FLOOD ROUTING ON RIVERS WITHOUT TRIBUTARIES

By

P. BAKONYI

Department of Hydraulic Engineering, Institute of Water Management and Hydraulic Engineering, Technical University, Budapest

(Received: November 1st, 1976)

(Presented by Prof. Dr. MIKLÓS KOZÁK)

When dealing with water management, flood control, river training etc., problems to be solved only by the unsteady flow theory are frequently faced. Computation of unsteady flows — despite their theoretical foundations being known for over more than a century — was developed only recently, due to the appearance of modern computer methods.

The present paper deals with the investigation of open-channel, unsteady, gradually varying flows (e.g flood waves). As solution the *implicit method* [1, 2, 3] was chosen, providing a rapid computation, easy to follow. In the discussion of the method some theoretical hydraulic knowledge is presumed, to be found in the literature referred to [1]. Our computations were carried out for rivers without tributaries, though theoretically there are no obstacles to take tributaries into account [4].

1. Fundamental equations

The unsteady, gradually varying open-channel flows are described by the continuity (1) and the dynamic (2) equations:

$$\frac{\delta Q}{\delta x} + \frac{\delta F}{\delta t} - q = 0 \tag{1}$$

$$\frac{\delta z}{\delta x} - \frac{\alpha' Q^2}{gF^3}\frac{\delta F}{\delta x} - \frac{\alpha'' Q}{gF^2}\frac{\delta F}{\delta t} + \frac{\alpha' Q}{gF^2}\frac{\delta Q}{\delta x} + \frac{\alpha''}{gF}\frac{\delta Q}{\delta t} + \frac{Q^2}{K^2} + \frac{\alpha'' Q}{gF^2} q = 0 \tag{2}$$

where:

$x[L]$ — is the co-ordinate of the section (increasing along the flow);
$t[T]$ — time;
$Q[L^3T^{-1}]$ — discharge;
$F[L^2]$ — wetted cross section area;

$q[L^2T^{-1}]$	—	the inflow, withdrawal, or incidental evaporation per unit length (henceforth termed as *linear load);*
$Z[L]$	—	elevation of the *water surface;*
$g[LT^{-2}]$	—	*gravity acceleration;*
$K[L^3T^{-1}]$	—	*water conveyance from* $Q = K\sqrt{J}$*;*
α'	—	*dispersion coefficient* of momentum;
α''	—	*dispersion coefficient* of local acceleration;
$[L]$, $[T]$	—	dimensions of length viz. time (e.g. m, cm, inch, sec. etc.).

Derivation and validity conditions of the fundamental equations are found in [1].

Eqs (1) and (2) form a partial differential equation system of hyperbolic type. A general solution of equations of this type cannot be obtained at the present state of mathematics, however, they can be approximated with numerical methods very well [1, 4].

2. Solution of the fundamental equations

Approximation of the general solution of Eqs (1) and (2) is carried out by the implicit method. As first step the partial differential Eqs (1) and (2) are transformed into a difference equation. For this purpose the investigated river without tributaries is divided into reaches each Δx long and the phenomenon is investigated at intervals Δt. Equidistant division is not required in either of the directions Δx and Δt. If the lines $x =$ constant, $t =$ constant are plotted in the co-ordinate system $x - t$, on the so-called wave plane (Fig. 1) a net is obtained, in the nodes of which the values of water level and discharge can be determined. Thus, the total interpretation range of the computation is reduced to a multitude of primary discreet ranges of dimensions Δx, Δt. Let us select a mesh situated in an arbitrary place at random (Fig. 2) and approximate the partial derivatives of the function $f(x, t)$ interpreted over the given mesh by means of the following scheme:

$$\frac{\delta f}{\delta x} \approx \frac{1}{2}\left(\frac{f_{i+1}^{j} - f_{i}^{j}}{\Delta x} + \frac{f_{i+1}^{j+1} - f_{i}^{j+1}}{\Delta x}\right) \tag{3}$$

$$\frac{\delta f}{\delta t} \approx \frac{1}{2}\left(\frac{f_{i}^{j+1} - f_{i}^{j}}{\Delta t} + \frac{f_{i+1}^{j+1} - f_{i+1}^{j}}{\Delta t}\right). \tag{4}$$

Constants in Eqs (1) and (2) are approximated by

$$f_c \approx \frac{1}{4}(f_i^j + f_{i+1}^j + f_i^{j+1} + f_{i+1}^{j+1}) \tag{5}$$

i.e. by the average function values in the node (subscripts i and j designating the place, and the time, resp.).

In the computations the instant $t = t_j$ is supposed already to be reached and now the condition at time $t = t_{j+1}$ has to be computed.

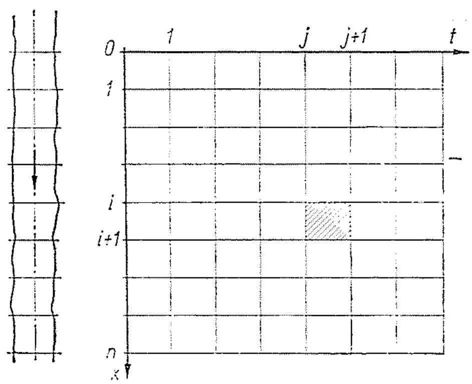

Fig. 1. Division of wave planes into elementary meshes

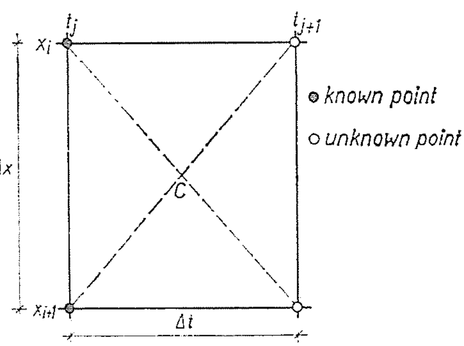

Fig. 2. Elementary mesh at an arbitrary place

Carrying out substitutions (3), (4) and (5) and expressing the values of the unknowns leads to the set of linear equations:

$$A_1 Z_i^{j+1} + A_2 Q_i^{j+1} + A_3 Z_{i+1}^{j+1} + A_4 Q_{i+1}^{j+1} = A_5$$
$$B_1 Z_i^{j+1} + B_2 Q_i^{j+1} + B_3 Z_{i+1}^{j+1} + B_4 Q_{i+1}^{j+1} = B_5 \,. \tag{6}$$

Writing for all meshes Eqs (6), $2 \cdot n$ equations are obtained, where the number of unknowns is $2 \cdot (n + 1)$. If in sections $x = x_0$, and $x=x_n$, also called upper and lower control sections, respectively; the value of each unknown is given, the equations can be solved.

The "equation constants" A_i, $B_i (i = 1, 2, \ldots, 5)$ depend on the value of the unknown because of the approximation according to the coefficients of Eq. (5). Therefore the set of equations (6) can only be solved by iteration.

3. Initial and boundary conditions

Above it was supposed that at the instant $t = t_j$ already all characteristics of the flow were known. Therefore at the beginning of the computation ($t = t_0$) beside the geometric and hydraulic characteristics the initial water stage and discharge distributions along the river should be given. This is the initial condition. The mathematical form of the initial conditions:

$$Z = Z(x, t_0)$$
$$Q = Q(x, t_0). \qquad (7)$$

Initial conditions refer physically to the history of the flow: they show prism storage of the channel and the prevailing hydraulic conditions thereof.

It must be noted that if the value of the linear load was not zero, Eqs (7) have to be completed by the equation

$$q = q(x, t_0) \; .$$

The determination of initial conditions is not discussed further, it may be found in [1].

The numbers of equations and unknows are only equal if the values of the two variables are given from one time cycle to another. These are the *boundary conditions*, to be described mathematically as in [1]:

$$Z = Z(x_0, t) \quad \text{or} \quad Q = Q(x_0, t)$$
$$Z = Z(x_n, t) \quad \text{or} \quad Q = Q(x_n, t). \qquad (9)$$

(As already pointed out, only one variable value has to be given for each of the control sections, and it is optional whether only the stage or only the discharge is chosen).

In case of linear load,

$$q = q(x_i, t) \qquad i = 1, 2, \ldots, n \qquad (10)$$

complete the boundary conditions.

Physically, the boundary conditions constitute the link between the investigated reach and its environment as a function of time.

4. Process of the computation

Major steps of the computation of a phenomenon of arbitrary duration is shown in a flow chart (Fig. 3). The usual plotting was abandoned to make more obvious the fitting into each other of the iterative solution of the set of equations and the time cycle.

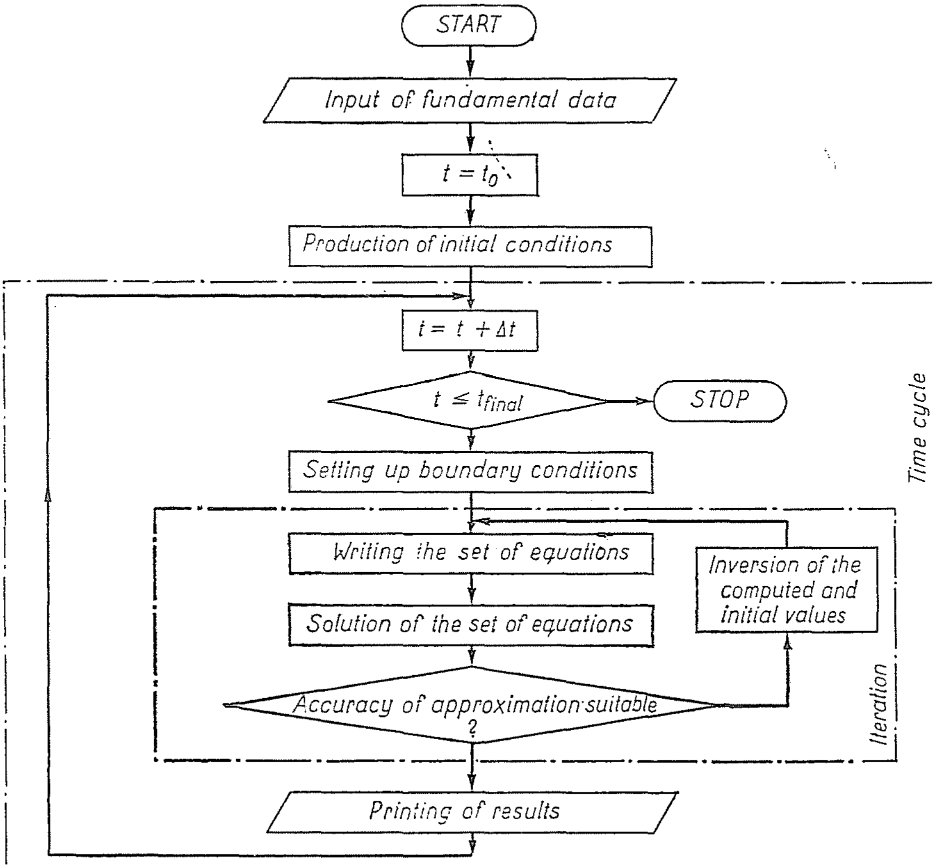

Fig. 3. Flow chart of the computation

5. Applications

From the countless application possibilities of the computation only some are stressed here:

— computation of unsteady flow developing because of the operation of a hydroelectric power station or a chain of such power stations;
— effect of river training and foreshore clearing upon runoff conditions;
— forecast of flood waves;
— effect of a level breach on the hydraulic conditions of the river;
— computation of flow conditions in consequence of a variable intake by irrigation canals . . . etc.

As an example we consider now how the filling of the storage space of a channel-storage hydroelectric power station occurs. To cut the computation work (running time), the phenomenon is investigated in a prismatic rectangular channel of 100 m width and 100 km length. The bottom slope is 10 cm/km and its smoothness factor (the reciprocal of Manning's n) $k = 40$ (Fig. 4). At $t_0 = 0$ and a water depth of 3 m, 240 m^3/s discharge flows down the river channel, having uniform motion.

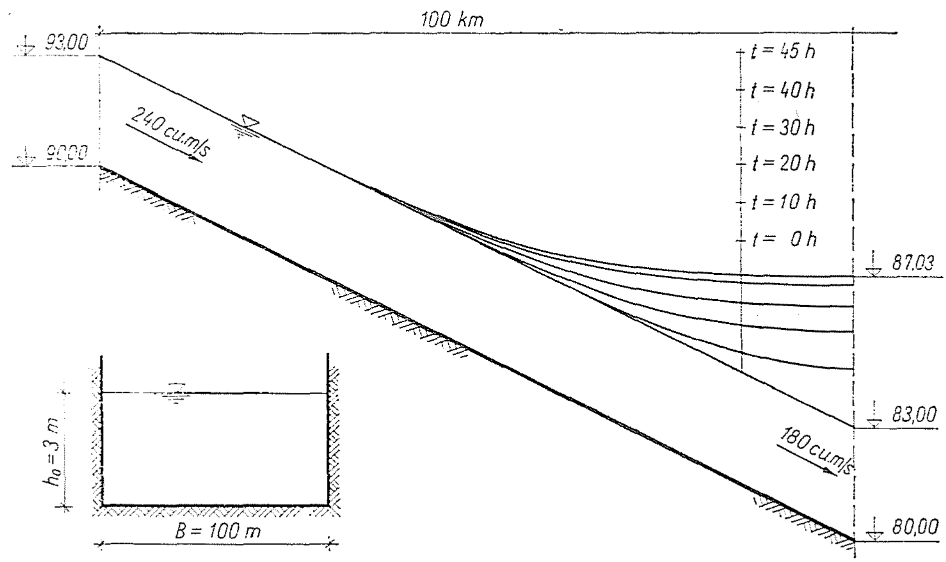

Fig. 4. Surface profils belonging to different instants of filling. $t = 45$h

As for the upper control section, it is supposed that a base flow of 240 m^3/s arrives during the whole period of the process. In the section of the hydroelectric power station, the effluent discharge is reduced to 180 m^3/s therefore discharge of 60 m^3/s is used to fill the storage space. In the storage area we want to obtain a headwater depth of 7,00 m and as soon as this is attained, the normal backwater operation begins. In the lower control section therefore a compound boundary condition has to be established:

— during the filling period $Q =$ const. $= 180$ m^3/s and

— in the backwater operation $Z_a =$ const. 87.00 m (Fig. 5).

The computation was carried out with mesh dimensions $\Delta x = 2000$ m and $\Delta t = 3600$ sec. When determining water level elevations, an iteration error of 1 mm was permitted.

Surface profiles belonging to the different instants are plotted in Fig. 4. The water reached the elevation of 87.00 m in the 45th hour, then automatically the boundary condition $Z_a =$ const. became valid. Subsequently, only changes in the order of cm occurred, therefore no further surface profiles were plotted.

In the section of the hydroelectric power station (Fig. 5) the change in time of the water elevation and the discharge are shown. It can be well seen on the figure that at the beginning of the filling period the rate of water rise is 30 cm/h, which decreases to 5 cm/h at the end of the filling. After reaching the specified backwater level the discharge soon assumes the original value of

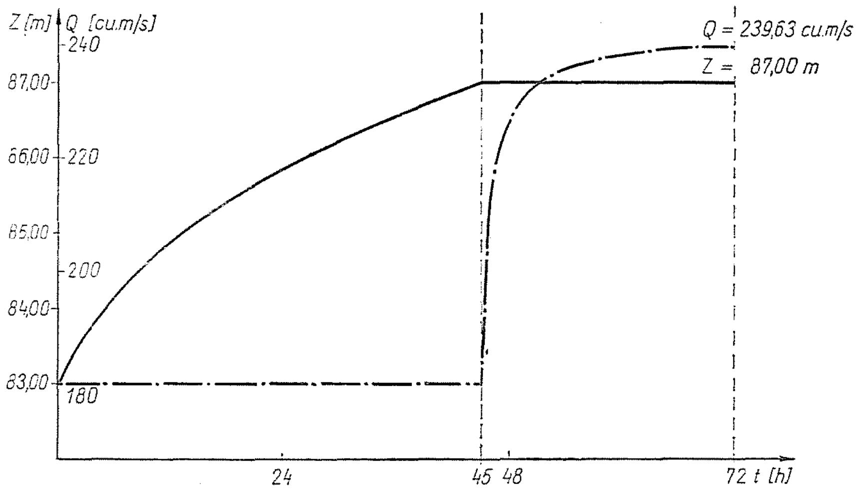

Fig. 5. Variation of water stage and discharge in the lower control section

240 m^3/s. Had the computations been carried out for a real channel it could be stated — based on the obtained results — whether the filling rate is suitable with regard to water bed erosion, bank stability, navigation, etc. or it has to be changed. Such computations yield important fundamental data for design and for establishing instructions of operation.

By means of this procedure, gradually varied, open-channel unsteady flows can be computed. A great advantage of the implicit method of differences is that for practically all practicable mesh dimensions it yields a reliably stable and rapid computation. For informative computations with large Δx, Δt and the iteration error given, the design situation is to be chosen for which more accurate but also more time-absorbing computations might be carried out.

The above example ($\Delta x = 2000$ m, $\Delta t = 1$ hour, $Z_{\text{error}} = 1$ mm, 50 reaches, 72 hour phenomenon) took 77 minutes on the medium-speed ODRA 1204 computer.

Summary

The paper describes computation of gradually varying unsteady flows. The two partial differential equations of the hyperbolic type describing the phenomenon are solved by means of the *implicit method of differences*. The ways of establishing initial and boundary conditions,

necessary for the solution, are discussed. A flow chart shows the computation steps. As an example of how to use the method, the filling of an in-channel reservoir in function of time is shown.

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Péter Bakonyi, H-1521 Budapest

* In Hungarian