DESIGN AND CRACK PREDICTION OF STEEL WIRE REINFORCED CONCRETE

By

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Received: March 15, 1977

1. General

Since long, the peculiar great difference between compressive and tensile strength of concrete (and in general, of rocks) — the latter being 5 to 20% of the former — has been attempted to equalize by incorporating steel bars, fabric, chippings, shot, wire, or fibrous non-metals such as asbestos, glass libres, blast furnace slag, mineral wool, synthetic fibres etc., or special technologies e.g. prestressing.

For instance, Monier introduced the reinforcement by steel bars or fabrics giving rise to reinforced concrete. By the turn of the century, patents have been granted and proposals presented for increasing the concrete strength characteristics by adding iron scrap, nails (Porter, 1910, Ficklin, 1914). In the early 'twenties, concrete with iron chippings had been applied in this country, and in 1950, the Hungarian engineer Rényi suggested to mix short steel wires to the concrete to improve its characteristics — a suggestion never axperimented. Various fibre reinforcements have been tested in Hungarian end foreign laboratories. In 1965, J. P. Romualdi (USA) applied for a patent on steel wire reinforced concrete already reported on by him in 1963. This patent referred to a two-phase material of concrete and straight steel wires. This new material was stated to have a tensile strength much superior to that of ordinary reinforced concrete, a low sensitivity to cracking, high deformability and flexibility, and a good resistance to thermal effects. Tests made in Hungary and abroad seemed to support this idea, but at the same time, both tests and production observations pointed to the difficulties of placing steel wire reinforced concretes, the lack of uniformity in steel wire distribution, and the inadequate orientation, eventual knotting of thinner steel wires, possibility of internal voids, inhomogeneous structure and problematic corrosion resistance.

2. Characteristics of steel wire distribution

Romualdi [1] assumed the average wire spacing S to be of generally random distribution:

$$S = \frac{13.80}{\sqrt{P_v}}\, d \,. \tag{1}$$

FEKETE [2] and SZABÓ [3] assumed the random distribution to be of tetrahedron bulk, and of dodecahedron type, respectively; such as:

$$S = \frac{22.2}{\sqrt[3]{P_v}} d \tag{2}$$

d being the steel wire diameter, and P_v the steel wire percentage by volume.

Our starting assumption is a cube model of a surface 6A considered to be of uniform $m.m$ wire distribution on each two opposite faces, and normally to these surfaces also of m_1 wires, wire gravity points being spaced at $t = \delta l$ where l is the wire length. Accordingly, spacing S:

$$S = \frac{8.86}{\sqrt{P_v}} \frac{d}{\sqrt{\delta}} \tag{3}$$

δ being ratio of projectional gravity points' spacings, with notations in Fig. 1:

$$\delta = \frac{1}{2} [\cos \alpha_1 \cos \gamma_1 (1 - 2\beta) + \cos \alpha_2 \cos \gamma_2 \tag{4}$$

where β is the overlapping ratio of parallel wires. For e.g.: $\alpha_1 = \alpha_2 = \gamma_1 =$ $= \gamma_2 = 0$

$$\delta_0 = 1 - \beta \tag{4a}$$

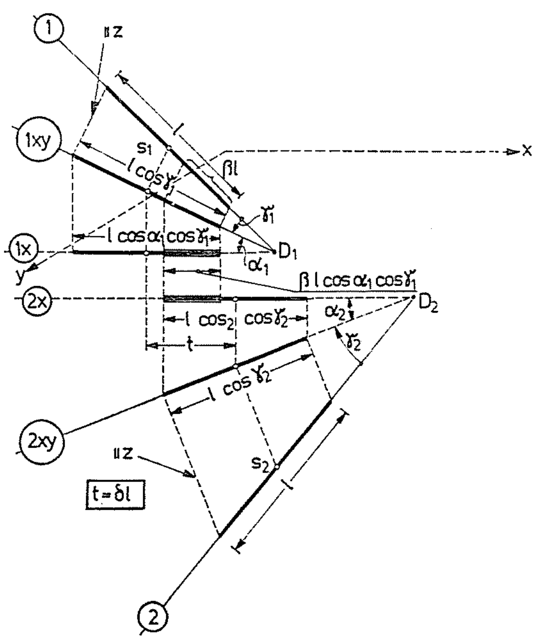

Fig. 1. Model of wire distribution and overlapping

and for $\beta = 0$ the wires are either not overlapping or continuous.

In a given plane, the wire efficiency, i.e. the characteristic of the efficient projection of the wire cross sections on that plane is:

$$\cos \varepsilon = \frac{\delta}{\delta_0} \tag{5}$$

then the effective wire cross section to be reckoned with in that plane:

$$A_v = \cos \varepsilon \; m^2 a_1 \;\; (\text{mm}^2) \tag{6}$$

where a_1 is the cross section area of a single wire.

$$a_1 = \frac{d^2\pi}{4}; \quad m = \frac{\sqrt{A}}{s}; \quad m_1 = \frac{\sqrt{A}}{\delta l}$$

and

$$m^2 = \frac{n}{m_1} = \frac{n}{\delta l}; \quad \text{or} \quad n = m^2 \; m_1 = \delta l m^2$$

Eq. (6) does not refer to two wires or two wires representing two groups of resultants but can be generalized for any m wires. The formula is easiest to apply by giving a realizable, practically statistical average for angles α and γ. Wire orientation depending on the placing method, various practical cases can be distinguished.

For wires parallel to the plane xz — one limiting case for tubes concreted by rolling —

$$\delta = \frac{1}{2}(\cos \gamma_1 + \cos \gamma_2) - \beta \cos \gamma_1 \; . \tag{7}$$

Observations show angles γ_1, γ_2 to slightly differ, and also the angle included with the plane xz to be small, 30 to 45° at most.

For $\gamma_1 = \gamma_2 = \gamma$

$$\delta = \cos \gamma (1 - \beta) \tag{7a}$$

and for $\gamma \sim 30$, 45° 0.866 $(1 - \beta)$ and 0.707 $(1 - \beta)$.

The β value depends on the size and length of wires, assuming identical dosage and proper workability, advisably assumed at least at 0.5 in case of straight, parallel wires.

The force transfer length l_v (in case of crack prediction, the likely spacing of an eventual subsequent crack) becomes [4]:

$$l_v = \frac{5d}{3c}\left(1 + \frac{1}{10\mu}\right) \tag{8}$$

where d is the reinforcement size, c the reinforcement type constant (for plain bars 1.0, for deformed bars 1.6), μ is the reinforcement percentage (A_v/A_c).

The anchorage length is about

$$l_f = \frac{d}{4}\frac{\sigma_r}{t_f} \tag{9}$$

where σ_v is the bar (wire) stress (yield point, tensile strength or ultimate stress), t_f the surface bond, function of the concrete tensile strength σ_t' and the reinforcement percentage, in general:

$$t_f = \sigma_t' \frac{3c}{2(1+10\mu)} \cdot \tag{10}$$

3. Design of the steel wire reinforced concrete cross section

In the uncracked condition of steel wire reinforced concrete, wires exhibit stresses low compared to their strength (5 to 10 times the concrete tensile stress), thus, the load capacity of cross sections (cracking moment, cracking force) can be determined from the usual ideal cross section (so-called stress state I). Calculations for a cracked cross section are based on stress state II — maybe making use of the concrete tensile strength. In general, the cracking moment of wire reinforced concretes can be stated to but slightly exceed that of plain concretes. Just as for r.c. beams, the cracking moment grows with increasing reinforcement percentage μ. Tests made abroad and in this country show no increase but for higher reinforcement percentages (over 0.4 per cent by volume).

The increase of cracking strength of steel wire reinforced concretes depends, in addition to the reinforcement percentage, on the wire space factor (l/d) and for the actual small sizes, on the bond strength also typical of the compaction method.

This is why the following formu of cracking strength σ_r has been suggested at the IIIrd International Con ess on Fracture in 1973:

$$\sigma_r = \sigma_t(1 - V_v) + 0.15 \cdot t \cdot V_v \cdot \frac{l}{d} \tag{11}$$

where: σ_t — concrete ultimate strength
t — bond stress
l/d — shape factor
V_v — wire volume.

3.1 *Design and checking wire reinforced concrete cross sections according to stress state I, by means of the ideal concrete cross section*

Ideal cross section in stress state I:

$$A_i = A_c(1 + n_t\mu). \tag{12}$$

Ideal cross section modulus:

$$K_i = K_c(1 + n_t\mu). \tag{13}$$

Flexural concrete and steel stress M in the extreme fibre σ_c and σ_v:

$$\sigma_c = \frac{M_r}{K_i}$$

$$\sigma_v = n_t\sigma_c. \tag{14}$$

Concrete bending-tensile strength σ_{bt} in the tensile extreme fibre, as characteristic:

$$n_t = \frac{E_v}{\nu_0 E_0} = \frac{n_0}{\nu_0} \tag{15}$$

$$\mu = \frac{A_v}{A_c}. \tag{16}$$

A_v is the effective wire cross section according to (6).

Conversion between percentages by weight and by volume can be made according to:

$$P_s = \frac{\varrho_s}{\varrho_c} \cdot P_v \sim \frac{10}{3} P_v \qquad (17$$

$$P_v = \frac{\varrho_c}{\varrho_s} \cdot P_s \sim 0.3 P_s \tag{18}$$

$$E_s = E_v = 2150 \text{ Mp/cm}^2 \text{ (2100 Mp/cm}^2\text{)}$$

$$n_t = \frac{E_v}{E_0 \nu_0} = \frac{n_0}{\nu_0}$$

where ν_0 is the ratio of the modulus of deformation E_x (belonging to the extreme stress in the concrete compressive zone σ_c) to E_0 and obtained from

$$\nu_0 = \frac{1}{2}\left[1 + \left(\frac{\sigma_c}{\sigma_p}\right)^{1/2}\right] \tag{19}$$

where σ_p is the bending-compressive strength of concrete, normally obtained by trial calculations.

3.2 *Design and checking the wire reinforced concrete cross section according to stress state II is advisably done taking the concrete bending-tensile strength into consideration.*

The basic stress diagram and notations are those in Fig. 2.
Starting relationships are:

$$N = N_c + N_v = \frac{1}{2}\xi ab\sigma_{c,r} + \frac{1}{2}\xi ab\mu\sigma'_{v,r}$$

$$H = H_v + H_c = \frac{1}{2}(1-\xi)\, ab\mu\sigma_{v,r} + \frac{1}{2}\alpha ab\sigma_{b,t}$$

$$\sigma'_{v,r} = n_t\sigma_{cr}; \quad N = \frac{1}{2}\xi ab\sigma_{c,r}(1 + n_t\mu)$$

$$\sigma_{v,r} = n_t\sigma_{c,r}\frac{1-\xi}{\xi}; \quad \sigma_{b,t} = \frac{\alpha}{\xi}\sigma_{c,r}; \quad \alpha = \frac{\sigma_{b,t}}{\sigma_{c,r}}\xi$$

$$H = \frac{1}{2}\frac{(1-\xi)^2}{\xi}abn_t\mu\sigma_{c,r} + \frac{1}{2}\frac{\alpha^2}{\xi}ab\,\sigma_{c,r}$$

$$H = N$$

$$\xi^2 + 2n_t\mu\xi - (n_t\mu + \alpha^2) = 0.$$

Based on principles detailed in [4], the relative neutral axis distance ξ of the cracked cross section is given by:

$$\xi = n_t\mu\left\{\left[1 + \frac{1}{n_t\mu} + \left(\frac{\alpha}{n_t\mu}\right)^2\right]^{1/2} - 1\right\}. \tag{20}$$

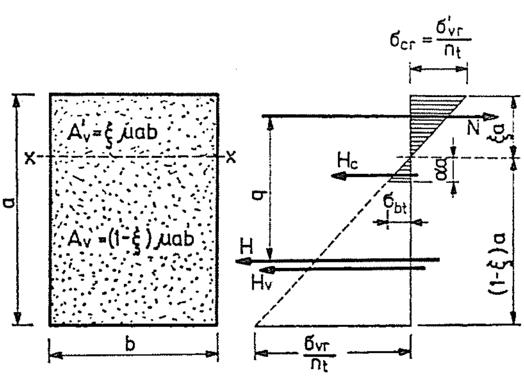

Fig. 2. Calculation of the cracked concrete cross section with wire reinforcement

equality

$$N = H = \frac{M}{(q)} \tag{21}$$

yields stresses

$$\sigma_{c,r} = \frac{N}{\frac{1}{2}\xi\, ab(1 + n_t\mu)} \tag{22}$$

$$\sigma'_{v,r} = n_t \sigma_{c,r} \tag{23}$$

$$\sigma_{v,r} = n_t \frac{1-\xi}{\xi} \sigma_{c,r} \tag{24}$$

$$H_v = H - \frac{1}{2}\alpha \, . \, ab\, \sigma_{b,t} \tag{25}$$

and

$$\sigma_{v,r} = \frac{H_v}{\frac{1}{2}(1-\xi)\, ab\, \mu} \, . \tag{26}$$

The approximate value can be taken as $0.66a$, the α value preestimated (at about 0.2ξ to $0.4\xi \sim 0.03-0.10$), applying the trial and error method.

4. Crack width calculations

Calculations are based on stress state II. Stresses from (26) yield crack width by means of the formula:

$$\Delta l = \psi \frac{\sigma_{v,r}}{2E_{v0}} l_r \tag{27}$$

where $\sigma_{v,r}$ is the ultimate wire stress in the cracked cross section, E_{v_0} is the modulus of elasticity of the steel wire, l_r is the critical force transfer length to be assumed at l_v to $2l_v$, ψ is the cross section constant, and l_v the likely distance of the possible subsequent crack, delivered by the following approximate relationships (see Chapter 8 in [4]):

$$l_v = \frac{d}{6c}(1 + 10\mu)\left(\frac{\sigma_{v,r}}{\sigma_{b,t}} - n_t\beta_t\right) \tag{28}$$

$$\beta_t \cong 0{,}73 \nu_0 \varrho^{-3/2} \tag{29}$$

$$\psi = \frac{1}{3}\left(1 + \frac{2\sigma_{b,t}}{\sigma_{v,r}} n_t\beta_t\right) \tag{30}$$

where $\sigma_{b,t}$ is the concrete bending-tensile strength and $\varrho = fc/(fc + 200)$, where fc is the concrete cube strength.

A moment M exceeding the cracking moment in the cracked cross section generates another crack at a distance l_v from the cracked cross section, and the crack width grows in conformity with the moment excess. As an approximation — since for the cracked cross section ξ is about the same as before, only stresses and rotations grow — the characteristic l_v of the crack width distribution will be multiplied by $M/M_r = k$, and the crack width by k^2, thus:

$$l'_v = kl_v \tag{31}$$

$$\Delta l'_{\max} = k^2 \Delta l_{\max} \frac{1}{\nu_v} . \tag{32}$$

The steel deformation modulus ν_v in (32), i.e. $E_{vx}/E_{v0} = \nu_v \cdot \nu_v$ depends on the steel wire stress. At the nominal yield stress σ_v in general:

$$\nu_v = \frac{1}{1 + \dfrac{4200}{\sigma_y}} . \tag{33}$$

for thin, high-tensile wires ($\sigma_y \sim 15\ 000$ kp/cm²) $\nu_v \sim 0.78$.

5. Example

As an illustration of the former, let:

$a = 12$ cm	$b = 100$ cm	
$f'_c = 400$ kp/cm²	$\sigma_p = 300$ kp/cm²	$\varrho = 0.667$
$\sigma'_t = 30$ kp/cm²	$\sigma_{b,t} = 60$ kp/cm²	$\sigma_y = 10\ 000$ kp/cm²
$E_0 = 360$ Mp/cm²	$E_s = E_v = 2150$ Mp/cm²	
$d = 0.6$ mm	$a_1 = 0.283$ mm²	
$P_v = 1.0\%$	$l = 50$ mm	
$\beta = 0.5$	$\gamma = 30°$	
$\delta = 0.433$	$\delta_0 = 0.5$	
$n_0 = 6.0$	$\cos \varepsilon = 0.866$	

$$S = \frac{8.86}{1.00} \cdot \frac{0.6}{0.433} = 8\text{mm} \quad m = \frac{1}{0.8} = 1.25\text{pc/cm}$$

$$m_1 = \frac{1}{0.433 \cdot 5.0} = 0{,}46\text{pc/cm} \quad n = 1.25^2 \cdot 0.46 = 0.72\text{pc/cm}^2$$

$$A_v = 0.866 \cdot 1.25^2 \cdot 0.283 = 0.385 \text{mm}^2$$

$$\mu\% = \frac{100 \cdot 0.385}{10 \cdot 10} = 0.385\%.$$

5.1 *Stress state I:*

$$\nu_0 = \frac{1}{2}\left[1 + \left(1 - \frac{60}{300}\right)^{1/2}\right] = 0.95; \quad n_t = \frac{6.00}{0.95} = 6.3$$

$$A_i = 12 \cdot 100\ (1 + 6.3 \cdot 0.00385) = 1229 \text{ cm}^2$$

$$K_i = 1200 \frac{12}{6} (1 + 0.024) = 2458 \text{ cm}^2$$

$$M_p = 2458 \cdot 60 = 148 \text{ Mpcm.}$$

$$\sigma_v = 6.2 \cdot 60 = 378 \text{ kp/cm}^2.$$

5.2 *Stress state II:*

$$\nu_0 \sim 0.785;\ n_t = 6/0.785 = 7.65;\ n_t\mu = 7.65 \cdot 0.00385 = 0.0294$$

$$\alpha \sim 0.045;\ \sigma_y = 10\,000 \text{ kp/cm}^2$$

$$\xi = 0.0294 \left\{\left[1 + \frac{1}{0.0294} + \left(\frac{0.045}{0.0294}\right)^2\right]^{1/2} - 1\right\} = 0.150$$

$$N \sim \frac{148\,000}{0.66 \cdot 12} = 18\,700 \text{kp}$$

$$\sigma_{c,r} = \frac{18\,700}{\frac{1}{2} 0.150 \cdot 12 \cdot 100 \cdot 1.0294} = \frac{18\,700}{92.65} = 202 \text{ kp/cm}^2$$

$$H_v = 18\,700 - \frac{1}{2} 0.045 \cdot 12 \cdot 100 \cdot 60 = 18\,700 - 1620 = 17\,080 \text{ kp}$$

$$\sigma_{v,r} = \frac{17\,080}{\frac{1}{2} 0.850 \cdot 12 \cdot 100 \cdot 0.0385} = 8700 \text{ kp/cm}^2$$

and

$$\sigma_{v,r} = 7.65 \frac{0.85}{0.15} 202 = 8700 \text{ kp/cm}^2.$$

5.3 *Crack width:*

$$\beta_t = 0.73 \cdot 0.667^{-3/2} \cdot 0.785 = 1.06; \; n_t\beta_t = 1.06 \cdot 7.65 = 8.15$$

$$\psi = \frac{1}{3}\left(1 + \frac{2 \cdot 60}{8700} \cdot 8.15\right) = 0.37$$

$$l_v = \frac{0.6}{6 \cdot 1.6} 1 + 10 \cdot 0.00385 \frac{8700}{60} - 8.15 = 8.9 \text{ mm.}$$

$$\Delta l_{\max} = 2 \cdot 0.37 \frac{8700 \cdot 1000}{2 \cdot 2150000} \cdot 8.9 = 13.4 \, \mu m$$

$$\Delta l_{\min} = 6.7 \mu m.$$

At failure, for

$$\sigma_{c,r} = \sigma_p = 300 \text{ kp/cm}^2, \quad k = \frac{300}{205} = 1.46$$

and

$$M = 1.46 \cdot 148 = 217 \text{ Mpcm}; \quad k^2 = 2.13; \quad \nu_v = 1 \Big/ \left(1 + \frac{4200}{10\,000}\right) = 0.7$$

$$\Delta l'_{\max} = 2.13 \cdot 1.34 \frac{1}{0.7} = 42.6 \, \mu m$$

$$\Delta l'_{\min} = 21.3 \, \mu m.$$

5.4 *Anchorage length:* ($\sigma_y = 10\,000$ kp/cm^2)

$$t_f = 30 \frac{3 \cdot 1.6}{2(1 + 0.0385)} = 69 \text{ kp/cm}^2 \quad l_f = \frac{0.6}{4} \frac{10\,000}{69} = 22 \text{ mm.}$$

Summary

Methods are given for designing steel wire reinforced concrete cross sections. and for predicting the crack width in concrete pipes with steel wire reinforcement. Design is based on stress states I and II, crack width on stress state II, taking the initial crack width due to the cracking moment into consideration, and observing the cracking behaviour under a force higher than the cracking one.

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* In Hungarian