UNSTEADY FLOW DEVELOPING UNDER THE EFFECT OF PEAK-HOUR OPERATION ON THE REACH OF THE DANUBE BETWEEN NAGYMAROS AND ADONY

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This paper presents a mathematical method yielding the hydraulic parameters of unsteady flow developing under the effect of the peak-hour operation of the two connected hydroelectric power stations of Gabčikovo and Nagymaros. A mathematical model adaptable to any other water-power system is presented, too. Finally, the results of the calculations carried out in connection with the reach of the Danube between Nagymaros and Adony are reported.

1. The train of barrages Gabčikovo — Nagymaros

Several conceptions have been proposed in recent years in connection with the exploitation of the Hungarian and Czecho-Slovakian common reach of the Danube. The intention is to produce under advantageous economical conditions as much energy as possible. As a result of estimating several design variants it was found that the above objective could only be realized by making use of a system consisting of the *Gabčikovo* hydraulic power station with a diversion canal fed by the *Dunakiliti* bank storage work and of the Nagymaros hydro-electric power station in co-operation with the above power plant.

The general layout of the power-station system planned is shown in Fig. 1. Near *Dunakiliti* at the River Station 1842 km in the right-hand side cut-off a *barrage* and a *temporary navigation lock* will be built. The function of the barrage will be to maintain the water-level of the reservoir *Dunakiliti-Hrušov* and to permit a temporary navigation also in the abandoned bed. The *Dunakiliti-Hrušov* reservoir being between flood dikes up to Bratislava assures the flood rate necessary to the peak-hour operation of the diversion-type power plant of Gabčikovo. The effective capacity of the reservoir is planned to be 60 million cubic metres.

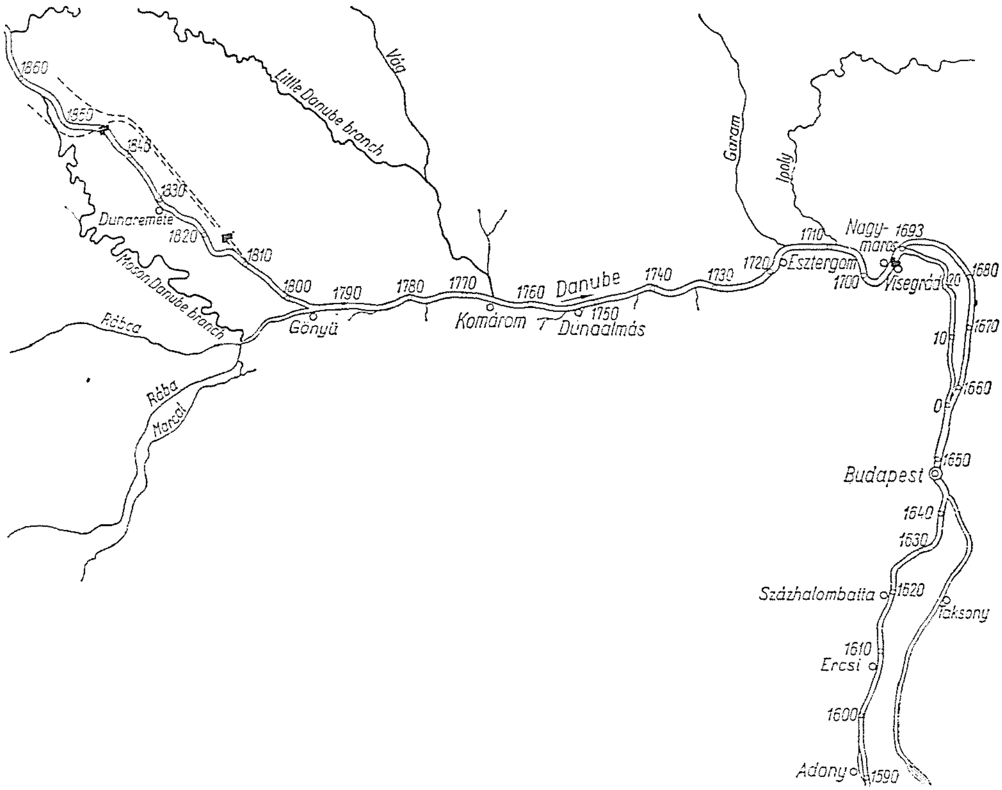

Fig. 1. General layout of the power-plant system Gabčikovo—Nagymaros

The new channel planned, i.e. the diversion canal will be built on the left-hand side of the Danube, partly in the flood plain, partly on the protected area. The 17 km long *headrace canal* conveys water with minimum loss of energy and water, and constitutes also the channel for navigation. The *tailrace* canal is built at its full length (8,2 km) in cut, in order to realize a useful head as high as possible.

The *Gabčikovo Barrage* will be constructed at River Station 17 km. At the hydro-electric power station of 700 MW nine vertical-shaft Kaplan turbines with 8 m diameter impellers will be installed. Two navigation locks, of a useful ground-surface 34×275 m each, permit the maintenance of navigation.

The *Nagymaros Barrage* will be constructed at River Station 1696,25 km. Water-power utilization and navigation will be assured by hinged-leaf radial gates situated in seven 24 m spans. The turbines of 273 m^3/s inverted capacity furnish 146 MW total rated output [6].

The construction of the water-power station system changes the runoff regime which presents new boundary conditions to the hydrological regime.

The forecast of this latter is indispensable both to planning, economical calculations and to working out operating instructions.

The detailed calculations are necessary considering the complex interaction of the hydroelectric power plants: the interaction and the peak-hour operation and natural hydrological regime, energetic conditions, changes in the design water-level with respect to the stability of the banks, bottom-current velocities and their duration, determination of the changes in space and with time of deposits due to backwater, effect of the backwater on the development of the flood runoff levels, effect of the backwater and peak-hour operation of special river training problems, etc. [3].

2. Mathematical model of the calculation

Purpose of the calculation: the following should be defined: change with time t of the water level Z and discharge Q in all of the calculation sections x_i of the river. Mathematically, the problem consists in determining at discrete points the following functions [1, 2]:

$$Q = Q(x, t) \tag{1}$$

$$Z = Z(x, t). \tag{2}$$

2.1 Governing equations of unsteady water flow

The hydraulic behaviour of open-channel gradually varied, unidimensional flows is described by a *dynamic equation:*

$$\frac{\delta z}{\delta x} - \frac{\alpha' Q^2}{gF^3}\frac{\delta F}{\delta x} - \frac{\alpha'' Q}{gF^2}\frac{\delta F}{\delta t} + \frac{\alpha' Q}{gF^2}\frac{\delta Q}{\delta x} + \frac{\alpha'' \delta Q}{gF\delta t} + \frac{Q^2}{K^2} + \frac{\alpha'' Qq}{gF^2} = 0 \tag{3}$$

and by a *continuity equation:*

$$\frac{\delta Q}{\delta x} + \frac{\delta F}{\delta t} - q = 0 \tag{4}$$

wherein the symbols of the variables not yet defined are as follows:

$F[\mathrm{m}^2]$	— wetted area perpendicular to flow,
$K[\mathrm{m}^3/\mathrm{s}]$	— water conveyance,
$q[\mathrm{m}^3/\mathrm{s}/\mathrm{m}]$	— specific transversal volume flow or linear load,
$g[\mathrm{m}/\mathrm{s}^2]$	— gravity acceleration,
α'	— coefficient of dispersion of the momentum taking into account non-uniform velocity distribution, and
α''	— coefficient of dispersion of local acceleration.

The first-order differential equations (3) and (4) belong to the class of the *pseudolinear partial differential equations of the hyperbolic type*. An exact solution to the above equations by direct integration in general form, according to the present state of mathematical knowledge is not possible, however, several solutions may be obtained with close approximation of the physical truth.

2.2 *Solution by the method of characteristics*

From among the solutions to Eqs (3) and (4) the method of characteristics [1] was chosen.

The essential of the solution is: Eqs (3) and (4) should be completed by the total differential of the independent variables Z and Q, and the system of the partial differential equations thus obtained solved as a system of linear equations. The system of ordinary differential equations of characteristics may be obtained with the aid of the method of determinants.

Along the identical characteristics the equations

$$\mathrm{d}x = W \cdot \mathrm{d}t \tag{5}$$

$$dz = \frac{dQ}{BM} - \left(P\frac{Q|Q|}{K^2} + D_M\right) dx \tag{6}$$

and along the reverse characteristics the equations

$$\mathrm{d}x = M \cdot \mathrm{d}t \tag{7}$$

$$dZ = \frac{dQ}{BW} - \left(P\frac{Q|Q|}{K^2} + D_W\right) dx \tag{8}$$

are valid where the symbols beside those already interpreted are as follows:

W[m/s] — celerity of waves of identical direction,
M[m/s] — celerity of waves of reverse direction,
B[m] — width of the water surface,
P, D_m, D_W — parameters in case of most general validity of equations [1], [2].

The numerical integration of the characteristic equations (5) to (8) may already be easily performed. For this purpose, the river has been subdivided to reaches of finite length Δx and the total period of calculation into time intervals Δt. The ordinary differential equations were approximated by difference equations and the approximate numerical integration carried out separately for each of the regions Δx and Δt (Fig. 2).

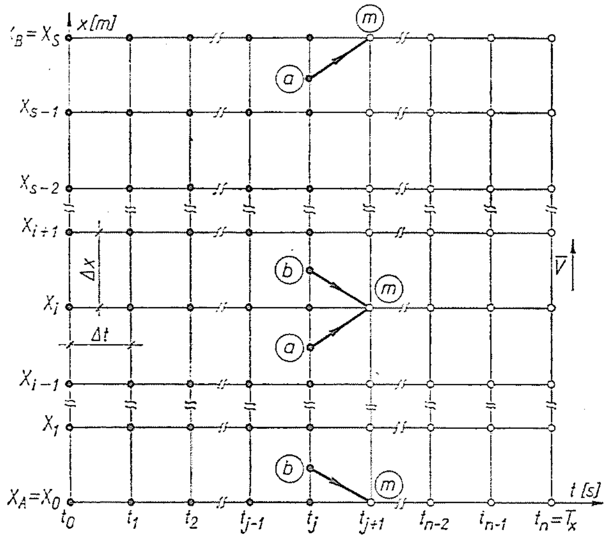

Fig. 2. Nodes of characteristics in case of an orthogonal network. Δx: length of design reach; Δt: design time interval; a m: identical characteristic; b m: reverse characteristic

2.3 Boundary conditions

From the theory of the differential equations it is known that for the solution of the system of equations of the characteristics the boundary conditions should be known. In the formulation of the theory of hydraulics these are *initial* and *boundary* conditions.

The *initial conditions* cover the momentary degree of repletion and the dynamic equilibrium of the channel, the distribution of the discharge and the surface profile at the beginning of the calculation in the form of the functions

$$Q = Q(x, t)_{t=t_0}$$

and

$$Z = Z(x,t)_{t=t_0}. \tag{9}$$

By knowing the basic data and the longitudinal distribution of the discharge, as a matter of fact, the determination of the surface profile constitutes the calculation of the initial condition. Assuming a non-uniform, gradually varying flow, this may be defined from the general energy equation (*Bernoulli*'s equation).

The *boundary conditions* are expressing the interaction and correlation of the system and its neighbourhood within the domain of interpretation [2]. In this particular case, this results in the establishment of the functions

$$Q = Q(x, t)_{x=x_{a,f}}$$

or

$$Z = Z(x, t)_{x=x_{af}} \tag{10}$$

in the lower and upper calculation sections. On a river controlled by water-power plants, in general, the change of discharge with time is given in the form of a peak-load schedule, steady-flow rating curve or constant mean daily discharge.

2.4 Node equations

Numerical integration and reduction of characteristic equations (5) to (8) yield the node equation [1].

The identical, semi-characteristic node equation is:

$$Z_m = Q_m\,(A|Q_m| + B) + C\ . \tag{11}$$

The reverse, semi-characteristic node equation is:

$$Z_m = Q_m(D|Q_m| + E) + F\ . \tag{12}$$

The general, bi-characteristic node equation, from (11) and (12), will be:

$$Q_m = \frac{F - C}{|Q_m^{r-1}|\,(A + B) + B - E}\ . \tag{13}$$

To the determination of Z_m either of Eqs (11) and (12) may be applied.

In this case, the determination of the discharge can only be carried out by successive approximation; the subscript r denotes the rth step of the trial-and-error method.

The values of the constants A, B, C, D, E and F in the node equations (11) to (13) resulted from Eqs (5) to (8) by integration.

Since the task is to work out a mathematical model which may be applied to other systems of river power plants, the effects of affluents or islands (if any) should be taken into account.

In Fig. 3 the characteristic fields of the adjoining node are seen in an enlarged representation.

The equations at the adjoining node at the mouth of the affluent or branch of the river at the lower end of the island. The formulae of iteration derived from the identical and reverse characteristic equations of the main arm — (11) and (12) — and from the identical equation (11) of the affluent or branch, to the calculation of the discharges are as follows:

$$Q_m^r = \frac{C_M - F_L + (C_K - C_M)\left(\frac{a}{b}\right)^{r-1}}{b^{r-1} - c^{r-1} + \left(\frac{bc}{a}\right)^{r-1}} \tag{14}$$

$$Q_K^r = \frac{Q_M^r\, c^r + C_M - C_K}{a^{r-1}} \tag{15}$$

$$Q_L = Q_K + Q_M\,. \tag{16}$$

The common water level $Z_K = Z_L = Z_M$ may be calculated from either of Eqs (11) and (12). The symbols entering in the relationships (14) to (16) are as follows:

A, B and C — constants obtained from identical characteristic equations,
D, E and F — constants obtained by integration of reverse characteristic equations,
K — subscript referring to section of main branch upstream the mouth,
L — subscript which refers to section of main branch downstream the mouth,
M — subscript which refers to mouth section of affluent or branch (Fig. 3)

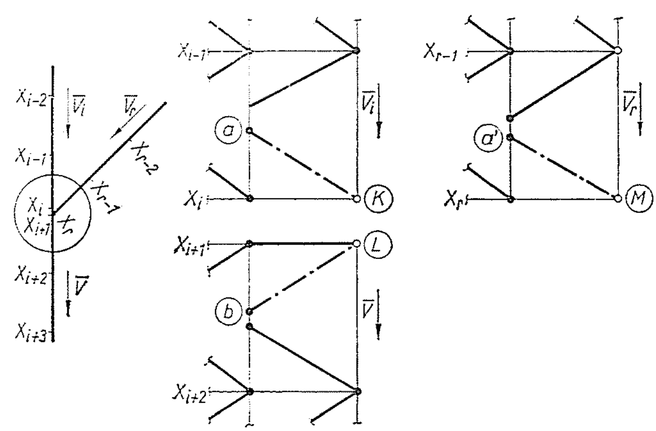

Fig. 3. Interpretation of the distribution of sections and of the characteristics at the mouth of affluent or river branch

r and $r-1$ — subscripts which designate number of internal steps of successive approximation; furthermore:

$$a = A_K \mid Q_K \mid + B_K \tag{17}$$

$$b = D_L \mid Q_K \mid + Q_M + E_L \tag{18}$$

$$c = A_M \mid Q_M \mid + B_M. \tag{19}$$

The equations of the adjoining node may be derived in a similar way for the end of the island at the affluent of the branch.

3. Computer program of the mathematical model

The calculation of the unsteady flow without electronic computers is practically impossible.

Computer programs of ALGOL-1204 representation were established for an ODRA-1204-type digital computer. Due to their large extent, the programs are here not detailed. Their form of construction adopts the process system, and as such, they are easily transformable. All of the significant parameters of the system may be changed, as for example, the geometric and hydraulic values, the initial and boundary conditions, Δx, Δt, the number of the water power stations, etc.

These possibilities of changing satisfy in a rather manifold way the practical requirements and permit the designers and investors to work out and to compare several concepts, to choose the most suitable solution and help in prescribing the operation schedule.

4. Application of the procedure

In this chapter the computation of the unsteady flow developing under the effect of the peak-hour operation on the reach of the Danube between Nagymaros and Adony by aid of the mathematical model and program will be described.

4.1 Basic geometric and hydraulic data

From the profiles drawn in the Atlas of Hydrography the *geometric basic data* of the calculation sections were determined. The geometric characteristics required to the mathematical approximation are illustrated on a typical cross section of Fig. 4.

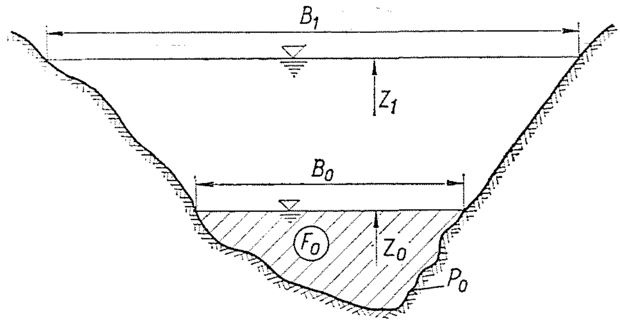

Fig. 4. Cross section geometry

The *Szentendre Danube branch* as well as the main channel are characterized by basic data.

The basic hydraulic values, the coefficients of smoothness k have been determined from the surface curve fixed in 1971 ($Q = 1000$ m^3/s, given by the Institute for Hydraulic Planning) and from the basic data. Along a dredged reach of the Danube branch of Vác $k = 40$ m$^{1/3}$/s and on the Szentendre Danube branch $k = 30$ m$^{1/3}$/s values have been assumed.

4.2 Initial conditions

In case of the *variant A-1* the regime before starting the peak-hour operation was assumed nearly permanent. Therefore, the discharge is, along the whole reach, the base flow of the Q–t diagram of the peak-hour operation at Nagymaros, i.e., $Q = 1000$ m^3/s. The initial conditions have been computed with the aid of the initial level, on the basis of the fundamental geometric and hydraulic data, the knowledge of the function $Q = Q(x,t)_{t=0}$ as well as the permanent rating curve of the gauging section at Adony. The values computed are represented together with the measurement results; the diagram corresponding to the hour $t = 0$ is shown in Fig. 8.

The assumption of a nearly steady character of the initial condition is an approximation; even in case of one peak a day, and after the runoff of a lasting constant discharge (at present 1000 m^3/s during about 19 hours). To eliminate the inexactitude caused by this approximation, in case of the *variant A-2*, the values $Z-Q$ calculated for the 24th hour of the previous day have represented the initial value.

4.3 Boundary conditions

The *upper boundary condition* is the discharge passing through the Nagymaros water power station, and varying with time. The diagram Q—t is given in Fig. 5. In order to investigate a satisfactorily long steady regime after the

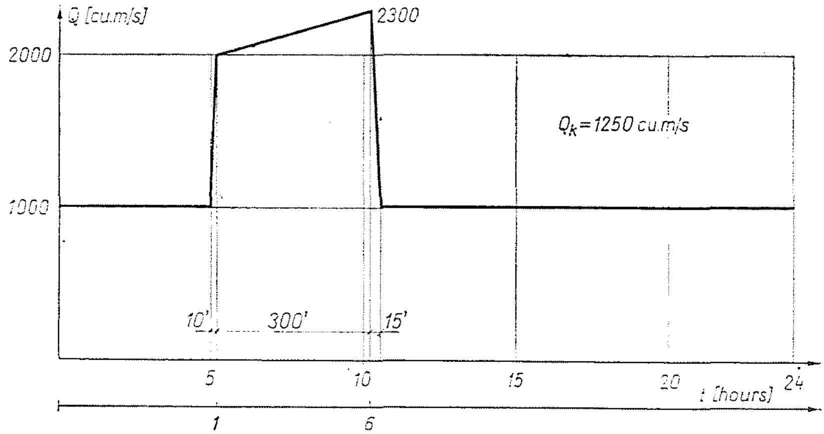

Fig. 5. Characteristic curves $Q = Q(x, t)_{x=1696,25}$ km and $Q_0 = Q(Z)$ of the boundary conditions

peak-hour operation, the axis x has been translated to the left, and the calculation carried out on the basis of the lower abscissa.

The *lower boundary condition* is the steady-flow discharge rating curve of the Adony gauging section, given by the Institute for Hydraulic Planning. The section of the discharge diagram (see Fig. 5) lying in the range of a discharge 716 to 1692 m^3/s has been approached by a second-order parabola. The values $Z-Q$ of the section of the boundary condition are yielded by a simultaneous solution of the node equation (11) and the discharge rating curve.

5. Evaluation of the computer results

With the help of the basic data, operation data and program detailed in the foregoing, the computation has been performed.

5.1 Evaluation of the results of variant A-1 ($t = 0$ to 24 hours)

To the calculation of the first-day phenomenon the quasi-steady initial condition has been assumed.

The characteristics $Z = Z(x, t)_{x=x_i}$ seen in Fig. 6 clearly show the progress and flattening out of the flood wave. The maximum rise of the tailwater level of the Nagymaros power station is 1,3 m (at the sixth hour) while at the lower mouth of the Szentendre Danube branch this value is not higher than 0,68 m (at 8,75 hours). At the Adony section (1594,65 km), due to a significant flattening out of the flood wave, the maximum rise of the water-level is 0,48 m (at 18,75 hours). After 24 hours, in the tailwater of the power station the water level of the initial condition has been restored with an insignificant difference, i.e., $\Delta Z = 1{,}7$ mm. In the Adony section, at the 24th hour, the

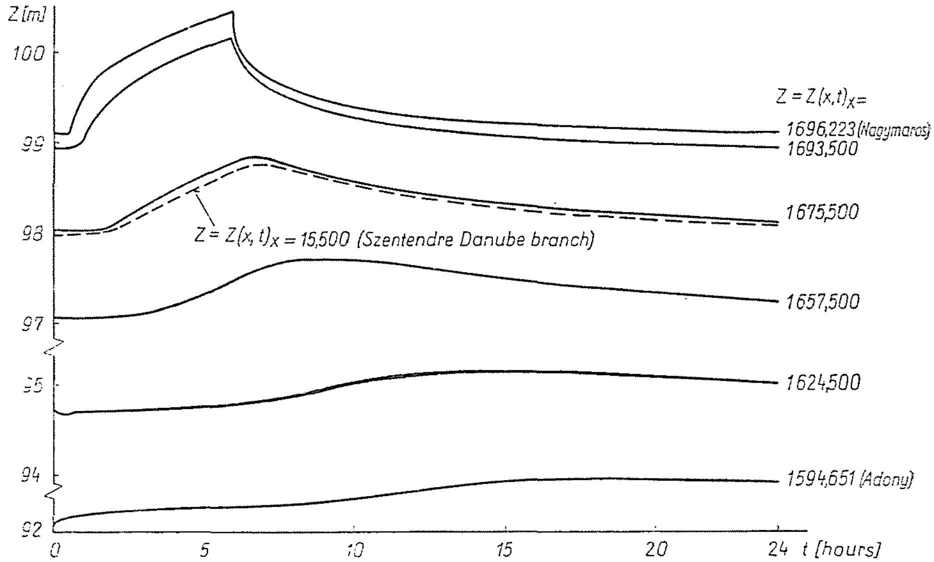

Fig. 6. Characteristic curves $Z = Z(x, t)_{X=x_i}$ in case of variant A-1

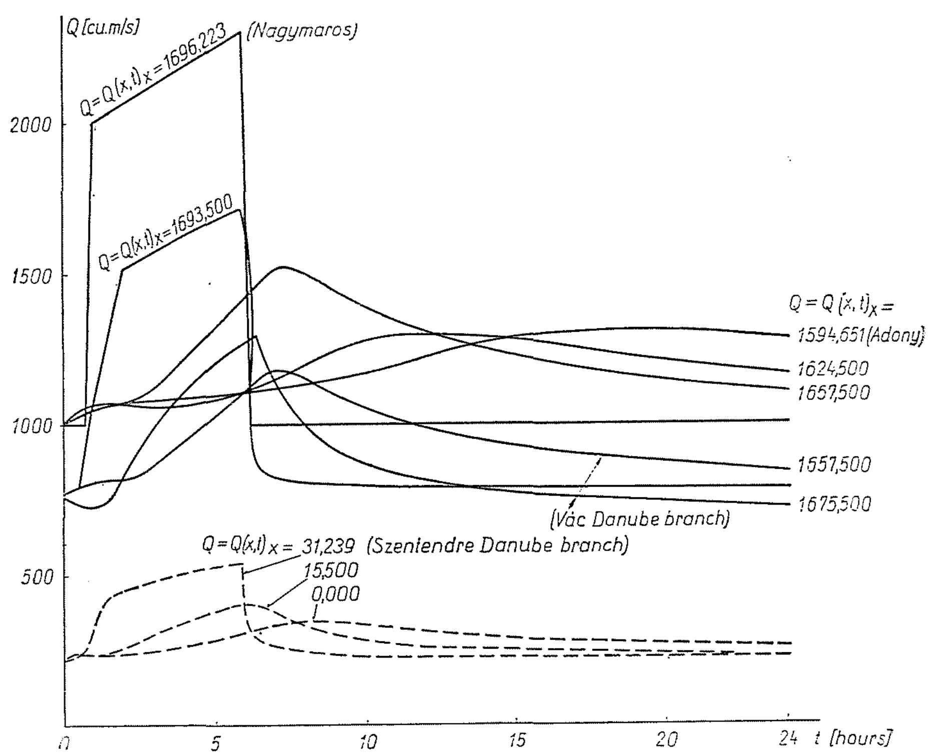

Fig. 7. Characteristic curves $Q = Q(x, t)_{X=x_i}$ in case of variant A-1

flow is not yet steady, on the descending limb of the flattened wave, 5 hours after the peak, a discharge of $Q = 1290$ m^3/s passes through, exceeding by 44 cm the initial stage.

In the characteristic cross sections the *change with time of the discharge* is represented in Fig. 7. It is worth while to draw the attention to two interesting observations. *One of them* is that on the main-branch side of the Szentendre island the progress of the wave is quicker, which may be ascribed to the higher celerity of the larger depths of water. The *second* observation refers to the approximate character of the calculation of the lower boundary condition. The peak flow of the Adony section, in agreement with considerations obtained from hydraulics, appears later than in a section by about 30 km upstream, however, its peak flood is higher by 13 m^3/s which contradicts the rules of flattening out. This contradiction results from the error committed in the lower boundary condition, i.e. from the assumption of the validity of the steady-state discharge rating curve. Namely, the actual discharge is higher on the rising limb of the flood wave and is lower on the descending limb than that calculated from the steady discharge rating curve.

Also the variation of the mean velocities in time was investigated. The maximum of the mean velocity is $v = 1.15$ m/s; this occurs in the tailwater of the water power station. It is worth while to consider also the velocities at the section of km station 1657.5. Partly, the dredging of the bed of the tailwater reach comes to an end here, partly the Szentendre Danube branch joins the main channel. Both increase the velocity. As a result, the mean velocity is all along over 0.8 m/s, and its maximum is 1.05 m/sec.

In Fig. 8 the *surface profiles* calculated for different times are seen. The deviation between the surface profiles related to $t = 0$ and $t = 24$ hours is at this variant not yet significant which may have several reasons. Certain deviation follows from the nature of the phenomenon: the falling limb of the wave does not yet leave the end section of the reach of about 102 km length, but from the initial condition assumed to be steady, also ensue some deviations.

This fact gave rise to the investigation of the next variant.

5.2 *Evaluation of the results of variant A-2* ($t = 24$ to 48 hours)

From the basic data and boundary conditions mentioned above and from the values $Z - Q$ obtained at the 24-th hour of the day before, used as initial conditions, another calculation has been carried out. In order to facilitate analysis of the deviation between the two calculations, time counting is started also in this case at 0 hour.

The maximum rise of the water level of the tailwater of the Nagymaros power plant is 1.33 m which means an insignificant difference in compari-

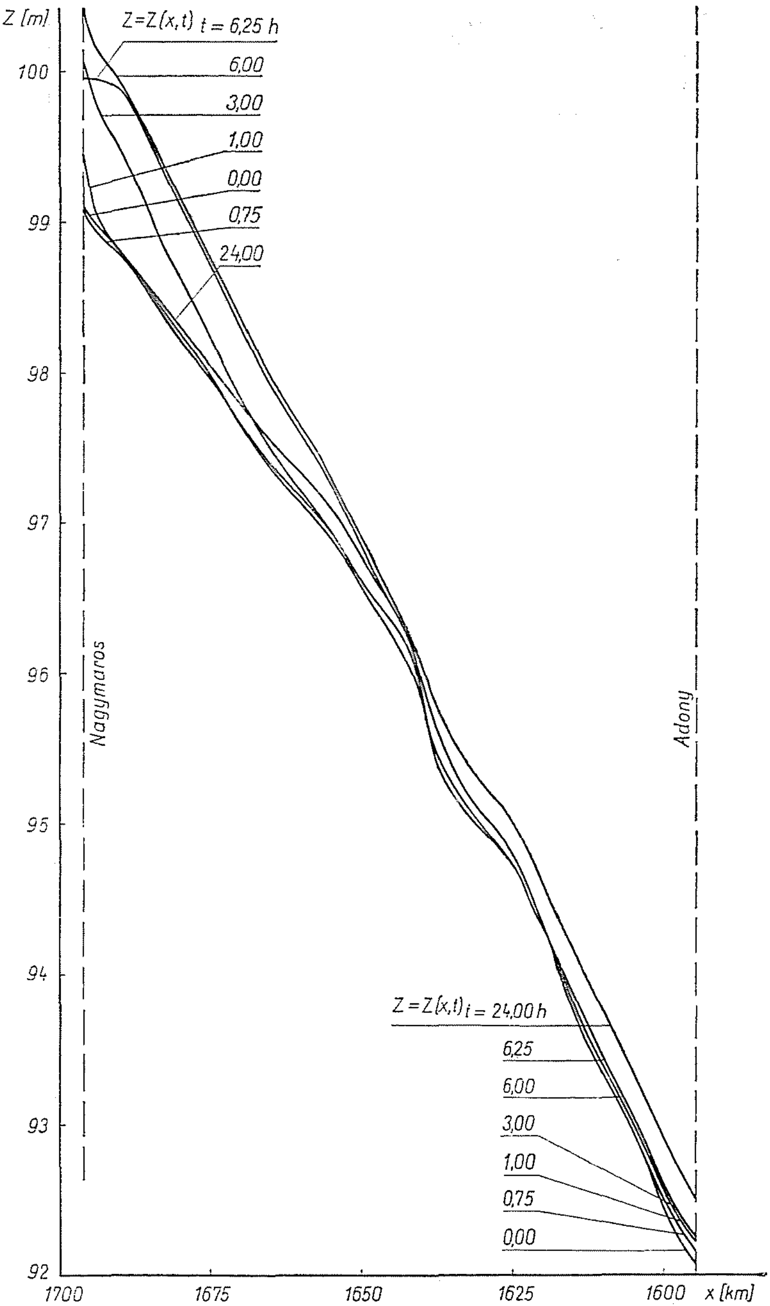

Fig. 8. Surface profiles $Z = Z(x, t)_{t=t_i}$ in case of variant A-1

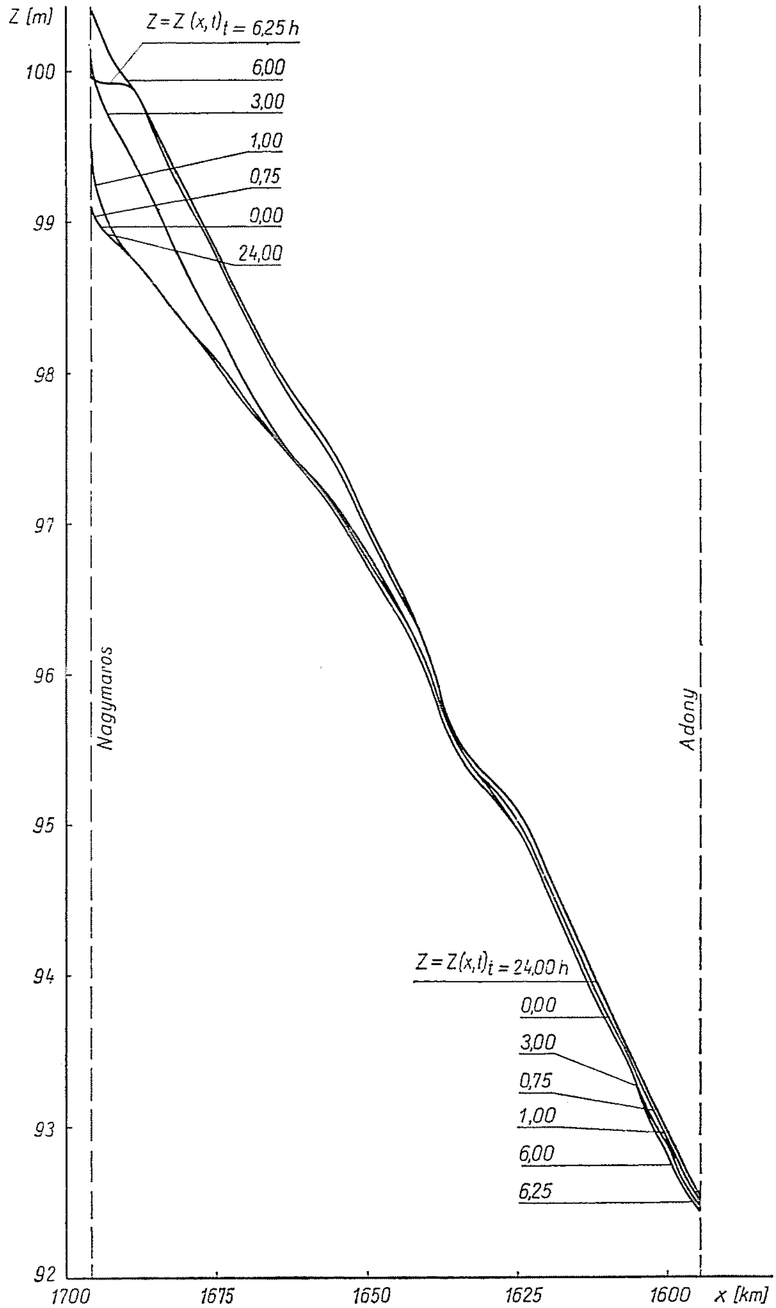

Fig. 9. Surface profiles $Z = Z(x, t)_{t=t_i}$ in case of variant A-2

son with the preceding variant. The difference between the absolute values of the two flood peaks is 2 mm. At the 48th hour in the tailwater of Nagymaros, the closing error, in comparison to the unsteady flow, is 9 mm. A significant deviation occurs downstream the km station 1633.

The maximum *rise of the water level* at the Adony section is 18.5 cm in comparison to the lowest level. The difference between the water levels of the 24th and 48th hour is only 4 cm which was 44 cm in case of the preceding variant.

A conspicuous deviation between the two variants may be found, in the first line, along the 1635 to 1594 km reach (Figs 8 and 9).

From the analysis of the *discharge* it is evident that due to the effect of the periodic peak-hour operation no steady regime is to develop, the longitudinal distribution of the discharge is unsteady. The maximum deviation between the discharges of the two variants does not attain 30 m^3/s, even in case of the peak discharges. In the discharge, the closing error is less than one per cent upstream the lower mouth of the Szentendre Branch of the Danube; it increases toward the Adony section, however, there either, it does not surpass 2 per cent.

As to the *mean velocities*, no significant differences may be observed in comparison with variant A-1.

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The mathematical model and its computer program described in the paper can be applied to compute the unsteady flow developing in the pools during the operation of other water power systems. Computation results, either presented here or not, furnish extremely significant information for the instance treated, for the whole reach of the Danube between Nagymaros and Adony, absolutely necessary for the designers and investors to perform economical river trainings and to establish a suitable operation manual for the sake of a safe and economical operation. As a matter of fact, these results hold only for the basic data, initial and boundary conditions given in the foregoing. In case of a change, however, the calculation has to be repeated.

Summary

The unsteady flow is calculated which develops on the Danube reach between Nagymaros and Adony affected by the peak-hour operation. To this end, the differential equations which describe this phenomenon have been solved, in general form, by making use of the *method of characteristics*. The way of presentation and the calculation of the geometric and hydraulic data are reported in short. The calculation results of a two-day phenomenon are given in the form of characteristics.

The mathematical model and its computer program may also be applied to the calculation of the unsteady flow developing in reaches during the operation of other water power stations.

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