THE OUT-OF-KILTER METHOD IN WATER MANAGEMENT PROBLEMS

By

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Both publications and our research results show network flow models well simulating the operation of water distribution and drainage systems. A lecture by VEN TE CHOW [14] and papers by J. KINDLER [8, 9, 10] made us aware of a solution method of network flow models, the out-of-kilter algorithm, successfully applied for two different water management operation problems.

A network flow model of the optimum distribution of water resources of the River *Tisza*, and another of the optimum regulation of plain-region drainage systems, solved by out-of-kilter method, will be presented.

Also classic linear programming suits to solve problems processed here by the out-of-kilter algorithm, this latter results, however, in substantial running-time saving for large-scale problems, since:

— the algorithm involves additive operations alone;

— no matrix inversion is needed.

In addition to running-time saving, the method is advantageous by permitting illustrative, graphic mapping of the investigated systems, the problem and the outputs are easy to survey and rough errors in modelling and data processing can be avoided.

These advantages recommend the out-of-kilter algorithm as an efficient method of process control in water management systems.

1. Mathematical background of the out-of-kilter method

Let us consider a network M with flow values f, meeting conditions (1) and (2), i.e.:

$$f(X,N) - f(N,X) = 0, \qquad \forall X \in N \tag{1}$$

where

N — tested node in the network;

X — nodes connected by an arc to node N;

and $\left.\begin{array}{l} f(X,N) \\ f(N,X) \end{array}\right\}$ total flow from the node (the sequence corresponding to the flow direction);

and

$$l(x,y) \leq f\ (x,y) \leq k(x,y) \qquad \forall (x,y) \in M \tag{2}$$

where

$l(x,y)$ and $k(x,y)$ lower and upper limit resp., for flow $f(x,y)$;

x,y nodes at the arc end points.

Conditions (1) and (2) have to be met for all nodes and arcs of the network, respectively.

A circulation flow at minimum cost, meeting capacity constraints, has to be found:

$$\text{MIN}\left(\sum_{(ij)\in M} C_{ij} f(i,j)\right) \tag{3}$$

where: $(i,j) \in M$ — summation to be made for each arc of the network;
C_{ij} — unit cost of the flow tending from i to j along arc (i,j).

Minimization of network flows described by (1), (2) and (3) is expediently done by an out-of-kilter algorithm. The model has been solved according to studies by Kindler [8, 9, 10] as well as to the book by Ford and Fulkerson [3].

2. Optimum distribution of River Tisza water resources by the out-of-kilter algorithm

In Hungary, research has been going on for years on optimum computer control of the *Tisza Valley Water Management System* (TVR). The Department of Water Management, Technical University, Budapest has been commissioned with the development of one part of it, termed the TIKI model [7].

The TIKI model was intended to distribute water resources optimally upstream the *Kisköre* barrage, to operate optimally the river barrages of *Tiszalök* and *Kisköre*. A stochastic [13] and a deterministic [7] model have been developed for distributing water resources. Most of the investigations have involved the deterministic model with stochastic input, easier to supply with data. Below, the deterministic model solved by an out-of-kilter algorithm will be presented.

Our investigations aimed essentially at developing an operation method for the barrages *Tisza I (Tiszalök)* and *Tisza II (Kisköre)*, and for the *Kisköre* reservoir, now under construction, with a head-water level to be raised gradually. Rather reliable hydrometeorological data sequences are available for afflux prediction but prediction of water demands is rather difficult due to the incertainty of the pace of irrigation development and to a varying degree of idleness of the irrigation facilities, depending also on other than hydrometeorological factors.

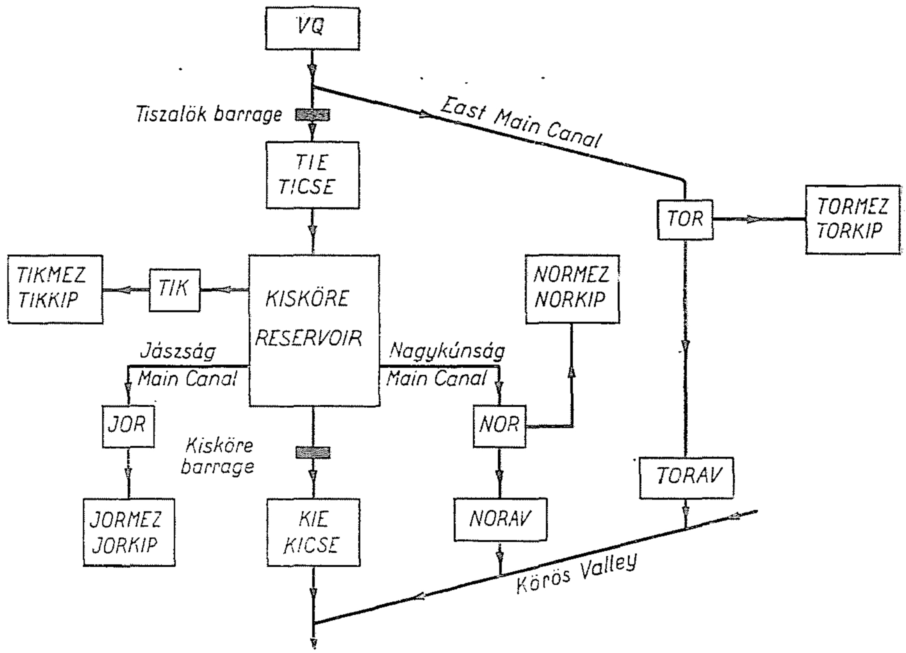

Fig. 1. Tisza Valley Water Management System

The *Kisköre* reservoir can be considered as seasonally balanced although the stored water quantity may be consumed within a month during the vegetation season in case of a low afflux, but the reservoir can always be replenished to early May, and thus from the end of the vegetation season to May, the reservoir operation is a mere hydraulic problem, restricted to reduce flood peaks.

The water management system investigated is shown in Fig. 1. Variables of the model affecting the objective function value are water volumes for different purposes. Symbols indicate the following targets:

MEZ — agricultural water consumption
KIP — communal and industrial water consumption
AV — discharge to another system
CSE — peak power production
E — base power production.

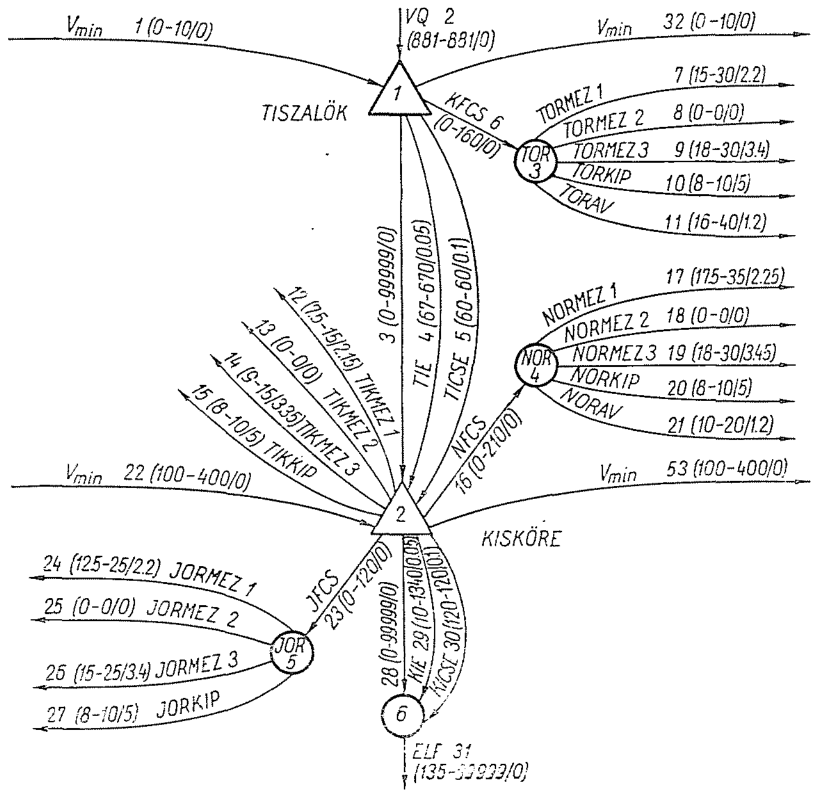

Fig. 2. One-period network flow model for the Tisza Valley Water Management System

Legend: 7 (15–30/2.2)

- 2.2 → Benefit from 1 cu · m of water consumption, Ft/m³
- 30 → Max. available water volume, million cu · m
- 15 → Min. water supply, million cu · m
- 7 → No. of arc
- (3) → No. of node

Variables have also been distinguished according to the place of water consumption as follows, indicated by marks always preceding the target symbol:

TOR — *Tiszalök Water Management and Irrigation Scheme* (supplied through *Eastern* and *Western Main Canals*),

JOR — *Jászság Irrigation Scheme* (supplied through *Jászság Main Canal*)

NOR — *Nagykunság Irrigation Scheme* (supplied through *Nagykunság Main Canal*),

TIK — Irrigation scheme between *Tiszalök* and *Kisköre*, supplied directly from the *Tisza* River,

Ti — *Tiszalök Hydroelectric Plant*,

Ki — *Kisköre Hydroelectric Plant.*

For the optimum operation of water distribution system in Fig. 1, a network flow model has been developed by systems analysis. Network flow models consist of nodes and arcs, where nodes represent characteristic points, or rather, various states of the system, and arcs the path of quantities conveyed from one point to the other, or from one state to another.

The part of the network flow model of the problem for one period is seen in Fig. 2, and the network flow model for four consecutive periods in Fig. 3.

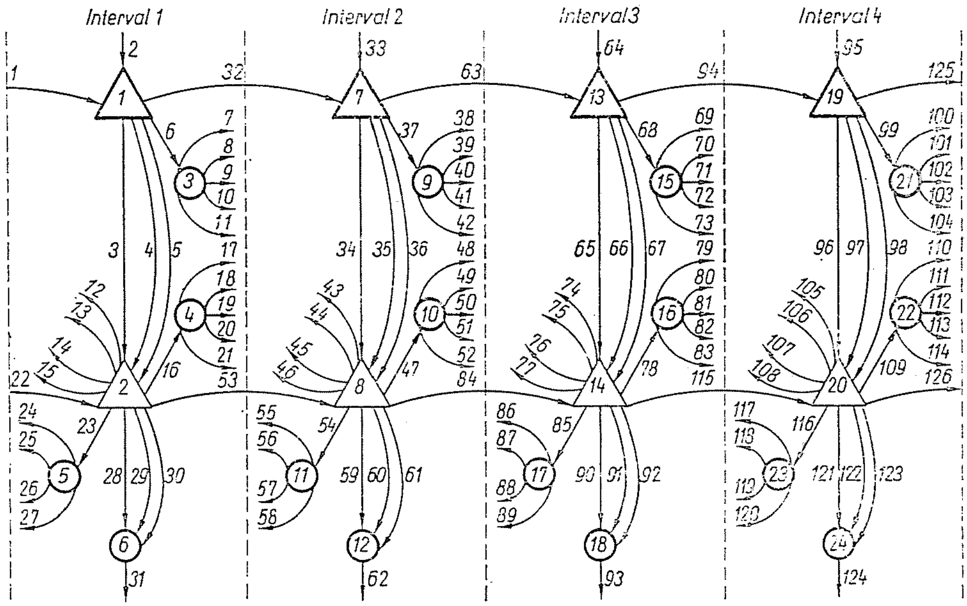

Fig. 3. Four-period network flow model for the Tisza Valley Water Management System
Legend:
32 No. of arc
③ No. of node

In Fig. 2, nodes 1 and 2 indicate the *Tiszalök* and the *Kisköre* reservoirs, nodes 3, 4 and 5 the irrigation channels, and node 6 the discharge from the *Kisköre* reservoir via the *Tisza* bed. Fig. 3 shows periods connected by arcs indicating residual water in reservoirs. Arcs pointing to the system indicate natural afflux and residual water left in the reservoir from the previous period, while arcs pointing outwards indicate water left in the reservoir for the next period, water used for various purposes, and that released unused through the *Tisza*. Arcs connecting nodes in a period illustrate the path of water volume flowing in the system.

Agricultural water use functions have been linearized by a piecewise method indicating linear sections of the functions by an arc each. Therefore, agricultural water uses are indicated by several arcs in Fig. 2.

Part of special arcs and nodes have been omitted from Fig. 3, introduced originally in order to make the total of water uses in given periods equal to the specified minimum.

The complete network flow model of the system had 144 arcs and 42 nodes. Solution of the model by linear programming required a running time of 45 min, and by out-of-kilter method 3 min, on an ODRA 1204 computer of Polish make. Originally, the coefficient matrix of the linear programming model had nearly 50,000 elements. The model was established automatically by the computer. To accelerate the linear programming algorithm, coefficient matrix dimensions have been reduced, taking condition peculiarities, and the advantage that the coefficient matrix had no elements but 0, +1, and —1, into consideration. Nevertheless, the indicated great running time difference in favour of the out-of-kilter method was maintained. Again, linear programming was made by the peripheral magnetic drum while the out-of-kilter algorithm involved only the central storage unit.

3. Optimum regulation of a drainage system by the out-of-kilter algorithm

The simple surface drainage system in Fig. 4 consists of three partial water catchments interconnected through by-canals, two main canal reaches, an emergency reservoir, and a drainage possibility by pumping and gravity.

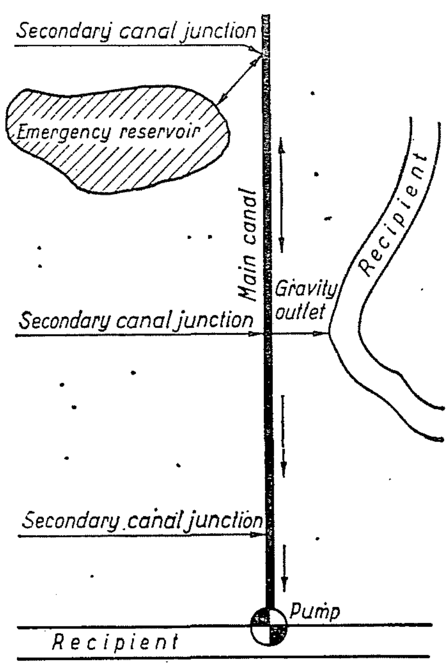

Fig. 4. Simple drainage system in a plain area

(In case of low water level in the recipient, the pump outlet can also be gravity operated.) The investigated period of excess water will be divided into three periods (decades).

The network-flow model of water management is shown in Fig. 5. Arcs starting from, or tending to, else than a node shown are pointing to a node 0 not indicated in the figure.

Operation of the system in the first period is illustrated by nodes enlisted in Table 1, and by arcs in Table 2.

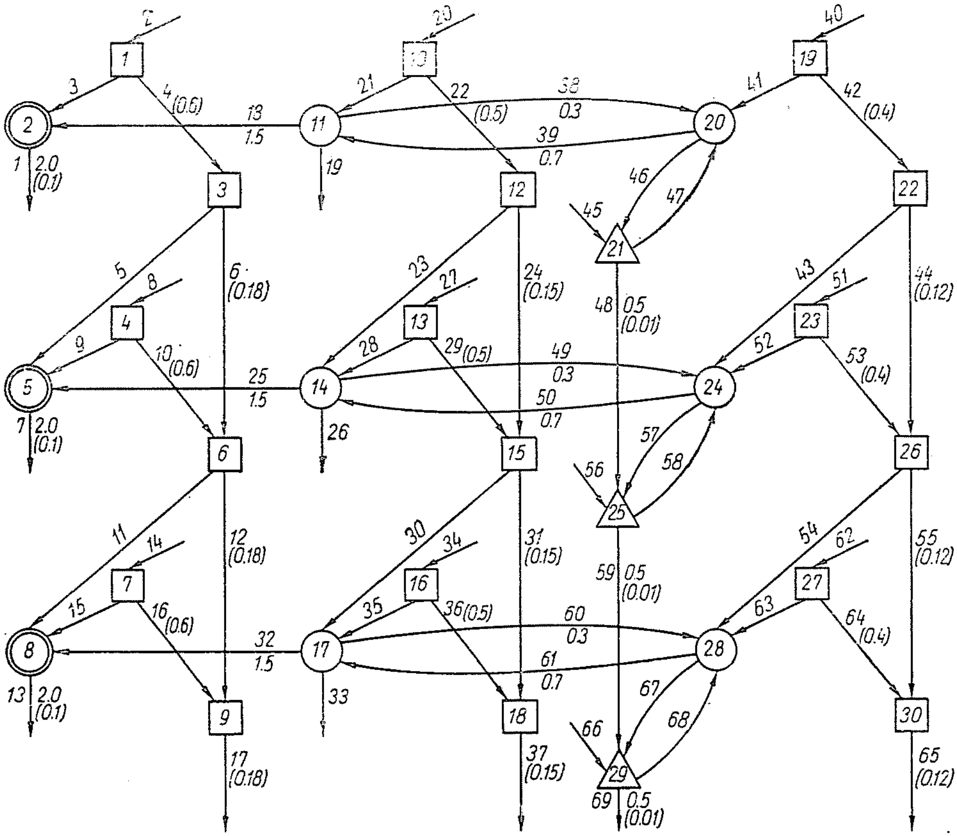

Fig. 5. Three-period network flow model of a simple drainage system in a plain area

Legend:

8 No. of arc

③ No. of node

2.0 Max. of potential discharge

(0.18) Damage due to 1 cu.m of residual water, Ft/m³

6 Catchment area

5 Pumping plant

7 Emergency reservoir

Table 1

Interpretation of network flow model nodes

Node	Interpretation
2	Pump station site
11	Gravity outlet point
20	Emergency reservoir
1, 10 and 19	Partial water catchments
21	Emergency reservoir

Table 2

Interpretation of network-flow arcs

Arc	Interpretation
2–0	Water pumped to the recipient
11–0	Gravitational water release to the recipient
1–2, 10–11 and 19–20	Water volume from partial catchments
1–3, 10–12 and 19–22	Residual water in partial catchments
20–21 and 21–20	Water inflow into the emergency reservoir or its effluent to the canal
0–1, 0–10 and 0–19	Harmful water volume in partial catchments
0–21	Water volume arising on the emergency reservoir surface
21–25	Water in the emergency reservoir by the end of interval 1
11–20 and 20–11	Water conveyed on the two-way main canal reach

The following approximations will be made:

— the investigated excess water period is divided into time intervals, by assuming uniformly distributed water volumes to be drained, within each interval;

— there is steady water flow in the drainage canals;

— damage due to flooding and to residual excess water is considered to be proportional to the water volume;

— only pump costs are considered as water transport costs, costs of gravity water transport — annual operation costs of canals, sluices etc. — being independent of water volume.

The model has been intended for the water management of surface drainage systems in plain areas, with large catchment basins (over 50 sq · km), where long rainy periods and snow melt are decisive for the discharge, hence the above approximations are acceptable.

Flow regulation possibilities are:
— gravity outlet of drainage water;
— water drained off either by pumping stations or by mobile pumps;
— water retention at the place of origin;
— gravity or pump feeding of water into emergency or other reservoirs;
— canal capacity increase by means of intermediate booster pumps;
— flow regulation and guidance by sluices.

Excess water quantities developing during examined periods in partial catchments are assumed to be known. Water management is intended to minimize the sum of flood damage and pumping costs. No prediction methods of inland water quantities will here be considered.

Canals

These convey water in one direction as a rule, though there are two-way canals, too. In the network-flow model, canal reaches have been represented by arcs between nodes (e.g. 18), two-way canals by two arcs (e.g. 38 and 39).

Nodes

Our models are constructed around the main drainage canal divided by nodes into reaches at the following points:
— canal junctions,
— change of profile (or of discharge capacity);
— major engineering structures (e.g. intersections with railways or roads or other linear structures that are at the same time artificial divides);
— divides between partial water catchments;
— likely jointing spots of reservoirs and emergency reservoirs,
— gravity outlets to recipients,
— pumps.

Canal nodes are those numbered 2, 11, 20, 17, 14 etc. in Fig. 5.

Reservoirs

Permanent reservoirs receiving and discharging water by gravity or pumping.

Emergency reservoirs, either pastures surrounded by small-size dikes, or natural depressions. Water is fed in by gravity (from the catchment area) or by pumping (from the canal), sometimes they may exclusively retain water accumulated in their own area.

Reservoirs are indicated by nodes 21, 25, 29 in Fig. 5.

Gravity water outlets

In case of a high water level in the recipient they can be totally or partially closed. They are represented by an arc each in the mathematical model (19, 26, 33).

Pumping plants

These may lie either at the recipient, or within the catchment area as booster pumps, or at the reservoirs as lifting pumps. Pumping plants may be either permanent or temporary. Two-way operation is in some cases possible.

Nodes 2, 5 and 8 in Fig. 5 are outlet pumps.

Partial catchments

Partial catchments supply water through minor canals or via the soil surface to the smallest canals investigated. Water that cannot be drawn off by the tested canals remains in the area.

Nodes 1, 10, 19, 3, 4 etc. in the example represent agriculturally utilized areas of partial catchments.

Water volumes conduct ed from the partial catchment to the network, and remaining in the area are represented by arcs 3, 21, 41, 3 etc., and 6, 12, 24 etc., respectively.

System input and output

System input and output mean water inlet into and outlet from the network.

For a period of several intervals, some inputs and outputs enter or leave the network before or after the given interval. Other inputs and outputs represent the water circulation between intervals (output of one interval is input to the other).

System input:

— harmful surface or subsurface water originating from rain or snow melt, evaporation and seepage to subjacent soil layers being also taken into account.

Input between intervals:

— water left from the preceding interval in the area and in reservoirs, to be calculated with regard to evaporation.

System output:

— water fed by gravity or by pumping to the recipient.

Output between intervals:

— water left in the area and in reservoirs for the next interval.

In the out-of-kilter model, arcs always connect nodes, therefore all input and output arcs of the system are connected to a node specially assumed to this end.

Excess water loads can be established as products of nodal areas by runoff per unit area.

Water stored in canals is negligible compared to that discharged during a ten-day interval, therefore it has been omitted.

Excess water loads of canals have been assumed at the nodes.

Examinations presented in Fig. 5 lasted for three ten-day periods. Initially, reservoirs were considered to be empty. Examinations may be prolonged beyond this time if needed, until all water has left all areas and reservoirs.

Cost and damage functions

Our model performs optimization on an economic basis, hence also economic impacts of operations are assessed, such as:

— damages,
— missing benefits,
— costs.

Economic effects have throughout been taken as proportional to the water volume, and indicated in Ft per cu·m units. Approximate ratios, rather than absolute values, have been assumed.

Assumptions of an economic nature used in the model are:

— processes, operations of negligible economic impact on optimization;
— gravity water flow;
— water left in the emergency reservoir in the first ten days;
— operation of regulators and sluices;
— maintenance of canals and hydraulic structures.

Costs

Pumping costs have been assumed to be constant, irrespective of the pump position, type, kind of power supply, and lifting head. The total does not include permanent costs.

Damages

Water left in agricultural areas causes significant damage, depending on:
- the size of the flooded area;
- the flooding time;
- the soil quality (the better the soil, the heavier the damage);
- the after-flood soil condition (soaked soil being difficult to cultivate by machinery);
- the season, or rather the stage of crop development;
- the type of the crop.

Flooding damage may be either total where no economic benefit may be expected at the end of the vegetation period; or partial, due to the following causes:
- untimely soil cultivation or
- partial decay of the crop,
- replacement of the destroyed crop or production of a less profitable crop imposed by delayed soil cultivation;
- manual soil cultivation must be recurred to.

The damage is worsened by water remaining in the cultivated area for the next period. Of course, the new damage is less than that due to the former flood. If the crop was destroyed by flood in the first ten-day period, the subsisting water cannot do harm to the crop but protracts soil drying, offsets cultivation and eventual sowing, involving further damage.

Specific damage due to water left in the area for the next ten-day interval has been assumed as 30%.

In exceptional cases, floods may be repeated. These are less harmful but because of being exceptional, are assumed to be as harmful as the first one, for the sake of simplicity.

Examples of an out-of-kilter model analysis

In addition to information in former items, Fig. 5 shows capacities of model elements in units of million cu · m per ten days, as well as costs and damages along each arc unit of Forint per cu · m per ten days.
Capacities available in the illustrated system are:

Pump outlet	2.0 million cu · m per ten days
Gravity outlet	0.8 million cu · m per ten days
Downstream canal reach	1.5 million cu · m per ten days
Upstream canal reach	0.7 million cu · m per ten days
or for counterflow	0.3 million cu · m per ten days
Emergency reservoir capacity	0.5 million cu · m.

Table 3

Computer inputs. Arc number = 69, Node number = 30

Network data

Arc No.	Source node	Sink node	Lower bound of flow	Upper bound of flow	Cost
1	2	0	.000	2.000	.100
2	0	1	.550	.550	.000
3	1	2	.000	999.999	.000
4	1	3	.000	999.999	.600
5	3	5	.000	999.999	.000
6	3	6	.000	999.999	.180
7	5	0	.000	2.000	.100
8	0	4	.350	.350	.000
9	4	5	.000	999.999	.000
10	4	6	.000	999.999	.600
11	6	8	.000	999.999	.000
12	6	9	.000	999.999	.180
13	8	0	.000	2.000	.100
14	0	7	.500	.500	.000
15	7	8	.000	999.999	.000
16	7	9	.000	999.999	.600
17	9	0	.000	999.999	.180
18	11	2	.000	1.500	.000
19	11	0	.000	.800	.000
20	0	10	1.430	1.430	.000
21	10	11	.000	999.999	.000
22	10	12	.000	999.999	.500
23	12	14	.000	999.999	.000
24	12	15	.000	999.999	.150
25	14	5	.000	1.500	.000
26	14	0	.000	.800	.000
27	0	13	.910	.910	.000
28	13	14	.000	999.999	.000
29	13	15	.000	999.999	.500
30	15	17	.000	999.999	.000
31	15	18	.000	999.999	.150
32	17	8	.000	1.500	.000
33	17	0	.000	.800	.000
34	0	16	1.300	1.300	.000
35	16	17	.000	999.999	.000

Contd.

Arc No.	Source node	Sink node	Lower bound of flow	Upper bound of flow	Cost
36	16	18	.000	999.999	.500
37	18	0	.000	999.999	5.10
38	11	20	.000	.300	.000
39	20	11	.000	.700	.000
40	0	19	.770	.770	.000
41	19	20	.000	999.999	.000
42	19	22	.000	999.999	.400
43	22	24	.000	999.999	.000
44	22	26	.000	999.999	.120
45	0	21	.220	.220	.000
46	20	21	.000	999.999	.100
47	21	20	.000	999.999	.000
48	21	25	.000	.500	.010
49	14	24	.000	.300	.000
50	24	14	.000	.700	.000
51	0	23	.490	.490	.000
52	23	24	.000	999.999	.000
53	23	26	.000	999.999	.400
54	26	28	.000	999.999	.000
55	26	30	.000	999.999	.120
56	0	25	.140	.140	.000
57	24	25	.000	999.999	.100
58	25	24	.000	999.999	.000
59	25	29	.000	.500	.010
60	17	28	.000	.300	.000
61	28	17	.000	.700	.000
62	0	27	.700	.700	.000
63	27	28	.000	999.999	.000
64	27	30	.000	999.999	.400
65	30	0	.000	999.999	.120
66	0	29	.200	.200	.000
67	28	29	.000	999.999	.100
68	29	28	.000	999.999	.000
69	29	0	.000	.500	.010

Table 4

Computer outputs. Optimum distribution of flows in the network

Arc	Flow	Arc	Flow	Arc	Flow
1	1.880	24	.000	47	.000
2	.550	25	.730	48	.290
3	.550	26	.800	49	.000
4	.000	27	.910	50	.620
5	.000	28	.910	51	.490
6	.000	29	.000	52	.490
7	1.080	30	.000	53	.000
8	.350	31	.000	54	.000
9	.350	32	1.200	55	.000
10	.000	33	.800	56	.140
11	.000	34	1.300	57	.000
12	.000	35	1.300	58	.130
13	1.700	36	.000	59	.300
14	.500	37	.000	60	.000
15	.500	38	.000	61	.700
16	.000	39	.700	62	.700
17	.000	40	.770	63	.700
18	1.330	41	.770	64	.000
19	.800	42	.000	65	.000
20	1.430	43	.000	66	.200
21	1.430	44	.000	67	.000
22	.000	45	.220	68	.000
23	.000	46	.070	69	.500

Optimum value of objective function: 0.48390

Costs and damages in the same units:

Pumping	0.1
Water retained in the emergency reservoir	0.01

Flood damages, when proceeding from the upper node downwards: 0.4—0.5—0.6. Starting data of the analysis of the system in Figs 4 and 5 have been compiled in Table 3, computation results in Table 4 and Fig. 6. This problem required 1 minute of running time on a computer ODRA 1204 of Polish make.

To illustrate the rapidity of the out-of-kilter method, the scheme of a major drainage system is shown in Fig. 7, and the systems theory scheme

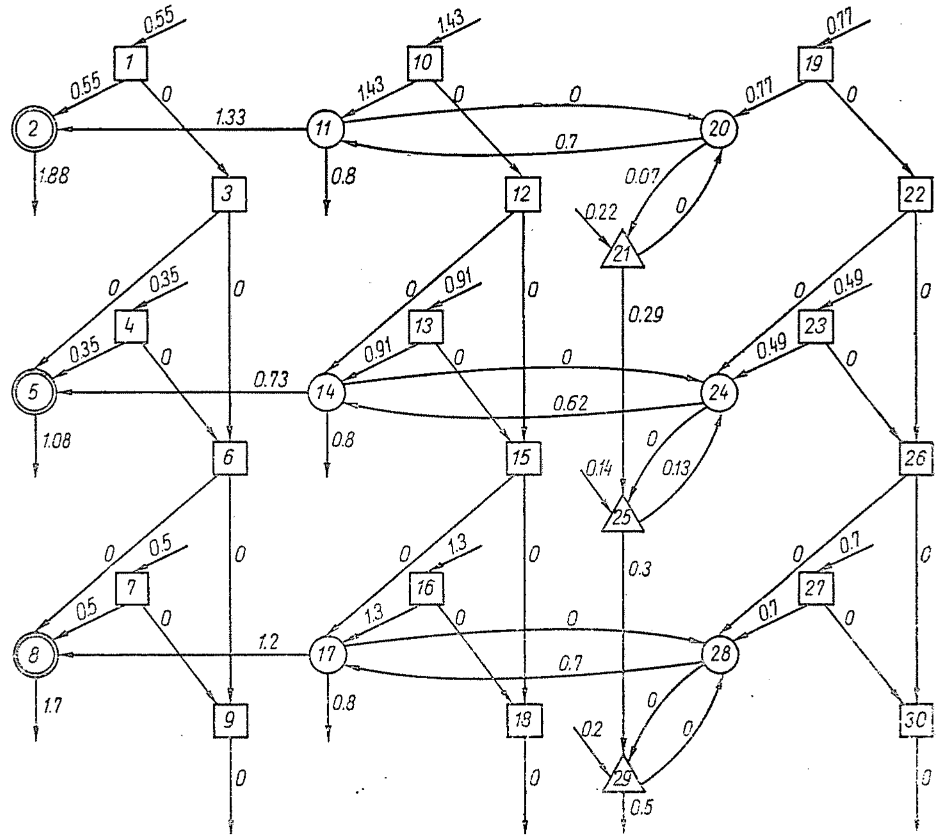

Fig. 6. Out-of-kilter optimization output (See Legend to Fig. 5).

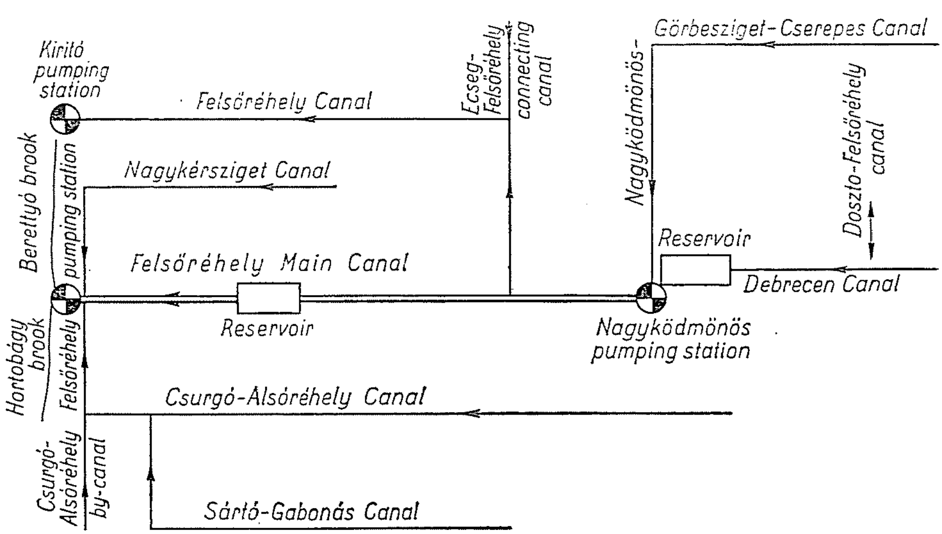

Fig. 7. Scheme of a drainage system in a plain area

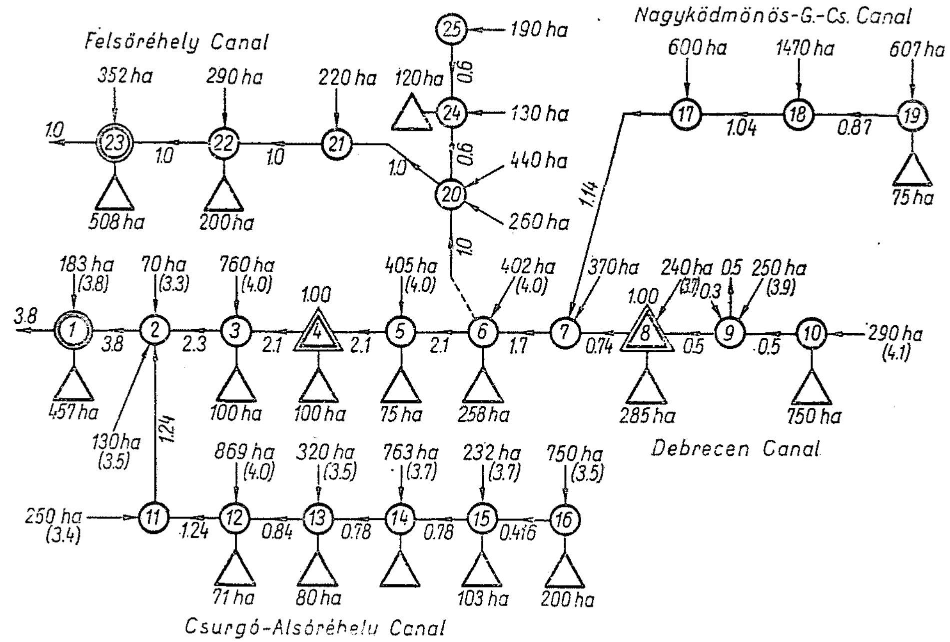

Fig. 8. Preliminaries to simulating a drainage system in a plain area by network flow model
Legend:
③ No. of canal node
290 ha Size of the area belonging to the node, in hectares
1.04 Max. canal capacity, million cu · m

for the development of the network flow model of the system in Fig. 8. The 3-interval network flow model of the system had 646 arcs and 279 nodes, the problem was solved within 16 min on an ODRA 1204 computer.

Summary

Operation of water management systems lends itself for network flow model simulation, as found recently by several researchers. Network flow models of two types of water management systems have been examined at the Institute of Water Management and Hydraulic Engineering, Technical University, Budapest. One concerned the optimum distribution of water reserves of the Tisza River, the other the optimum control of a canal network, pumping plants and reservoirs of water drainage systems in the plain. Models have been solved by an out-of-kilter algorithm. Some problems were also optimized by linear programming of which the out-of-kilter method was found to be ten to twenty times faster. Experience shows the out-of-kilter algorithm to be an efficient method of process control in water management systems.

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