STIFFENING SYSTEM OF MULTI-STOREY BUILDINGS BY THE CONTINUUM MODEL

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1. Introduction

Multi-storey buildings are popular and fashionable structures in our time, but their design raises special problems. Beside structural, heating, ventilating and building physical questions, design of the load bearing structure is subject to certain specific viewpoints.

The higher the building, the more the wind load prevails over loads by wind gusts affecting the building not only statically, but also dynamically. In addition to wind forces, also seizmic forces act as horizontal loads.

Whereas the vertical forces increase linearly with the number of storeys, moments caused by horizontal forces increase quadratically with height. One of the main structural problems for multi-storey buildings is the substantial increase of the effect of horizontal forces.

Horizontal loads result in deformations not to be neglected, manifest above all in deflection of the entire building. The increase of horizontal displacement with height still exceeds that of moments, namely in proportion to the fourth power of height. Above a certain degree this displacement or better sidesway due to wind gusts is uncomfortable for occupants and may even impair the stability of less elastic structures — e.g. panel constructions. Therefore certain specifications set a limit to the rate of displacement.

Thus for multi-storey buildings an adequately designed stiffening system is needed to take up horizontal loads and to limit deformations. These recent-type structural systems still demand much research work and improved analysis methods.

2. Stiffening structures of multi-storey buildings

A structure of adequate rigidity for supporting horizontal loads may be developed — depending on the architecture and on the chosen building system — with framework, shear walls, wind bracing (for instance in steel skeleton buildings), with a box-like inner wall structure (core) or with a grid or Vierendeel-system box structure, placed peripherally in the façade planes.

With increasing number of storeys, it is more and more difficult, and above a certain number of storeys, technically impossible to have wind loads taken up only by frames. Stiffness of medium-rise skeleton buildings can be successfully increased by designing floor plans involving the lift shafts and staircase walls in the stiffening system.

With a higher number of storeys, shear walls are not only designed at places following from the floor plan, but certain frameworks are replaced by walls with or without perforations explicitly to stiffen the building.

With further increase of the number of storeys, still more shear walls are needed to result, instead of the framework, in a structural system purely of longitudinal and transverse walls, exemplified by buildings with cast in-situ walls or of precast large panels.

The stiffening system of the building consists of the spatial structure developed of the above mentioned stiffening elements in the vertical plane and of floor diaphragms in the horizontal plane (Fig. 1).

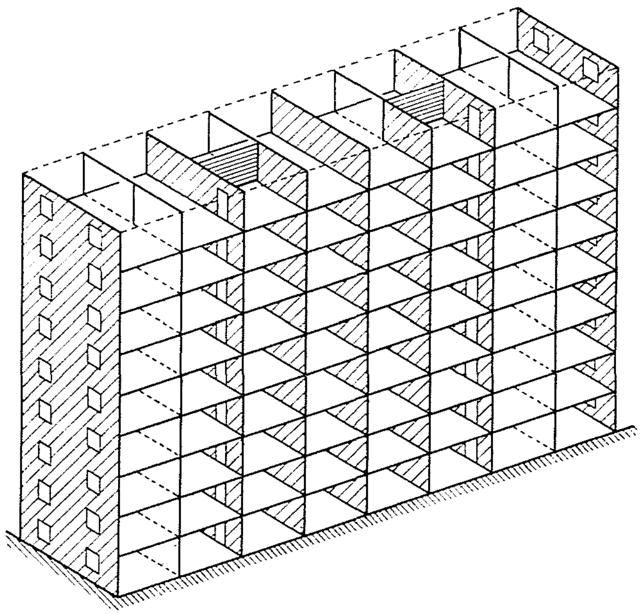

Fig. 1

In this spatial structure, floors act as load distributors, i.e. they function as deep beams in the horizontal plane, therefore it is advisable to design the floors as rigid diaphragms. Advantageous properties of the applied building technology cause the cast in-situ reinforced concrete floors to be extremely suitable to distribute horizontal loads. The joints between panels and precast reinforced concrete floors have to be designed so as to be efficient diaphragms.

Some typical alternatives for stiffening structures in the vertical plane are shown in Fig. 2.

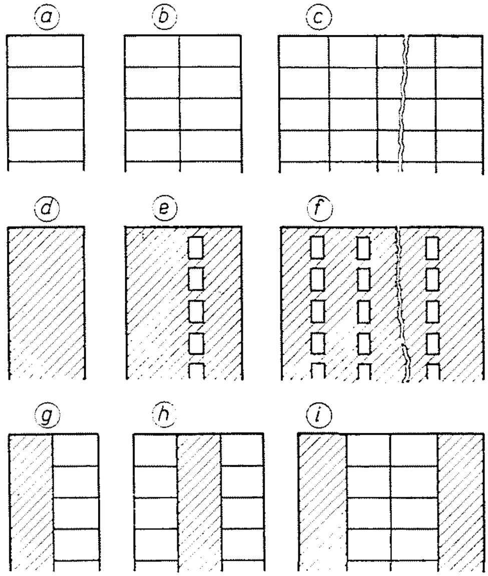

Fig. 2

3. Design assumptions

Recently published analyses for stiffening systems of multi-storey buildings are to be chosen of, according to the structural system to be developed and the accuracy required.

In this theme the best known authors are *Rosman*, *Drozdov*, *Kositzin*, *Sigalov*, *Horáček*, etc.

All of these analyses are more or less of an approximative character, depending on the assumptions and on what kind of structural model is applied.

In research work of new analyses in general, the scope is to follow the effect of loads as exactly as possible by means of a possibly simple calculation. These two opposite requirements are rather difficult to satisfy by traditional manual calculation means, in most cases a computer has to have recourse to.

As a result of research done to exactly determine the horizontal loads, stresses and deformations in the stiffening system, the author developed a method of analysis, involving the following assumptions of the theory of elasticity;

— wind pressure and suction due to wind load are transmitted from the façade planes to the stiffening system in the plane of floors;
— floor diaphragms are infinitely rigid in their plane, normal to their plane they are, however, perfectly flexible;
— geometry and material characteristics of stiffening elements in the vertical plane are constant throughout the elevation;

— torsion rigidity of each stiffening element is negligible compared to the bending rigidity;
— in the analysis, the stiffening system is substituted by a continuum model;
— determination of deformations of the continuum model of the stiffening system involves, beside the bending moment, also effects of normal and shearing forces, as well as elastic deformations of the foundation.

The first six items contain the usual assumptions, whereas the last item offers the possibility of an increased accuracy in following the real forces of the structure.

4. Construction of the continuum model

In analyses of multi-storey building frameworks it is advisable to determine internal forces of the vertical elements of the stiffening system by assuming a continuum model instead of the usual discrete models. In this case the unknown internal force components can be expressed as a function of height.

As continuum model of the framework (Fig. 3a) a cantilever beam elastically supported against twist along its length first applied by CSONKA [3] is considered. At a difference from the original model, now it is clamped in an elastically bedded footing (Fig. 3d). A further deviation from the original Csonka model is that here also the effect of column deformations due to normal force is taken into consideration. This model, however, describes the actual load effects correctly only if the investigated framework satisfies rigidity conditions of the so-called "proportional frame" [2], otherwise the results are only approximations.

Derivation of the continuum model is shown in Fig. 3, where as first step a substituting framework [11] is produced (Fig. 3b), transformed into a cantilever beam bedded elastically point-wise (Fig. 3c), finally a continuum beam on continuous elastic bedding is developed (Fig. 3d). In this case the horizontal load originally consisting of concentrated nodal forces is transformed into a distributed load to be expressed by functions.

Elastic characteristics of the continuum model are given in terms of the following stiffnesses:

K_{Mi} — bending stiffness of the substituting cantilever beam;
K_{Ni} — bending stiffness in connection with column strains due to normal forces;
K_{Ti} — shear stiffness of the model;
R_i — bedding factor of the elastic bedding, substituting the stiffening effect of beams in the frame.

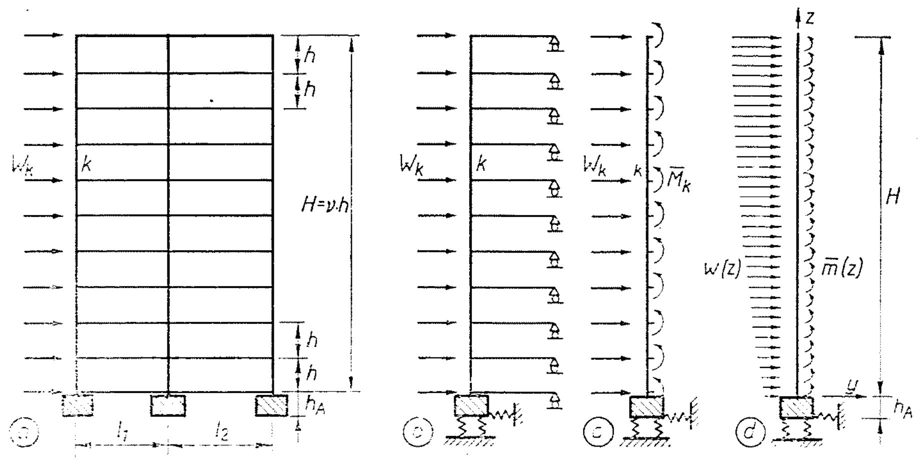

Fig. 3

Foundation body of the model is considered as infinitely rigid, but elastic deformations of the soil are taken into account, thus in the plane of the foundation the following rigidities are taken into consideration:

K_{MAi} — bending stiffness at the foundation level;
K_{NAi} — bending stiffness against normal forces at the foundation level;
K_{TAi} — shear stiffness at the foundation level;
R_{Ti} — bedding factor of the upper edge stiffness;
R_{Ai} — bedding factor of the lower transom connecting footings.

Subscript i indicates the stiffening element in question. Each of the above stiffnesses means a load derived from a unit movement and is calculated by elementary structural methods.

Perforated shear walls may also be considered as frameworks with the difference that only frame beam positions corresponding to the width of perforations are elastic, remaining parts are infinitely rigid (Fig. 4).

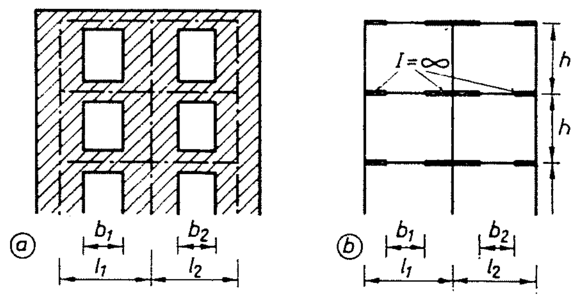

Fig. 4

For the sake of uniform discussion, also shear walls without perforations will be considered as frameworks. Shear walls without perforations are assumed to be divided by a vertical parting line into two, about equal wall zones, connected at floor level by frame beams considered as infinitely rigid (Fig. 5). In this way a shear wall is obtained where the width of the openings is $b = 0$, and in this case the model has a bedding factor $R = \infty$.

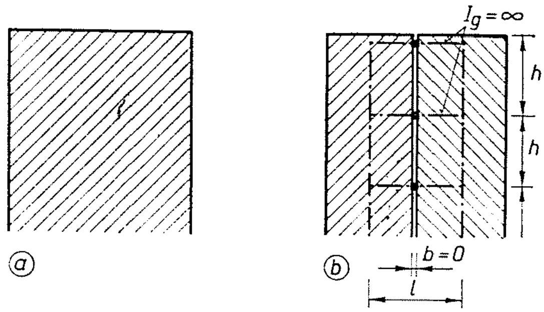

Fig. 5

The continuum model of the complete stiffening system is obtained by continuously connecting throughout the building height the continuum models of vertical elements substituting the load distribution effect of the floors each storey.

5. Computation of symmetrical stiffening systems

Vertical elements are supposed to be in parallel planes and their floor planes to be symmetrical, further the horizontal load resultant to be in the symmetry plane (Fig. 6). Stiffening elements are numbered in a way that elements of identical geometry get the same number.

To determine forces of the stiffening system, equilibrium and deformation conditions are written.

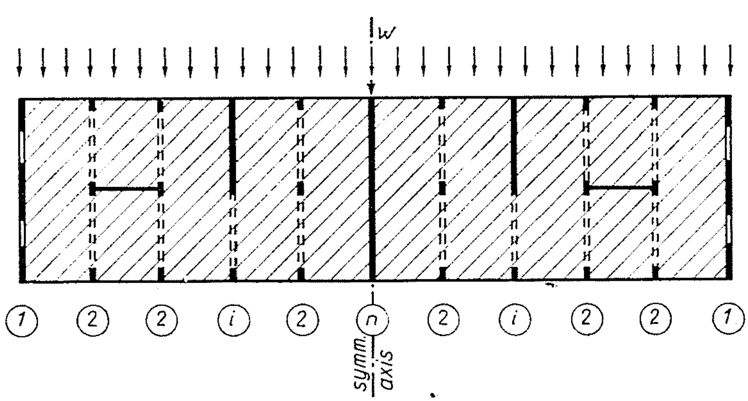

Fig. 6

The moment equilibrium condition for the cross section at any height z of the stiffening system is:

$$\sum_{i=1}^{n} n_i M_i^\circ = M^\circ \tag{1}$$

where

M° — moment in a vertical cantilever beam of height H clamped at the bottom, obtained from the horizontal load $w = w(z)$, acting on the entire building;

M_i° — moment on a beam i calculated as above from the load $w_i = w_i(z)$ acting on the beam because of load distribution;

n_i — number of elements i in the stiffening system.

Deformation equations express the condition that floor diaphragms considered as infinitely rigid and from symmetry causes able only to displacements parallel to their plane produce identical deformations in all elements. Deformation conditions expressing identity of displacements of structural cross sections at any height z are:

$$\delta_1 = \delta_i \tag{2}$$
$$(i = 2, 3, \ldots, n).$$

Above statement involves $n - 1$ independent equations, thus (1) and (2) represent an equation system suitable to determine altogether n unknowns.

As against processes known till now, taking either bending or shear deformation alone into account, deformation of the continuum model is produced as the sum of bending and shear deformations (Fig. 7b), therefore horizontal displacement of element i is expressed by

$$\delta_i = \delta_{Mi} + \delta_{Ti} \tag{3}$$
$$(i = 1, 2, \ldots, n).$$

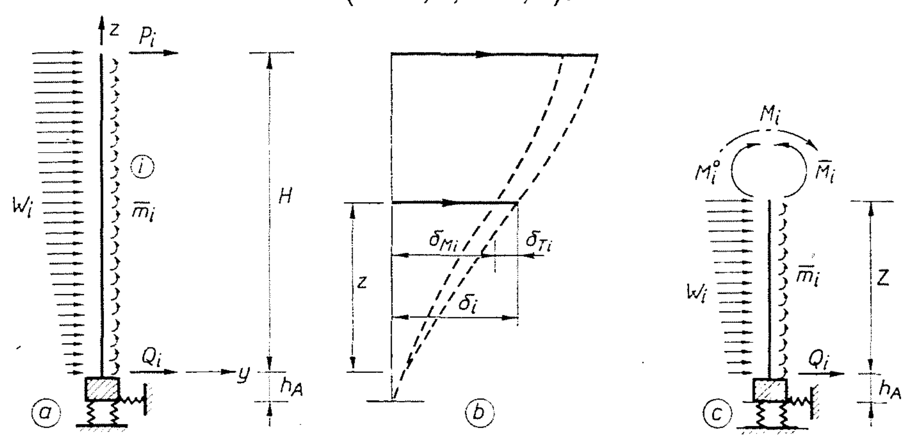

Fig. 7

The moment equilibrium condition for the cross section of element i at height z (Fig. 7c):

$$M_i = M_i^\circ - \overline{M}_i \tag{4}$$

where

M_i — the resultant moment;
M_i° — cantilever moment calculated from horizontal load w_i;
$\overline{M}_i$ — resultant of specific bedding resistance moments $\overline{m}_i$.

Resultant of resistance moments:

$$\overline{M}_i = \int_z^H \overline{m}_i \, dz \, . \tag{5}$$

Specific bedding resistance moments are calculated from frame beam end rotations ϑ_{Mi} and axis rotations ψ_{Ni}:

$$\overline{m}_i = R_i(\vartheta_{Mi} - \psi_{Ni}) \, . \tag{6}$$

In the following the n moment functions M_i are considered as unknown quantities, to be determined from the sets of Eqs (1) and (2). For this purpose Eqs (2) are derived twice to have relationships in terms of moments, then Eqs (3) and (4) and other known elasticity relationships will be used to write the equations as a function of the unknown moments $\overline{M}_i$, resulting in the following inhomogeneous fourth-order linear differential equation system of n unknowns with a constant coefficient applying the notation $d\overline{M}_i/dz = \overline{M}_i'$:

$$\left.\begin{aligned} &\sum_{i=1}^{n} (-a_i \overline{M}_i'' + b_i \overline{M}_i) = M^\circ \\ &c_1 \overline{M}_1'''' - d_1 \overline{M}_1'' + e_1 \overline{M}_1 = c_i \overline{M}_i'''' - d_i \overline{M}_i'' + e_i \overline{M}_i \, . \\ &(i = 2, 3, \ldots, n) \end{aligned}\right\} \tag{7}$$

Coefficients of the differential equation system are given by rigidities of the continuum models, thus they are constant:

$$\left.\begin{aligned} &a_i = n_i \frac{K_{Mi}}{R_i}, \quad c_i = \frac{K_{Mi}}{K_{Ti}}, \quad e_i = \frac{1}{K_{Ni}} \\ &b_i = n_i \left(1 + \frac{K_{Mi}}{K_{Ni}}\right), \quad d_i = \frac{1}{R_i} + \frac{1}{K_{Ti}} \left(1 + \frac{K_{Mi}}{K_{Ni}}\right) . \end{aligned}\right\} \tag{8}$$

Solution of the differential equation system consists of the general solution of the homogeneous equation system and of a particular solution of the inhomogeneous equation system:

$$\overline{M}_i = \overline{M}_{i,\,\text{hom}} + \overline{M}_{i,\,\text{part}} \tag{9}$$

Having the general solution of the homogeneous differential equation system in the form

$$\overline{M}_i = \varkappa_i \cdot e^{\lambda z} \tag{10}$$

and making certain formal changes in the usual Euler's solution to ease computer processing, the following expression is obtained as a solution:

$$\overline{M}_{i,\,\text{hom}} = \sum_{k=1}^{m} \varkappa_{i,k}(A_{2k-1} \cdot e^{-\alpha_k(H-z)} + A_{2k} \cdot e^{-\alpha_k z}) \tag{11}$$

$$(i = 1, 2, \ldots, n)$$

where

$$\varkappa_{i,k} = \frac{c_1\omega_k^2 - d_1\omega_k + e_1}{c_i\omega_k^2 - d_i\omega_k + e_i} \; . \tag{12}$$

Values ω_k and α_k are determined from the characteristic equation of the differential equation system

$$D(\lambda) = p(\lambda) = \sum_{k=0}^{m} r_k \lambda^{2k} = 0 \tag{13a}$$

or applying substitution $\omega = \lambda^2$, from

$$D(\omega) = p(\omega) = \sum_{k=0}^{m} r_k \omega^k = 0 \, . \tag{13b}$$

Namely $\omega_k (k = 1, 2, \ldots, m)$ stands for roots of polynomial $p(\omega)$ of order m delivering values of α_k as follows:

$$\alpha_k = \left| \sqrt{\omega_k} \right| . \tag{14}$$

A_{2k-1} and A_{2k} in the term in brackets mean the integration constants.

Expression of the particular solution of the inhomogeneous differential equation system depends on the moment function M^0. Supposing load function $w = w(z)$ to be always given in polynomial form, also the moment function $M^0 = M^0(z)$ is a polynomial, i.e. it can be written as:

$$M^0 = M^0(z) = \sum_{k=0}^{r} m_k \cdot z^k \tag{15}$$

In this case also the particular solutions can be written as rth-degree polynomials:

$$\overline{M}_{i,\text{part}} = \overline{M}_{i,\text{part}}(z) = \sum_{k=0}^{r} \mu_{i,k} \cdot z^k. \quad (i = 1, 2, \ldots, n) \tag{16}$$

Coefficients $\mu_{i,k}$ are determined by substituting functions (15) and (16) into the differential equation system (7).

For the determination of the integration constants, equilibrium and deformation conditions are written for $z = 0$ and $z = H$. These so-called boundary conditions are the following:

Equilibrium at $z = H$ in case of a free top edge ($R_{Ti}=0$, and $R_i \neq \infty$):

$$\overline{M}_{i(z=H)} = 0 \quad (i = 1, 2, \ldots, n) \tag{17a}$$

and for a braced top edge:

$$\overline{M}'_{i(z=H)} = 0. \quad (i = 1, 2, \ldots, n) \tag{17b}$$

The deformation equations at $z=H$ become:

$$\delta''_{1(z=H)} = \delta''_{i(z=H)}. \quad (i = 1, 2, \ldots, n) \tag{18}$$

At $z = 0$, the specific bedding resistance moment for $R_i \neq \infty$ and $R_{Ai} \neq \infty$ is of a value

$$\overline{M}'_{i(z=0)} = R_i(\psi_{NAi} - \vartheta_{MAi}) \quad (i = 1, 2, \ldots, n) \tag{19a}$$

and for $R_i \neq \infty$ and $R_{Ai} = \infty$ (case of strip foundation):

$$\overline{M}'_{i(z=0)} = 0. \quad (i = 1, 2, \ldots, n) \tag{19b}$$

Identity of rotations at $z = 0$:

$$\vartheta_{MA1} + \vartheta_{T1(z=0)} = \vartheta_{MAi} + \vartheta_{Ti(z=0)}. \quad (i = 2, 3, \ldots, n) \tag{20}$$

Identity of displacements at $z = 0$:

$$\vartheta_{MA1} \cdot h_{A1} + \delta_{TA1} = \vartheta_{MAi} \cdot h_{Ai} + \delta_{TAi}. \qquad (21)$$
$$(i = 2, 3, \ldots, n)$$

Subscript A indicates displacements of the footing at $z = 0$.

The equilibrium of forces Q_i concentrated in the upper plane of the footings at the lower edge of the continuum model (Fig. 7c) is:

$$\sum_{i=1}^{n} n_i Q_i = 0. \qquad (22)$$

The system of the above linear equations yields the integration constants.

In knowledge of resistance moment functions $\overline{M}_i$ from (9), resultant moments M_i are obtained from:

$$M_i = -\frac{K_{Mi}}{R_i} \overline{M}_i'' + \frac{K_{Mi}}{K_{Ni}} \overline{M}_i. \qquad (23)$$

Knowing $\overline{M}_i$ and M_i, the external moment M_i^0 is given by Eq. (4). In knowledge of the above moment functions, moment diagrams of the frameworks can be calculated [3], [11].

Using other, well-known differential relationships, shear and load functions of the continuum models can be computed as:

$$\left.\begin{aligned} T_i^{\circ} &= -M_i^{\circ\prime} \\ w_i = M_i^{\circ\prime\prime} &= -T_i^{\circ\prime} \end{aligned}\right\}. \qquad (24)$$

According to (3), displacement function of the continuum model is composed of bending and shear deformations.

In possession of the moment function M_i, the bending displacement is obtained by integrating twice as follows:

$$\delta_{Mi} = \frac{1}{K_{Mi}} \iint M_i \, dz \cdot dz + B_i \cdot z + C_i \qquad (25)$$

where B_i and C_i are integration constants and are given by the following boundary conditions for $z = 0$:

$$\vartheta_{Mi(z=0)} = \vartheta_{MAi} \qquad (26)$$

$$\delta_{Mi(z=0)} = \vartheta_{MAi} \cdot h_{Ai}. \qquad (27)$$

Shearing displacement is obtained by integrating shear force function T_i^0:

$$\delta_{Ti} = \frac{1}{K_{Ti}} \int T_i^0 \, dz + D_i . \tag{28}$$

Also integration constant D_i is given by boundary condition at $z = 0$:

$$\delta_{Ti(z=0)} = \delta_{TAi} . \tag{29}$$

6. Computation of an asymmetrical stiffening system

Elements in an asymmetrical ground plan according to Fig. 8a are numbered 1 to n, hence now also elements of identical geometry get different subscripts. The co-ordinates of each element x_i as well as the co-ordinate of the horizontal load resultant $w = w(z)$ are determined in the orthogonal co-ordinate system x, y, z of origin O at an optional point — where the y-axis is parallel to the plane of the stiffening elements.

Now the bending moment equilibrium condition becomes:

$$\sum_{i=1}^{n} M_i^0 = M^0 . \tag{30}$$

The torque equilibrium is

$$\sum_{i=1}^{n} T_i^0 \cdot x_i = T^0 \cdot x_w = M_c^0 . \tag{31}$$

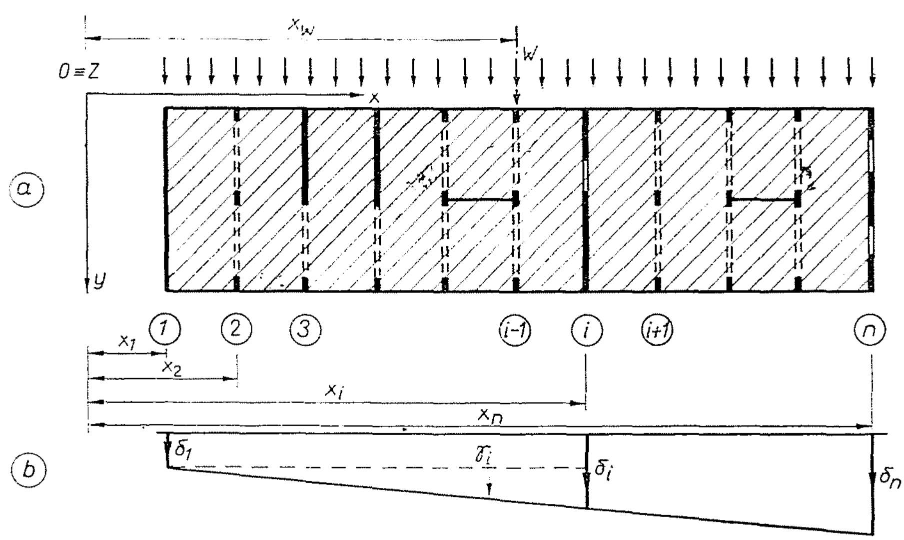

Fig. 8

The deformation equations express the condition that the infinitely stiff floor diaphragms are subject not only to displacement but also to torsion (Fig. 8b) and the rotation angle tangent between any two elements must be equal:

$$\gamma_i = \gamma_n \tag{32}$$
$$(i = 2, 3, \ldots, n-1)$$

where

$$\gamma_i = \frac{\delta_i - \delta_1}{x_i - x_1}\,. \tag{33}$$
$$(i = 2, 3, \ldots, n)$$

$n - 2$ of the deformation equations (32) can be written; adding Eqs (30) and (31), an equation system suitable for determining n unknowns is obtained. With operations according to the previous chapter, the following differential equation system is obtained:

$$\left.\begin{aligned} &\sum_{i=1}^{n} U_i = M^o \\ &\sum_{i=1}^{n} U'_i \cdot x_i = -M^o_c \\ &q_i\,(V_i - V_1) = q_n(V_n - V_1) \\ &(i = 2, 3, \ldots, n-1) \end{aligned}\right\} \tag{34}$$

where

$$U_i = -a_i\overline{M}''_i + b_i\overline{M}_i \tag{35}$$

$$U'_i = -a_i\overline{M}'''_i + b_i\overline{M}'_i \tag{36}$$

$$V_i = c_i\overline{M}''''_i - d_i\overline{M}''_i + e_i\overline{M}_i \tag{37}$$

$$q_i = \frac{1}{x_i - x_1}\,. \tag{38}$$

The inhomogeneous 4th-order differential equation system (34) with n unknowns and constant coefficients is solved similarly to (7), but the general solution of the homogeneous differential equation system is somewhat modified because of the change in character of the characteristic equation such as:

$$D(\lambda) = \lambda \cdot p(\lambda) = \lambda \sum_{k=0}^{m} r_k \cdot \lambda^{2k} = 0 \tag{39}$$

and with substitution $\omega = \lambda^2$

$$D(\lambda, \omega) = \lambda \cdot p(\omega) = \lambda \sum_{k=0}^{m} r_k \cdot \omega^k = 0. \tag{40}$$

According to this the modified solution:

$$\overline{M}_{i,\,\mathrm{hom}} = \sum_{k=1}^{m} \varkappa_{i,k}(A_{2k-1} \cdot e^{-\alpha_k(H-z)} + A_{2k} \cdot e^{-\alpha_k z}) + \varkappa_{i,m+1} \cdot A_{2m+1}. \tag{41}$$

Proportionality factors $\varkappa$ are obtained by substituting the characteristic equation roots into the homogeneous linear equation system and solving it. Further on the solution and computation of loads and deformations follows the course in the previous chapter.

Summary

The new computation method for stiffening systems of multi-storey buildings permits an exacter determination of forces and deformations of the structure by taking — beside flexural deformation of the elastically bedded cantilever beams applied as structural model — also the deformation due to shear and normal force, further, the footing movements due to elastic deformations of the soil under the base plane into consideration. A program made for a digital computer facilitates numerical computations. The discussed method can be developed for stiffening systems with more complicated ground plan arrangements.

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