — according to 4.32 — is a function of α as well:

$$\alpha = \frac{1}{1+\left(\dfrac{2f}{D}\right)^2}\left[H\lambda_y^2 - \frac{N}{N_p}\frac{r_f^2}{m^2}\left(\frac{\lambda_y}{100}\right)^2\right].$$

Above-mentioned design rules give results similar to those suggested by British authors [7] [9], with the following essential differences:

— The use of amplifying factor $\psi = \dfrac{1}{1-\dfrac{N}{N_{E,x}}}$ leads to severer requirements, and

— in the case of lateral-torsional buckling around a restrained axis and of a uniform bending moment, design rules are more conservative than those in [10].

Summary

A series of tests have been made with full-scale steel frames in preparation of the Hungarian Specifications for Plastic Design. Details of four experiments are given, with special care for the problem of lateral-torsional buckling of beam-columns. Further experiments and more detailed discussion will be described in a publication to follow.

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