FLUID VELOCITY MEASUREMENT BY MEANS OF LASER-DOPPLER ANEMOMETRY

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The laser beam measurement of velocity introduced by YEH and CUMMINGS in 1964 is a perfectly new way of measuring point velocities in flowing media. Ever since, an extremely intensive research and development work is going on, giving rise to many publications. Research covers every detail of the complex such as effect of optical arrangement, modes of signal detection, ways of signal processing, systematic errors etc. Relevant research is being performed in the Laboratory of this Institute to be described in the following, together with the principle underlying laser beam velocity measurements.

Formation of the Doppler frequency

Laser velocity measurement — usually referred to as LDV, LDA, LV, from initials of Laser Doppler Velocitymeter or Anemometer — is based on the Doppler-effect-induced frequency variation of light scattered from micron-size particles moving with the flowing medium the variation of which is proportional to the particle velocity. Thus, the measured medium is required to contain alien particles small enough to move together with it and producing adequate light diffusion. Ordinary tap water was seen to contain enough of such particles.

The formation of a Doppler signal can be deduced in different ways. For the sake of simplicity, and to ease understanding for inexperts, the Doppler effect will be deduced from optical geometry, a possibility due to the fact that the examined velocity range is much below that of light propagation so the phenomenon is not a relativistic one.

In case of a relative displacement between a light source and the observer, this latter will perceive a frequency different from the original one at the signal sources. The phenomenon is similar when both the light source and the sensor are in a mutually fixed position but the signal is reflected by a moving object. This also holds for light waves. Let us calculate the change of frequency for an arrangement according to *Fig. 1a*, where

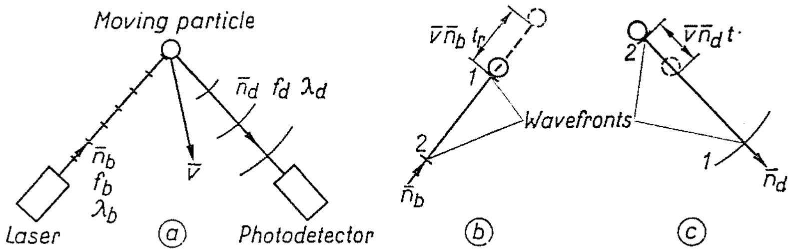

Fig. 1

f_b frequency of incident laser beam
λ_b wave length of incident laser beam
$\bar{n}_b$ unit vector of direction of incident laser beam
f_d frequency of scattered light
f_d wave length of scattered light
$\bar{n}_d$ unit vector of sensing direction
$\bar{V}$ particle velocity vector.

A standing particle would receive

$$f_b = \frac{c}{\lambda_b} \tag{1}$$

wave fronts during unit time (at light velocity c). Assuming the particle to be just attained by wave front 1 at time t_0 *(Fig. 1b)*, wave front 2 is exactly at a distance λ_b from it, to reach it after the interval t_r. During this time, the particle will be displaced by a distance $\bar{V}\cdot\bar{n}_b\cdot t_r$, hence wave front 2 will travel $\lambda_b + \bar{V}\cdot\bar{n}_b\cdot t_r$ during the interval t_r to reach the particle:

$$t_r = \frac{\lambda_b + \bar{V}\bar{n}_b \cdot t_r}{c}\,. \tag{2}$$

Its reciprocal is the frequency of light source as seen from the moving particle:

$$f_r = \frac{1}{t_r} = \frac{c}{\lambda_b + \bar{V}\bar{n}_b t_r} = \frac{c - \bar{V}\bar{n}_b}{\lambda_b}\,. \tag{3}$$

This light of frequency f_r is scattered also towards the sensor *(Fig. 1c)*. Hence, two wave fronts leave the particle at intervals t_r. During this time, the particle approaches wave front 1 by a distance $\bar{V}\cdot\bar{n}_d\cdot t_r$ yielding the wave length of scattered light as the spacing between two neighbouring wave fronts:

$$\lambda_d = \lambda_r - \bar{V}\cdot\bar{n}_d\cdot t_r\,. \tag{4}$$

Substituting $\lambda_r = c/f_r$ and $t_r = 1/f_r$, Eq. (4) yields for the wave length of scattered light:

$$\lambda_d = \frac{c - \overline{V}\overline{n}_d}{f_r} . \tag{5}$$

Substituting Eq. (3) for f_r one has:

$$\lambda_d = \frac{c - \overline{V}\overline{n}_d}{c - \overline{V}\overline{n}_b} \cdot \lambda_b = \lambda_b \frac{1 - \frac{V\overline{n}_d}{c}}{1 - \frac{Vn_b}{c}} \tag{6}$$

and thus the frequency of scattered light will be

$$f_d = \frac{c}{\lambda_d} = \frac{c}{\lambda_b} \cdot \frac{1 - \frac{\overline{V}\overline{n}_b}{c}}{1 - \frac{\overline{V}\overline{n}_d}{c}} . \tag{7}$$

The Doppler frequency is the difference between the respective frequencies of incident and scattered light:

$$f_D = f_d - f_b = \cdot \frac{c}{\lambda_b} \left[\frac{1 - \frac{\overline{V}\overline{n}_b}{c}}{1 - \frac{\overline{V}\overline{n}_d}{c}} - 1 \right] . \tag{8}$$

By multiplying the term in brackets by c and by reducing it to a common denominator one obtains

$$f_D = \frac{1}{\lambda_b} \frac{\overline{V}\overline{n}_d - \overline{V}\overline{n}_b}{1 - \frac{\overline{V}\overline{n}_d}{c}} . \tag{9}$$

For $V \ll c$, always true in practice, the second term in the denominator of (9) can be neglected, and thus, the Doppler frequency is obtained as:

$$f_D = \frac{1}{\lambda_b} \cdot \overline{V}(\overline{n}_d - \overline{n}_b). \tag{10}$$

The determination of the Doppler frequency seems to be difficult at the first glance, namely the practically possible frequency change between 10^6 and 10^8 Hz is rather slight when compared to that of the light beam being around 10^{15} Hz. Determination of the Doppler signal is much simplified by optical mixing. This means essentially nothing else but to conduct two signals of different frequencies upon an element with non-linear characteristic, the output of which will display, together with other components, a signal component of a frequency equal to the frequency difference between both signals. Hence, in the actual case, one signal may be the original laser beam itself, and the other a diffuse light of a frequency differing from that of the former by f_D according to (10). This procedure, of the arrangement shown in *Fig. 2*, is called the reference beam method. Laser beam is decomposed in two parts by means of the partially reflecting mirror T_1, then reflected by mirrors T_2

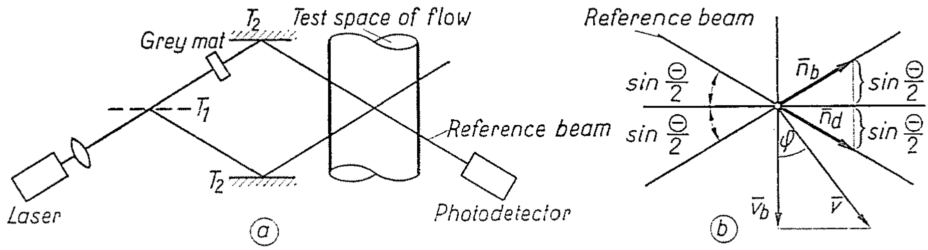

Fig. 2

to the measuring point. The reference beam falls directly on the detector surface, while the other beam is scattered by a particle passing through the intersection point. Applying (10) to *Fig. 2b*, the two unit vectors differ by $2.\sin\,\theta/2$, and in the occurrence of an isosceles triangle, they are normal to the bisector between the two beams. Essentially, the scalar product $\overline{V}(\overline{n}_d - \overline{n}_b)$ is the component of the velocity vector V normal to the bisector:

$$\overline{V}_B = \overline{V} \cdot \cos\varphi \tag{11}$$

and this substituted into (10) will yield:

$$f_D = \frac{2\overline{V}}{\lambda_b} \sin\frac{\theta}{2} \cdot \cos\varphi = \frac{2\overline{V}_B}{\lambda_b} \cdot \sin\frac{\theta}{2}\,. \tag{12}$$

The reference beam is seen to be on the same straight line as the sensing direction, hence their unit vectors differ by zero, the frequency of the reference beam remains inaltered.

Another possible measuring arrangement is the so-called dual-beam method, similar to that in *2a*, but with the substantial difference that the sensing direction does not coincide with that of the reference beam but has an arbitrary direction $\overline{n}_e$ *(Fig. 3)*. Taking the direction $\overline{n}_d$ of the incident beam and that of the sensing n_e into consideration, (10) yields the diffuse light frequency as:

$$f_1 = f_b + \frac{1}{\lambda_b} \cdot V(\overline{n}_e - \overline{n}_d)\,. \tag{13}$$

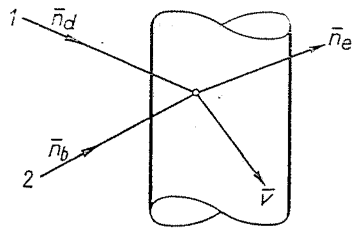

Fig. 3

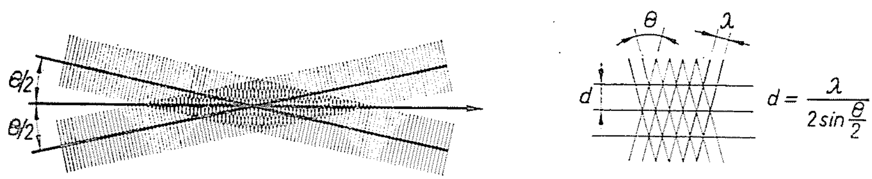

Fig. 4

Similarly, for the beam $\bar{n}_b$ one may write:

$$f_2 = f_b + \frac{1}{\lambda_b} \cdot \bar{V}(\bar{n}_e - \bar{n}_b). \tag{14}$$

On the detector output there appears the difference of the two frequencies:

$$f_k = f_1 - f_2 = \frac{1}{\lambda_b} \cdot \bar{V}(\bar{n}_b - \bar{n}_d),$$

not containing the unit vector of sensing direction $\bar{n}_e$. Obviously, the detector output frequency is independent of the sensing direction but is unambiguously determined by the position of the two incident beams. Thereby, the scattered light can be collected by a lens, improving the signal/noise ratio.

The dual-beam method is clearly illustrated by *Fig. 4*.

At the intersection point of both beams, interference fringes appear, spaced, according to the figure, at:

$$d = \frac{\lambda}{2\sin\frac{\theta}{2}}. \tag{15}$$

The particle crossing this intersection point also crosses these fringes and scatters more or less of light. For a given arrangement, the frequency of the light intensity variation depends on the particle velocity normal to the bisector:

$$f = \frac{\bar{V}_b}{d} = \frac{2 \cdot \bar{V}_b}{\lambda} \cdot \sin\frac{\theta}{2} \tag{16}$$

which is the same as (12).

Processing the Doppler signal

Two methods are in use for processing the Doppler signal output of the photodetector. One is the so-called counting method, consisting essentially in counting and timing a given number (e.g. 10) of impulses of the Doppler signal. One tenth of this time is the signal period, and its reciprocal is the signal frequency.

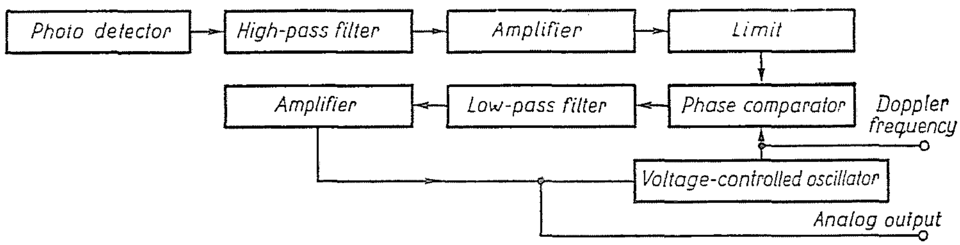

Fig. 5

The other method is based on the so-called PLL circuit, built up according to *Fig. 5*. Detector signal is conducted through a filter and an amplifier to the PLL circuit. In this circuit the voltage-controlled local oscillator signal is compared with the detector signal by means of a phase-difference indicator circuit. If their frequencies differ, then a voltage will appear on the output of the phase-difference indicator, tuning, after filtering and amplifying, the voltage-controlled oscillator to a frequency equalling that of the input signal. Thus, the voltage controlling the oscillator becomes proportional to the Doppler frequency, hence also to the fluid velocity.

Laboratory experiments

Laser velocity measurement was seen to need two beams from the same laser. There are different methods to produce these two beams. Our tests involved the arrangement shown in *Fig. 6*.

The original beam was transformed by the biconvex lenses L_1 and L_2 into a parallel beam of about 40 mm dia. Its path was cut by a mat made of an opaque material, pierced by two 3 mm holes 20 mm apart. Hence, behind the mat we had two parallel beams projected by lens L_3 to the measuring spot. The scattered light has been collected by lens L_4 and projected upon the cathode of the photo-electron multiplier. The signal emitted by the photo-electron multiplier passed through a band-pass filter and then got amplified. *Photo 1* is a Doppler signal displayed on the storage oscilloscope screen.

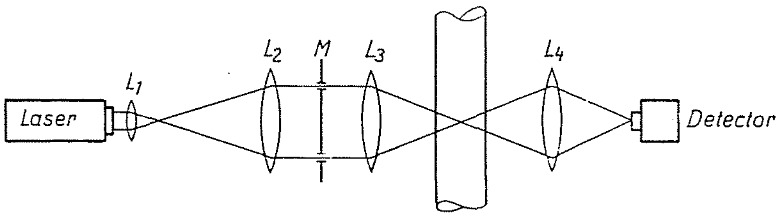

Fig. 6

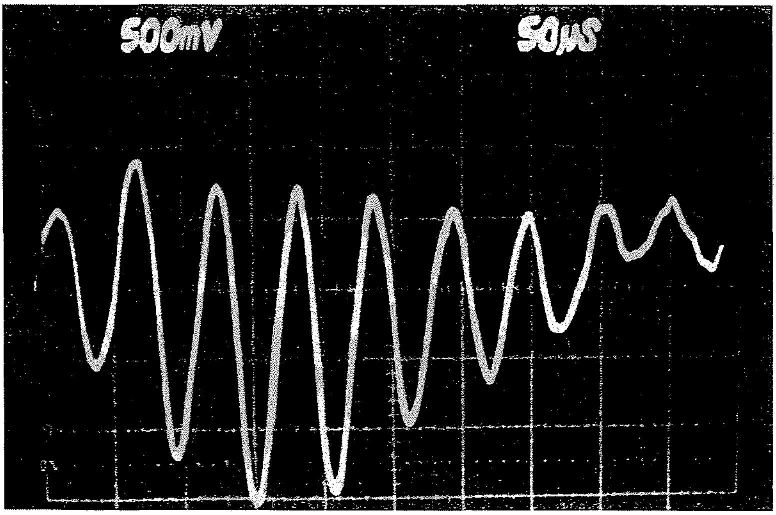

Photo 1

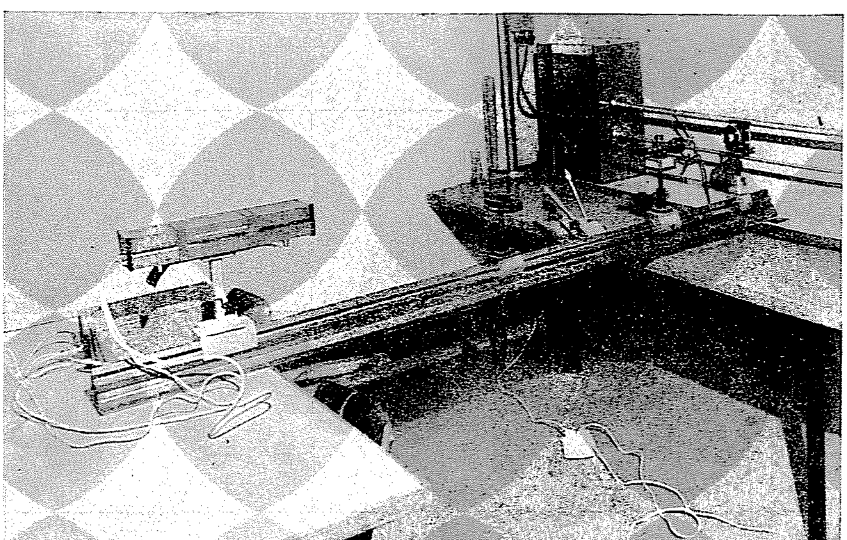

Photo 2

Our equipment has been applied to record the velocity distribution of laminar flow in the glass tube 20 mm dia. of the laboratory Reynolds apparatus. Measurement arrangement is seen in *Photo 2* (with sensing lens and detector placed behind the model, receiving light through a circular hole in the back side).

Measured and theoretically computed velocity profiles were in fair agreement. To produce the corresponding D-signal, a slight amount of talcum powder was added to water. Tests were made with a 5 mW He—Ne gas laser made by MOM Hungarian Optical Works.

Summary

The generation of the Doppler signal has been derived by means of considerations borrowed from optical geometry. Two fundamental methods — those of interference fringes and of dual beams — have been described and their difference explained. The dual-beam method is also explained by the development of interference fringes. Various possibilities of signal processing are described, together with laboratory LDV tests.

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