THERMAL EFFECTS IN REINFORCED CONCRETE SILOS

By

Á. OROSZ

Department of Reinforced Concrete Structures,
Technical University, Budapest

(Received September 15, 1977)

1. Introduction

Analysis of thermal effects in engineering structures looks back to a few decades. Namely, resultant stresses have been underestimated, — anyhow, engineering education cared little for details — if not in structures exposed to particular thermal effects (storage of hot materials etc.).

Analysis of circular symmetric r.c. reservoirs under thermal effects has been considered in detail by KILIÁN and BALÁZS [1]. KORDINA and EIBL [2] suggested a useful, rather accurate method for calculating single circular cylinder silo cells, valid also for uneven solar heating. They stressed the effect of boundary disturbances and hinted to the loss of extension and bending stiffness of r.c. reservoir walls with cracking, seriously affecting behaviour under thermal stresses. Their statements were, however, not supported by numerical data.

In the following, an improved method will be presented for the case of homogeneous, uncracked r.c. silo walls, extended — by approximation — to the cracked condition.

Namely, building codes tolerate in silos a predetermined crack width for the sake of economy, provided it is of no risk for the durability of the silo.

By limiting the crack width under working and thermal loads, anticorrosive protection of reinforcement is provided for the useful life of the structure.

Determination of the reinforcement percentage requires, however, exact knowledge of stresses.

In this respect it is of interest to mention the work of PALOTÁS [3], the first to tackle with the analysis of shrinkage and inherent thermal stresses demonstrating the likelihood of a short-time cooling by about 15°C to exhaust the concrete extensibility, hence to cause cracking, even without external loads.

The awareness of important tensions due to outer loads in silo structures superposed by internal forces due to thermal effects leads to the conclusion that their combined effect unavoidably leads to cracking. Limitation of crack width requires to know the magnitude of these effects.

Fundamentals of the determination of thermal stresses will be surveyed on the basis of publications, followed by an approximation method fitting silos, based on our research work.

Variation of the outer temperature causes deformation in the silo wall, and the stored material exposed to expansion and contraction cycles follows the wall movements, becomes increasingly elastic upon compaction, resulting in force effects acting as tensile forces in the cell wall.

THEIMER [4] was the first to determine pressure increment due to thermal effects in circular cylindrical metal silos. His analyses of plane stress state will now be extended and generalized for:

— spatial stress state,
— short-time temperature variation,
— permanent temperature variation, taking creep of the storage material into consideration,
— circular cylindrical and rectangular reinforced concrete silos.

This method involves the following approximations:

— temperature distribution is assumed to be circular symmetric, omitting temperature differences between sunny and shaded sides;
— applied spatial stress relationships are assumed to be valid for granular material.

2. Consideration of the spatial stress state

The well-known stress–strain relationships can be written for the spatial stress state of the stored material in polar co-ordinates as: [5]

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu\sigma_\varphi - \nu\sigma_z\right] \tag{1}$$

$$\varepsilon_\varphi = \frac{1}{E}\left[\sigma_\varphi - \nu\sigma_r - \nu\sigma_z\right] \tag{2}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\sigma_r - \nu\sigma_\varphi\right]. \tag{3}$$

Symmetry requires horizontal stresses to be identical in any direction:

$$\sigma_r = \sigma_\varphi = \sigma_h.$$

The highest strain value is obtained in plane stress state, hence for $\sigma_z = 0$, exempt of the inhibition by the vertical stress–strain,

$$\varepsilon_{r,\max} = \frac{\sigma_h}{E}\left[1 - \nu\right]. \tag{4}$$

The other extreme value belongs to the hydrostatic stress state, where $\sigma_r = \sigma_\varphi = \sigma_z = \sigma_h$ i.e.:

$$\varepsilon_{r,\,\min} = \frac{\sigma_h}{E}\,[1 - 2\nu]\,.$$

The stress state hence deformation of the stored material is between both.

To determine the strain in the actual stress state, let us zero the vertical deformation of the stored material.

For $\varepsilon_z = 0$, (3) becomes:

$$0 = \frac{1}{E}\,[\sigma_z - 2\nu\sigma_h]$$

yielding the relationship between vertical and horizontal pressures:

$$\sigma_z = 2\nu\sigma_h.$$

Substitution into (1) gives:

$$\varepsilon_r = \frac{1}{E}\,[\sigma_h - \nu\sigma_h - 2\nu^2\sigma_h] = \frac{\sigma_h}{E}\,[1 - \nu(1 + 2\nu)]\,. \qquad (6)$$

The same results from (2):

$$\varepsilon_r = \varepsilon_\varphi\,.$$

Correctness of this result — at least by order of magnitude — may be checked in another way.

The relationship between vertical and horizontal stresses (pressures) borrowed from the theory of earth pressure yields the vertical stress increment (pressure) due to horizontal stress increment:

$$\sigma_z = \lambda\sigma_h\,.$$

Substituted into (1):

$$\varepsilon_r = \frac{1}{E}\,[\sigma_h - \nu\sigma_h - \nu\lambda\sigma_h] = \frac{\sigma_h}{E}\,[1 - \nu(1 + \lambda)]\,. \qquad (7)$$

Comparison with (6) shows:

$$\lambda \cong 2\nu$$

acceptable from the aspect of magnitude.

Eqs (6) and (7) yield results between extreme values from Eqs (4) and (5) hence are more realistic.

Further analyses will apply (6), on the safety side compared to (7).

Remark that THEIMER started from (4) referring to the plane stress state in determining the pressure increment, at an important difference to the detriment of safety, compared to the spatial stress state.

Hitherto the silo wall has been assumed to be horizontally displaced and the stored material to be essentially subject during contraction to a condition similar to passive earth pressure. The passive earth pressure is known to be the multiple of the active one, hence constriction by the silo wall may impose significant stresses on the material. In the suggested relationship, pressure conditions similar to the passive earth pressure may be reckoned with by the modulus of elasticity E, of a special importance to be determined and known. Unfortunately, to now, little attention has been paid to it and to the Poisson's ratio. Perspectively, however, the safety of structures will require a better knowledge of the physical characteristics of the stored material.

Introducing the symbol $\sigma_h = p_h$ for the radial strain in spatial stress state yields

$$\varepsilon_r = \frac{\sigma_h}{E}\left[1 - \nu(1 + 2\nu)\right] = \frac{p_h}{C_g}$$

where

$$C_g = \frac{E}{1 - \nu(1 + 2\nu)} \tag{8}$$

a fictitious strain coefficient, introduced to take spatiality, hence a stress state similar to passive earth pressure, into consideration.

For plane stress state, THEIMER deduced from Eq. (4) the fictitious strain coefficient

$$C_g' = \frac{E}{1 - \nu}\,. \tag{9}$$

Eqs (8) and (9) are rather different, manifest in subsequent computation results by the safety loss upon assuming plane stress state.

3. Effect of short-time cooling on a circular cylindrical cell

Determination of pressure increment due to daily temperature fluctuations or cooling for a few days (or weeks) may ignore the creep of the stored material to consider elastic properties alone.

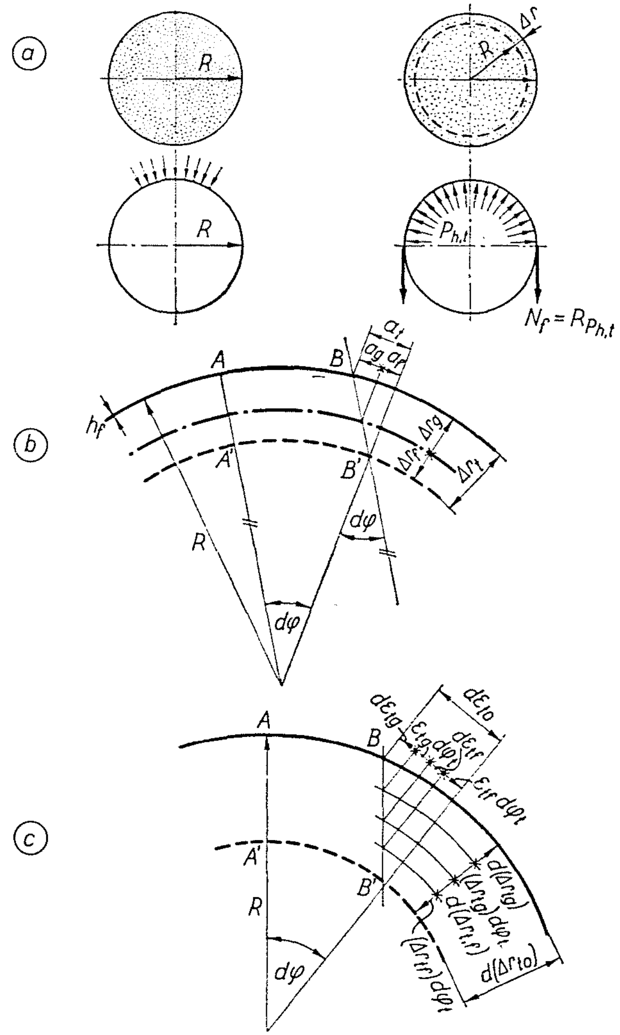

Fig. 1. **Deformations of the circular cylinder**

External cooling causes the silo wall to contract, to compress the stored material developing a pressure increment, imposing, in turn, a tension on the silo wall causing strain (Fig. 1). Conditional equation for radial deformation is, with notations in Fig. 1b:

$$\Delta r_t - \Delta r_g - \Delta r_f = 0, \tag{10}$$

where

Δr_t — radial elastic thermal deformation of the unclamped, unloaded cell wall,

Δr_g — elastic deformation due to compression of the stored material,

Δr_f — elastic deformation of the cell wall due to the force effect.

The same conditional equation can be written for the annular deformations:

$$a_t - a_g - a_f = 0\,. \tag{11}$$

Radial and annular deformations are related by:

$$\Delta r_i \,\mathrm{d}\varphi = a_i\,.$$

Annular strain and radial displacement are known to be related by:

$$\varepsilon_\varphi = \frac{\Delta r_i}{R}\,.$$

Deformations may be determined as follows:

Radial thermal *displacement* of the unclamped, unloaded *cell wall*:

$$\Delta r_t = \varepsilon_\varphi R = \alpha \Delta t_\alpha R \tag{12}$$

where

α — coefficient of thermal expansion of the cell wall;

$\Delta t_á$ — temperature variation of the wall.

Radial compression of the stored material, using Eq. (6):

$$\Delta r_g = \int_0^R \varepsilon_r \,\mathrm{d}r = \frac{p_{h0}}{C_g}\int_0^R \mathrm{d}r = \frac{p_{h0}}{C_g} R = \varepsilon_{\varphi,\,g} R\,. \tag{13}$$

Radial elastic deformation due to the cell wall force:

$$\Delta r_f = \varepsilon_{\varphi,f} R = \frac{N_{f0}}{E_f h_f} R = \frac{p_{h,0} R}{D_f} R \tag{14}$$

namely, according to the boiler formula:

$$N_{f_0} = p_{h_0} R$$

and $D_f = E_f h_f$ — extension stiffness of the cell wall;

$$\varepsilon_{\varphi,f} = \frac{p_{h0} R}{D_f}\,.$$

Substituting into the conditional equation for elastic deformations yields:

$$\alpha \Delta t_á R - \frac{p_{h0}}{C_g} R - \frac{p_{h0} R^2}{D_f} = 0$$

divided by R:

$$\alpha\, \Delta t_{\acute{a}} - \frac{p_{h0}}{C_g} - \frac{p_{h0} R}{D_f} = 0 \tag{15}$$

the conditional equation in terms of annular elastic strains:

$$\varepsilon_{\varphi,t} - \varepsilon_{\varphi,g} - \varepsilon_{\varphi,f} = 0 .$$

After arranging, the thermal pressure increment in the silo wall becomes:

$$p_{h,0} = \frac{\alpha\, \Delta t_{\acute{a}}\, D_f}{R + \dfrac{D_f}{C_g}} = T_{f,0}\,\frac{1}{R + \dfrac{D_f}{C_g}} = \frac{T_{f,0}}{R_{h,0}} \tag{16}$$

where

$T_{f,0}$ — force in the wall under perfectly inhibited deformation;

$R_{h,0}$ — radius of an imaginary substituting circular cylinder, involving the effect of compressibility of the stored material, e.g. $C_g = \infty$, i.e. a stiff material leads to the well-known boiler formula.

This relationship fits determination of short-time pressure increment under rapid cooling but omits the stored material creep under permanent load, and the resulting pressure decrease.

4. Effect of permanent cooling in a circular cylindrical cell

4.1. *Equilibrium equation of deformation*

The pressure increment due to permanent cooling causes creep in the stored material, reducing the initial pressure increment with time.

The creep of stored material will be calculated by the Dischinger theory, fitting engineering analyses. The exacter theories for creep are complexer, yet at little gain in final result, reliability due to incertainty, standard deviation of material constants.

Thus creep and elastic deformation are related by:

$$\varphi_t = \frac{\varepsilon_l}{\varepsilon_r} = \varphi_n(1 - e^{-\beta t}) . \tag{17}$$

Besides, this method of analysis involves the following assumptions:

a) The modulus of elasticity E_g of the stored material is constant in time.

b) Permanent cooling is separated from daily and short-time thermal variations.

c) Permanent cooling is considered as an average during several weeks or months from the time of filling.

d) Pressure increment causing the cıeep is pıoportional to cooling and grows from zero to its final value. The cooling strain exhibits a variation affine to the creep function:

$$\frac{\varepsilon_t}{\varepsilon_{t,n}} = \frac{\varphi_t}{\varphi_n}$$

an approximate assumption questionable especially in the initial condition.

Nevertheless, over prolonged periods and concerning the final result, it is acceptable.

e) Creep functions of cell wall and stored material are considered to be identical.

Annular strains due to the timely variation of pressure increment under permanent cooling follow the conditional equation: (Fig. 1c)

$$\frac{d\varepsilon_{t0}}{dt} - \varepsilon_{t,f}\frac{d\varphi_t}{dt} - \frac{d\varepsilon_{t,f}}{dt} - \varepsilon_{t,g}\frac{d\varphi_t}{dt} - \frac{d\varepsilon_{t,g}}{dt} = 0 \tag{18}$$

where

$\frac{d\varepsilon_{t,0}}{dt} = \frac{\alpha\, \Delta t_{\dot{a}}}{\varphi_n}\frac{d\varphi_t}{dt}$ — cooling strain with creep up to a time $t + dt$,

$\varepsilon_{t,f}\frac{d\varphi_t}{dt} = \frac{p_{h,t}\, R}{D_f}\frac{d\varphi_t}{dt}$ — strain due to cell wall tension up to a time $t + dt$,

$\frac{d\varepsilon_{t,f}}{dt} = \frac{dp_{h,t}\, R}{dt\, D_f}$ — cell wall extension during a time dt,

$\frac{\varepsilon_{t,g}}{dt} = \frac{p_{h,t}}{C_g}\frac{d\varphi_t}{dt}$ — storage material strain at a time $t + dt$,

$\frac{d\varepsilon_{t,g}}{dt} = \frac{1}{C_g}\frac{dp_{h\cdot t}}{dt}$ — storage material strain during a time dt.

4.2. *Creep of the stored material*

Compared to the creep of the stored material, that of the cell wall is practically negligible. The relevant conditional equation can be written (for $\varepsilon_{t,f} = 0$) as:

$$d\varepsilon_{t,0} - d\varepsilon_{t,f} - \varepsilon_{t,g}\, d\varphi_t - d\varepsilon_{t,g} = 0$$

i.e.:

$$\frac{\alpha \Delta t_{\acute{a}}}{\varphi_n} \mathrm{d}\varphi_t - \mathrm{d}p_{h,t} \frac{R}{D_f} - p_{h,t} \mathrm{d}\varphi_t \frac{1}{C_g} - \mathrm{d}p_{h,t} \frac{1}{C_g} = 0,$$

arranged and multiplied by $C_g/\mathrm{d}\varphi_t$:

$$\frac{\mathrm{d}p_{h,t}}{\mathrm{d}\varphi_t}\left(1 + \frac{RC_g}{D_f}\right) + p_{h,t} - \frac{\alpha \Delta t_{\acute{a}} C_g}{\varphi_n} = 0. \tag{20}$$

Introducing notation

$$\beta_g = \frac{1}{1 + \frac{RC_g}{D_f}} = \frac{\frac{D_f}{C_g}}{\frac{D_f}{C_g} + R} = \frac{\frac{D_f}{C_g}}{R_{h,0}}$$

the differential equation becomes:

$$\frac{\mathrm{d}p_{h,t}}{\mathrm{d}\varphi_t} + \beta_g p_{h,t} - \beta_g \frac{\alpha \Delta t_{\acute{a}} C_g}{\varphi_n} = 0. \tag{21}$$

Its solution

$$p_{h,t} = \frac{\alpha \Delta t_{\acute{a}} C_g}{\varphi_n}(1 - e^{-\beta_g \varphi_t}), \tag{22}$$

is simplified by notation $\alpha \Delta t_{\acute{a}} D_f = T_{f,0}$ into the pressure increment:

$$p_{h,t} = \frac{T_{f,0}}{\varphi_n \frac{D_f}{C_g}}(1 - e^{-\beta_g \varphi_t}). \tag{22a}$$

4.3. *Creep of the cell wall*

For the sake of completeness, let us consider the case where the creep of the stored material is unimportant and that of the cell wall counts, a case possible for plastic silos liable to important creep though less subject to thermal effects than are metal silos.

Now, the conditional equation becomes ($\varepsilon_{t,g} = 0$):

$$\mathrm{d}\varepsilon_{t,0} - \varepsilon_{t,f} \mathrm{d}\varphi_t - \mathrm{d}\varepsilon_{t,f} - \mathrm{d}\varepsilon_{t,g} = 0 \tag{23}$$

i.e.:

$$\frac{\alpha \Delta t_{\acute{a}}}{\varphi_n} \mathrm{d}\varphi_t - \mathrm{d}p_{h,t} \frac{R}{D_f} - p_{h,t} \mathrm{d}\varphi_t \frac{R}{D_f} - \mathrm{d}p_{h,t} \frac{1}{C_g} = 0$$

multiplied by $\frac{D_f}{R}\,\mathrm{d}\varphi_t$ and denoting $\beta_f = \dfrac{R}{R + \dfrac{D_f}{C_g}}$ yields the differential equation:

$$\frac{\mathrm{d}p_{h,t}}{\mathrm{d}\varphi_t} + \beta_t\, p_{h,t} - \beta_f \frac{\alpha\, \Delta t_a\, D_f}{R\varphi_n} = 0 \tag{24}$$

with the solution for the pressure increment:

$$p_{h,t} = \frac{\alpha\, \Delta t_a\, D_f}{R\varphi_n}\left(1 - e^{-\beta_f \varphi_t}\right) = \frac{T_{f,0}}{R\varphi_n}\left(1 - e^{-\beta_f \varphi_t}\right). \tag{25}$$

This formula is of little practical importance, since creep of the stored material much exceeds that of the cell wall .

5. Effect of short-time cooling in rectangular silo cells

The analysis method presented for cylindrical silo cells can be extended to rectangular cells. Corner cells of a silo block of rectangular cells are in the worst position because of the simultaneous cooling and shortening of two outer walls.

CHINILIN, Y. Y. [6] developed a method based on beams on an elastic foundation for computing the pressure increment in rectangular corner cells, though ignoring the creep of storage material.

A sufficient approximation is given by the analysis of the broken-line frame skeleton clamped both ends in Fig. 2.

Determination of the pressure increment involves the following assumptions and approximations:

— Only the mean temperature decrease is examined, hence only the wall midline temperature variation is reckoned with.

— Pressure increment due to stored material compression is proportional to the displacement normal to the wall surface, with the approximation that the pressure increment variation replacing the real wall deformation is considered to be linear (Fig. 2b).

According to Fig. 2, the conditional equation for the deformations:

$$a_{t,1} - a_{f,1} - a_{g,1} = 0 \tag{26}$$

where:

$a_{t,1} = \alpha\, \Delta t_a\, l_1$ — contraction of wall 1 around an empty cell;

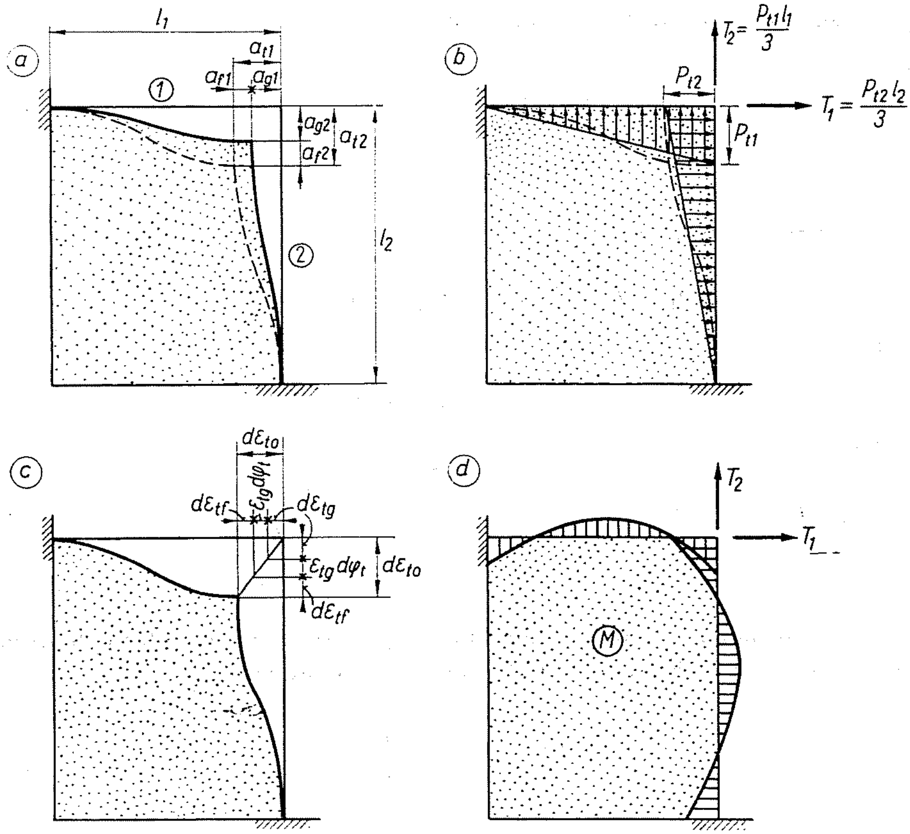

Fig. 2. **Deformations of a rectangular cell wall**

$a_{f,1} = \frac{T_1}{D_{f,1}} = \frac{p_{t,2} l_2 l_1}{3 D_{f,1}}$ — extension of wall 1 due to tension from pressure increment affecting wall 2;

$a_{g,1} = \frac{p_{t,2}}{C_g} l_1$ — compression of the stored material due to pressure increment.

Accordingly, the conditional equation becomes:

$$\alpha \Delta t_{\acute{a}} l_1 = p_{t,2} \frac{l_2 l_1}{3 D_{f,1}} + p_{t,2} \frac{l_1}{C_g} \tag{27}$$

yielding, after arrangement, the pressure increment affecting wall 2:

$$p_{t,2} = \alpha \Delta t_{\acute{a}} D_{f,1} \frac{1}{\frac{l_2}{3} \frac{D_{f,1}}{C_g}} . \tag{28}$$

By analogy, pressure increment affecting wall 1:

$$p_{t,1} = \alpha \Delta t_{\acute{a}} D_{f,2} \frac{1}{\frac{l_1}{3} + \frac{D_{f,2}}{C_g}} \,. \tag{29}$$

For a cell square in plan:

$$l_1 = l_2 = l \quad \text{and} \quad D_{f,1} = D_{f,2} = D_f$$

then:

$$p_{t,1} = p_{t,2} = p_{t,0}$$

that is:

$$p_{t,0} = \alpha \Delta t_{\acute{a}} D_f \frac{1}{\frac{l}{3} + \frac{D_f}{C_g}} = T_{f,0} \frac{1}{\frac{l}{3} + \frac{D_f}{C_g}} = \frac{T_{f,0}}{R_{h,0}} \,. \tag{30}$$

These formulae are of the same built-up as those for circular cylindrical shells, excepted for the radius of the imaginary cylinder:

$$R_{h,0} = \frac{l}{3} + \frac{D_f}{C_g} \tag{31}$$

that is, the greatest pressure increment equals the pressure increment in a circular cylinder of radius $R = \frac{l}{3}$.

6. Effect of permanent cooling in a rectangular cell

6.1. *Consideration of the stored material creep*

Conditional equation describing the timely variation of the pressure excess due to permanent cooling can be written according to Fig. 2b to take the creep of stored material into consideration as:

$$\mathrm{d}\varepsilon_{t,0} - \mathrm{d}\varepsilon_{t,f} - \varepsilon_{t,g} \mathrm{d}\varphi_t - \mathrm{d}\varepsilon_{t,g} = 0 \tag{32}$$

or detailed:

$$\frac{\alpha \Delta t_{\acute{a}}}{\varphi_n} \mathrm{d}\varphi_t - \mathrm{d}p_{h,t} \frac{l}{3D_f} - \frac{p_{h,t} \mathrm{d}\varphi_t}{C_g} - \frac{\mathrm{d}p_{h,t}}{C_g} = 0 \,.$$

Arranged and multiplied throughout by $C_g/\mathrm{d}\varphi_t$:

$$\frac{\mathrm{d}p_{h,t}}{\mathrm{d}\varphi_t}\left(1+\frac{lC_g}{3D_f}\right)+p_{h,t}-\frac{\alpha\,\Delta t_{\dot{a}}\,C_g}{\varphi_n}=0\,.$$

Introducing notation

$$\beta_g=\frac{1}{1+\dfrac{lC_g}{3D_f}}=\frac{\dfrac{D_f}{C_g}}{\dfrac{D_f}{C_g}+\dfrac{l}{3}}=\frac{\dfrac{D_f}{C_g}}{R_{h,0}}$$

yields the differential equation:

$$\frac{\mathrm{d}p_{h,t}}{\mathrm{d}\varphi_t}+\beta_g p_{h,t}-\beta_g\frac{\alpha\,\Delta t_{\dot{a}}\,C_g}{\varphi_n}=0 \tag{33}$$

solved to deliver the pressure increment:

$$p_{h,t}=\frac{\alpha\,\Delta t_{\dot{a}}\,C_g}{\varphi_n}\left(1-e^{-\beta_g\varphi_t}\right). \tag{34}$$

Denoting

$$T_{f,0}=\alpha\,\Delta t_{\dot{a}}\,D_f$$

yields:

$$p_{h,t}=\frac{T_{f,0}}{\varphi_n\dfrac{D_f}{C_g}}\left(1-\mathrm{e}^{-\beta_g\varphi_t}\right). \tag{34a}$$

The differential equation and its solution are the same as for the circular cylinder but constants are different.

In knowledge of the pressure increment, the resulting tensile force T and the bending moment can be determined (Fig. 2c), to be added to stresses computed from lateral pressure due to storage.

Numerical examples demonstrate stresses due to pressure increment in the uncracked cell wall to be about 30% of stresses due to lateral pressure during storage in case of cooling by as little as $\Delta t_{\dot{a}}=10$°C. Even cracked cell walls exhibit about 20% stress increment, thus, the pressure increment is important and by no means negligible.

7. Singular rectangular silo cells

For the rectangular silo cell in Fig. 2c assuming *uniform* pressure distribution, wall extensions are obviously expressed by

$$a_{f,1} = \frac{p_{t,2}\, l_2 l_1}{2D_{f,1}}$$

and

$$a_{f,2} = \frac{p_{t,1}\, l_1 l_1}{2D_{f,2}} .$$

7.1. *Short-time cooling*

Pressure increments in terms of deformational equations are:

$$p_{t,2} = \alpha\, \Delta t_{\acute{a}}\, D_{f,1} \frac{1}{\frac{l_2}{2} + \frac{D_{f,1}}{C_g}} = \frac{T_{f,1}}{R_{h,1}} \tag{35}$$

and

$$p_{t,1} = \alpha\, \Delta t_{\acute{a}}\, D_{f,2} \frac{1}{\frac{l_1}{2} + \frac{D_{f,2}}{C_g}} = \frac{T_{f,2}}{R_{h,2}} . \tag{36}$$

Radii of substituting circular cylinders are represented by those of inscribed circles:

$$R_{h,1} = \frac{l_2}{2} + \frac{D_{f,1}}{C_g}$$

or

$$R_{h,2} = \frac{l_1}{2} + \frac{D_{f,2}}{C_g} .$$

For a square cell: $l_1 = l_2 = l$ and $D_{f,1} = D_{f,2} = D_f$

$$R_{h,0} = \frac{l}{2} + \frac{D_f}{C_g} .$$

7.2. *Permanent cooling*

The relevant differential equation differs by its constants from that for the corner cell. The formula for a square cell:

$$\beta_g = \frac{\dfrac{D_f}{C_g}}{\dfrac{l}{2} + \dfrac{D_f}{C_g}} = \frac{\dfrac{D_f}{C_g}}{R_{h,0}} \tag{38}$$

fundamental equations being the same.

8. Approximation with the substituting modulus of elasticity

The items above have presented the exact calculation of the pressure increment in cooling lasting some weeks, taking the stored material creep into consideration.

Beside the presumably exact method involving differential equations, also the substituting modulus of elasticity may lead to useful results. Now, the substituting modulus of elasticity of the stored material is given by the well-known formula:

$$E_{g,l} = \frac{E_g}{1 + a\varphi_n}$$

coefficient a indicating time dependence of the creep effect.

According to practical calculations,

$$a \cong \frac{2}{3}$$

yields a fair approximation.

Accordingly, the substituting modulus of elasticity of the stored material, taking creep into consideration, is given by:

$$C_{g,l} = \frac{E_{g,l}}{1 - \nu_g(1 + 2\nu_g)} \, . \tag{40}$$

Thereby relationships valid for a short-time thermal effect may be applied for a time $t = \infty$ involving the substituting moduli of elasticity.

9. Numerical examples, pressure increment due to silo wall cooling

9.1. *Metal silo* (Example by THEIMER [4])

Circular cylindrical cereal bin

Data: $R = 1355$ cm; $h_f = 0.952$ cm

$E_f = 2.1 \times 10^6$ kp/cm²; $\alpha = 1.2 \times 10^{-5}$

A Stored material: cereal

$E_g = 700$ kp/cm² $\nu_g = 0.4$

The DIN lateral pressure: $p_h = 4350$ kp/m²

a) *Case of short-time, abrupt cooling*:

$$\Delta t_{\acute{a}} = 30°\text{C}.$$

1) Application of the original Theimer formula for plane stress state

Auxiliary magnitudes:

$$C'_{g0} = \frac{E_g}{1 - \nu_g} = \frac{700}{1 - 0.4} = 1167 \text{ kp/cm}^2;$$

$$D_f = E_f h_f = 2.1 \times 10^6 \cdot 0.952 = 1.9992 \times 10^6 \text{ kp/cm};$$

$$\frac{D_f}{C'_{g0}} = \frac{1.9992 \times 10^6}{11167} = 1713 \text{ cm};$$

$$T_{n,0} = \alpha \Delta t_a D_f = 1.2 \times 10^{-5} \times 30 \times 1.9992 \times 10^5 = 720 \text{ kp/cm}.$$

Pressure increment:

$$p_{h,0} = \frac{T_{n,0}}{R + \dfrac{D_f}{C'_{g0}}} = \frac{720}{1355 + 1713} = 2346 \text{ kp/m}^2.$$

(The original Theimer example indicates 2445 kp/m², obviously a misprint.) Thus, the pressure increment is 54% of the silo pressure.

2) Consideration of the spatial stress state (formula 8)

Auxiliary magnitudes:

$$C_{g0} = \frac{E_g}{1 - \nu(1 + 2\nu)} = \frac{700}{1 - 0.72} = 2500 \text{ kp/cm}^2;$$

$$\frac{D_f}{C_{g0}} = \frac{1.9992 \times 10^6}{2500} = 800 \text{ cm}.$$

Pressure increment:

$$p_{h,0} = \frac{720}{1355 + 800} = 3340 \text{ kp/cm}^2 .$$

Accordingly, the pressure increment may be as high as 77% of the silo pressure.

The relationship for spatial stress state yields a value much higher than does the assumption of a plane stress state.

Thus, short-time, abrupt cooling causes an excessive pressure increment in metal silos, likely to exhaust the safety and to induce failure of the silo wall.

b) *Case of permanent cooling*

Be $\Delta t_{\dot{a}} = 30°C$ (as for the short-time cooling).

1) Exact calculation. Consideration of the creep of stored material in spatial stress state by the differential equation

Auxiliary magnitudes:

$$C_{g0} = 2500 \text{ kp/cm}^2 : \quad \varphi_n = 3 ; \quad \frac{D_f}{C_{g0}} = 800 \text{ cm}$$

$$\beta_g = \frac{\dfrac{D_f}{C_{g0}}}{\dfrac{D_f}{C_{g0}} + R} = \frac{800}{800 + 1355} = 0.3712 .$$

Pressure increment:

$$p_{h,t} = \frac{\alpha \, \Delta t_{\dot{a}} \, C_g}{\varphi_n} (1 - e^{-\beta_g \varphi_n}) = \frac{1.2 \times 10^{-5} \times 30 \times 2500}{3} \cdot 0.672 = 2016 \text{ kp/m}^2 .$$

This is the full initial value, hence 60% of the pressure increment for abrupt cooling (3340 kp/m²). Thus, pressure increment is much lower in creep.

Remark that assumption of $\Delta t_{\dot{a}} = 20°C$ for permanent cooling would be more realistic, resulting in a pressure increment:

$$p_{h,t} = \frac{20}{30} \times 2016 = 1344 \text{ kp/m}^2$$

31% of the silo pressure and acting for a long time.

2) Approximate calculation. Application of a relationship derived by introducing the substituting modulus of elasticity in spatial stress state

Auxiliary magnitudes: $\varphi_n = 3$; $\Delta t_{\dot{a}} = 20°C$

$$C_g l = \frac{C_{g0}}{1 + \frac{2}{3}\varphi_n} = \frac{2500}{1 + \frac{2}{3}3} = 833.3\ \text{kp/cm}^2;$$

$$\frac{D_f}{C_g l} = \frac{1.9992 \times 10^6}{833.3} = 2400\ \text{cm};$$

$$T_{n,0} = \alpha\, \Delta t_{\dot{a}}\, D_f = 1.2 \times 10^{-5} \times 20 \times 1.9992 \times 10^{-5} = 480\ \text{kp/cm}.$$

Pressure increment:

$$p_{h,0} = \frac{T_{n,0}}{R + \frac{D_f}{C_g l}} = \frac{480}{1355 + 2400} = 1278\ \text{kp/m}^2.$$

Still acceptable deviation by -5.6% from the exact value (1344 kp/m²).

9.2. *Reinforced concrete circular cylindrical silo*

Data: $R = 375$ cm; $h_f = 18$ cm
Materials: concrete B 280
reinforcement B.60.40.

$E_{b0} = 200.000\ \text{kp/cm}^2$

$E_{bl} = 110.000\ \text{kp/cm}^2$

$\alpha = 1.10^{-5}$

$E_a = 2{,}100.000\ \text{kp/cm}^2$

$\mu_{\text{opt}} = 4 \times 0.2 = 0.8\%$

$$f_{a,\text{opt}} = 0.8 \frac{F_b}{100} = 0.8 \frac{18}{100} = 0.142\ \text{cm}^2/\text{cm}.$$

Wall thickness equivalent to the reinforcement:

$$h_{a,\min} = 0.072\ \text{cm}.$$

Stored material: cereal — $E_g = 400\ \text{kp/cm}^2$, $\nu_g = 0.4$;

$$C_{g0} = \frac{E_g}{1 - \nu_g(1 + 2\nu_g)} = \frac{400}{1 - 0.4(1 + 2 \times 0.4)} = 1429\ \text{kp/cm}^2.$$

Extension rigidities:

before cracking: $D_{f0} = E_{b0}h_f = 2 \times 10^{-5} \times 18 = 36 \times 10^5$ kp/cm

after cracking: $D_{fr} = E_a h_a\, 1/\psi_a = 2.1 \times 10^6 \times 0.142 = 3.02 \times 10^5$ kp/cm (assuming $\psi_a \cong 1$).

The DIN lateral pressure value:

$$p_h = 3530 \text{ kp/m}^2.$$

a) *Case of short-time, abrupt cooling*

Auxiliary magnitudes: $\Delta t_{\dot{a}} = 15°\text{C}$

before cracking:

$$\frac{D_{f0}}{C_{g0}} = \frac{36 \times 10^5}{1429} = 2519 \text{ cm};$$

$$T_{n0,0} = \alpha\, \Delta t_{\dot{a}} D_{f0} = 1 \times 10^{-5} \times 15 \times 36 \times 10^5 = 540 \text{ kp/cm}$$

after cracking:

$$\frac{D_{fr}}{C_{g0}} = \frac{1.51 \times 10^5}{1429} = 105.7 \text{ cm}$$

$$T_{n0,r} = \alpha \Delta t_{\dot{a}} D_{fr} = 1 \times 10^{-5} \times 15 \times 3.02 \times 10 = 45.3 \text{ kp/cm}$$

Pressure increment:

before cracking:

$$p_{h,0} = \frac{T_{n0,0}}{R + \dfrac{D_{f0}}{C_{g0}}} = \frac{540}{375 + 2519} = 1866 \text{ kp/m}^2$$

55% of lateral pressure,

after cracking:

$$p_{h,0,r} = \frac{T_{n0,r}}{R + \dfrac{D_{fr}}{C_{g0}}} = \frac{45.30}{375 + 211.4} = \frac{45.30}{486.4} = 756 \text{ kp/m}^2$$

21.4% of lateral pressure.

Thus, in uncracked condition there is an important pressure increment entraining cracking. In cracked condition there is a pressure increment of about 20%, not to be neglected.

b) *Case of permanent cooling*

$$\Delta t_{\acute{a}} = 15°\mathrm{C}$$

1) Exact calculation by a differential equation, taking the storage material creep and the spatial stress state into consideration

1.1 Uncracked condition $\varphi_n = 3$

Auxiliary magnitudes:

$$D_{f0} = E_{b0} h_f = 2.0 \times 10^5 \times 18 = 36 \times 10^5 \ \mathrm{kp/cm^2};$$

$$\frac{D_{f0}}{C_{g0}} = \frac{36.0 \times 10^5}{1429} = 2520 \ \mathrm{cm};$$

$$\beta_g = \frac{\dfrac{D_{f0}}{C_{g0}}}{\dfrac{D_{f0}}{C_{g0}} + R} = \frac{2520}{2520 + 375} = 0.870 .$$

Pressure increment:

$$p_{h,t} = \frac{\alpha \, \Delta t_{\acute{a}} \, C_{g0}}{\varphi_n} (1 - e^{-\beta_n \varphi_n}) = \frac{1 \times 10^{-5} \times 15 \times 1429}{3} \cdot 0.927 = 663 \ \mathrm{kp/m^2},$$

18.8% of silo pressure.

1.2 Cracked condition $\varphi_n = 3$

Auxiliary magnitudes:

$$D_{fr} = 3.02 \times 10^{-5}, \quad \frac{D^0_{fr}}{C_{g0}} = 211.4 \ \mathrm{cm}.$$

In cracked condition:

$$\beta_g = \frac{211.4}{375 + 211.4} = 0.36.$$

Pressure increment:

$$p^0_{h,t} = 715 = 0.6604 = 472 \ \mathrm{kp/m^2},$$

13.4% of silo pressure.

In the uncracked r.c. circular cylindrical silo permanent cooling causes a pressure increment of about 20%, and even in the cracked one, about 13%.

2) Approximate calculation
Substituting modulus of elasticity:

$$C_{gl} = \frac{C_{g0}}{1 + \frac{2}{3}\varphi_n} = \frac{1429}{3} = 476.3 \text{ kp/cm}^2 .$$

2.1 Uncracked condition

$$\frac{D_{f0}}{C_{gl}} = \frac{36 \times 10^5}{476.3} = 7558 \text{ cm},$$

$$T_{n0,\,0} = 540 \text{ kp/cm} .$$

Pressure increment:

$$p_{h.\,t} = \frac{T_{n0,\,0}}{R + \frac{D_{f0}}{C_{gl}}} = \frac{540}{375 + 7558} = 681 \text{ kp/cm}^2 ,$$

deviation by +2.8% from the exact value (663 kp/m²).

2.2 Cracked condition

$$\frac{D_{fr}}{C_{gl}} = \frac{3.02 \times 10^5}{476.3} = 634 \text{ cm}$$

$$T_{n0,\,r} = 45.3 \text{ kp/cm}$$

Pressure increment:

$$p_{h,\,t} = \frac{45.3}{375 + 634} = 450 \text{ kp/m}^2$$

deviation by −5% from the exact value (472 kp/m²).

9.3. *Rectangular r.c. corner cell*

Data: $l = 300$ cm $\quad h_f = 15$ cm

Materials: concrete: B 280

reinforcement: B.60.40

$E_{b0} = 200.000$ kp/cm^2;

$E_{bl} = 110.000$ kp/cm^2;

$\alpha = 1.10^{-5}$;

$E_a = 2.1 \times 10^6$ kp/cm^2;

$\mu_{\text{opt}} = 4 \times 0.2 = 0.8\%$;

$$f_a = 0.8 \frac{Fb}{100} = 0.8 \frac{15}{100} = 0.12 \text{ cm}^2/\text{cm};$$

Stored material: cereal — $E_g = 200$ kp/cm^2; $\nu_g = 0.4$;

$$C_{g0} = \frac{E_g}{1 - \nu_g(1 + 2\nu_g)} = \frac{200}{0.28} = 714.3 \text{ kp/cm}^2.$$

Extension stiffnesses:

before cracking:

$$D_{f0} = E_{b0} \quad h_f = 2 \times 10^5 \times 15 = 30 \times 10^5 \text{ kp/cm},$$

after cracking:

$$D_{fr} = E_a h_a \frac{1}{\psi_a} = 2.1 \times 10^6 \times 12 \times 10^{-5} = 2.52 \times 10^5 \text{ kp/cm}.$$

The DIN lateral pressure value during storage:

$$p_{h,t} = \gamma \frac{F}{K\mu_t} = 800 \frac{9}{12.0,414} = 1450 \text{ kp/m}^2.$$

a) *Short-time abrupt cooling*

$$\Delta t_a = 10°\text{C}$$

Auxiliary magnitudes:

Uncracked:

$$\frac{D_{f0}}{C_{g0}} = \frac{30 \times 10^5}{714.3} = 4200 \text{ cm};$$

$$T_{f.o} = \alpha \, \Delta t_{\acute{a}} \, D_{f0} = 10^{-5} \times 10 \times 30 \times 10^5 = 300 \text{ kp/cm}.$$

Cracked:

$$\frac{D_{fr}}{C_{g0}} = 352.8 \text{ cm}; \quad T_{fr} = 25.2 \text{ kp/cm}.$$

Pressure increments:

Uncracked:

$$p_{h,t} = T_{fo} \frac{1}{\frac{l}{3} + \frac{D_{f0}}{C_{g0}}} = 300 \frac{1}{\frac{300}{3} + 4200} = 700 \text{ kp/m}^2,$$

48.3% of the lateral pressure during storage.

Cracked:

$$p^0_{h,t} \frac{25.2}{100 + 352.8} = 556 \text{ kp/m}^2,$$

38.3% of the lateral pressure during storage.

b) *Case of permanent cooling*

$$\Delta t_{\dot{a}} = 10°\text{C}$$

1) Exact calculation by a differential equation, taking the stored material creep and the spatial stress state into consideration

1.1 Uncracked condition:

$$C_g = 714.3 \text{ kp/cm}^2; \quad \varphi_n = 3$$

$$\frac{D_{f0}}{C_{g0}} = \frac{30}{714.3} = 4200 \text{ cm}$$

$$\beta_g = \frac{\frac{D_{f0}}{C_{g0}}}{\frac{l}{3} + \frac{D_{f0}}{C_{g0}}} = \frac{4200}{100 + 4200} = 0.976.$$

Pressure increment:

$$p_{h,t} = T_\infty \frac{1}{\varphi_n \frac{D_{f0}}{C_{go}}} (1 - e^{-\beta_g \varphi_t}) = 300 \frac{1}{3 \times 4200} 0.947 = 225 \text{ kp/m}^2,$$

15.5% of lateral pressure.

1.2 Cracked condition:

$$\frac{D_{fr}}{C_{g0}} = \frac{2.52 \times 10^5}{714.3} = 352.8 \text{ cm}$$

$$\beta_g = \frac{353}{100 + 353} = 0.779.$$

Pressure increment:

$$p_{h,t} = \frac{25.2}{3 \times 352.8} 0.904 = 215 \text{ kp/m}^2$$

14.8% of the silo pressure.

2) Approximate calculation

Substituting modulus of elasticity:

$$C_{gl} = \frac{C_{g0}}{1 + \frac{2}{3}\varphi_n} = \frac{714.3}{1 + \frac{2}{3} 3} = 238 \text{ kp/cm}^2.$$

2.1 Uncracked condition:

$$\frac{D_{f0}}{C_{gl}} = \frac{30 \times 10^5}{238} = 12{,}600 \text{ cm}.$$

Pressure increment:

$$p_{h,t} = T_\infty \frac{1}{\frac{l}{3} \frac{D_{f0}}{C_{gl}}} = \frac{300}{100 + 12{,}600} = 236 \text{ kp/m}^2$$

deviation by +4.9% from the exact value.

2.2 Cracked condition:

$$\frac{D_{fr}}{C_{gl}} = \frac{2.52 \times 10^5}{238} = 1058 \text{ cm}.$$

Pressure increment:

$$p^0_{h,t} = \frac{25.2}{100 + 1058} = 218 \text{ kp/m}^2$$

deviation by 1.1% from the exact value (215 kp/m²).

9.4. *Circular cylindrical silo made of a synthetic material*

Silo wall of glass fibre reinforced polyester

Modulus of elasticity:	$E_f = 5 \times 10^5$ kp/cm²
Coefficient of thermal expansion:	$\alpha = 2 \times 10^{-5}$
	$\Delta t_á = 10°C$
Radius of the circular cylinder:	$R = 600$ cm
Substituting wall thickness:	$h_f = 1.0$ cm
Stored material: cereal	$\gamma = 800$ kp/m²; $\varphi = 30°$
	$E_g = 400$ kp/cm²
	$\nu = 0{,}4.$

Substituting modulus of elasticity:

$$C_{g0} = \frac{E}{1 - \nu(1 + 2\nu)} = \frac{400}{0.28} = 1429 \text{ kp/cm}^2 .$$

Auxiliary magnitudes:

$$\frac{D_{f0}}{C_{g0}} = \frac{E_f \cdot h_f}{C_{g0}} = \frac{5 \times 10^5 \times 1}{1429} = 350.0 \text{ cm}$$

$$R_{h0} = 600 + 350 = 950 \text{ cm} .$$

Pressure increment:

$$p_{h,0} = \frac{\alpha \, \Delta t_á \, D_{f,0}}{R + \dfrac{D_{f0}}{C_{g0}}} = \frac{T_{f,0}}{R_{h,0}} = \frac{100}{950} = 0.1053 \text{ kp/cm}^2 = 1053 \text{ kp/m}^2 .$$

The DIN silo pressure during storage:

$$p_h = \gamma \frac{F}{K\mu_t} = \gamma \frac{R}{2\mu_t} = 0.8 \times 3 \frac{1}{0.414} = 5.790 \text{ kp/m}^2 .$$

Considering the possible heating to 40 to 60°C of fibre reinforced polyester in summer because of the high heat absorption of glass, the thermal difference may be estimated at

$$\Delta t_á = 30 \div 40°C ,$$

pressure increments being:

$$3160 \text{ kp/m}^2 \text{ and } 4220 \text{ kp/m}^2$$

54.5% and 72.6%, respectively, of the storage pressure.

This load increment much reduces the safety.

10. Evaluation

Decrease of ambient temperature is accompanied by cooling of the adjoining structures, walls. Contraction of the wall causes pressure increment in the storage bin, accessible to the derived relationships. The presented calculation method is valid both for short-time and long-time cooling. In permanent cooling, stored material creep due to pressure increment may be taken into consideration. This method lends itself for structures either with uncracked or with cracked r.c. walls. Numerical examples have been presented for estimating the order of magnitude of pressure increments.

Theoretical considerations and analysis of numerical examples lead to the following conclusions:

1) Short-time, abrupt cooling of silo walls causes a pressure increment in the stored material too high to be neglected.

2) Pressure increment has to be calculated by taking spatial stress state in the stored material into consideration, at an about 40% excess related to the plane stress state.

3) Pressure increment due to abrupt cooling in *metal* and *plastic* silos may be as high as 70% of the lateral pressure in the stored material, at an important loss of safety, likely to induce failure in adverse cases.

4) Even permanent cooling may produce about 30% pressure increment in *metal* silos, imposing to be reckoned with.

5) Pressure increment values for *r.c. silos* significantly differ between uncracked and cracked walls.

6) Pressure increment due to abrupt cooling in uncracked *r.c. silos* may be as high as 30 to 50% of the lateral pressure, likely to much decrease after cracking but still amounting to 15–20% of storage pressure.

7) Pressure increment during permanent cooling of uncracked *r.c. silos* is 15 to 20%, and after cracking, 10 to 15% of the lateral pressure.

The expected 15 to 50% pressure increment in uncracked silo walls upon cooling explains for months after filling in the stored material to pass until the first, visible vertical cracks appear upon a short-time or permanent external thermal effect.

Lateral pressure of the stored material, combined with thermal effect, may be high enough to produce cracks.

Summary

Determination of pressure increment due to outdoor temperature decrease has to reckon with spatial stress state of the stored material. Relationships are given for short-time cooling of circular cylindrical and rectangular silo cells. Differential equation yields permanent cooling, taking the creep of the stored material into account. There is a possibility to take the cracked condition into consideration. In addition to the exact method, an approximate method is presented, with numerical examples showing 40 to 70% pressure increments in metal silos, and 10 to 30% in r.c. silos, to occur upon cooling.

References

1. Kilián, J.—Balázs, Gy.: Inherent Forces in Circular Symmetrical Rotational Bins Due to Thermal Gradients.* Mélyépítéstudományi Szemle, 2. 1962. 76—82.
2. Kordina—Eibl: Zur Frage der Temperatur-Beanspruchung von kreiszylindrischen Stahlbetonsilos. Beton- u. Stahlbetonbau, H. 1. 1964. 1—11.
3. Palotás, L.: Inherent Thermal Stresses in Concrete.* Mélyépítéstudományi Szemle, 8. 1970. 333—338.
4. Theimer, O.: Bersten von Stahlsilos bei tiefen Temperaturen. Der Bauingenieur, 42, Nr. 3. März 1961.
5. Hartmann, F.: Die Berechnung des Ruhedruckes in kohesionslosen Böden bei waagerechter Oberfläche. Die Bautechnik, 11. Nr. 4. 1967.
6. Чинилин, Ю. Ю.: Расчёт стен квадратных силосов на одностороннее температурное воздействие. Иссл. и расчёт элеваторных сооружений. ст. 108—113. МОСКВА изд. КОЛОС. 1976.

Prof. Dr. Árpád Orosz, H-1521 Budapest

* In Hungarian.