CALCULATION METHODS OF DYNAMIC PRESSURE ACTING UPON SLUICE GATES

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Introduction

Fluid pressure distribution on sluice gates is differing from the hydrostatic one if either water is passing below a slightly lifted sluice gate or if there is an overfall along the top of the gate. For some of such cases approximate solutions have already been derived, based upon two-dimensional potential flow theory. Now, a more exact method, taking real boundary conditions into account, will be described and compared with an approximate method already known.

Case of a slightly lifted sluice gate

By assuming a sluice gate slightly lifted so as to have a gap of height a below the gate open to flow, then a pressure distribution differing from the hydrostatic one will be encountered, that is usually described by means of an approximate method owing to KULKA [1], [2].

The Kulka method is based upon the assumption that there is a sink at the point of intersection between the plane of the sluice gate and that of the bottom. The upstream surface of the sluice gate is considered as a streamline along which a velocity distribution, corresponding to that of a sink, will develop *(Fig. 1)*. The velocity distribution being thus known, the pressure distribution may be calculated by using the Bernoulli theorem, as

$$\frac{p}{\gamma} = (h_0 - h)\left(1 - \frac{a^2}{h^2}\frac{h_0 + h}{h_0 + a}\right) . \tag{1}$$

Some of the boundary conditions assumed in the Kulka method are in an obvious contradiction to reality, since the water surface $y = h_0$ is a streamline itself and so are all lines $y =$ const if far enough from the sluice gate in the upstream direction. All this shows but a very far likeness to a sink.

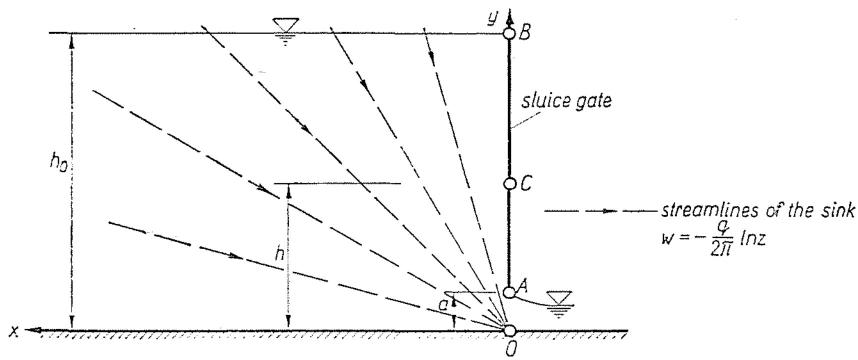

Fig. 1

Let us assume the water surface $y = h_0$, the sluice gate $x = 0$ and the channel bottom $y = 0$ to be streamlines. These conditions are fulfilled by the mapping function

$$w = \frac{-q}{2\pi} \ln \sinh \frac{\pi z}{2h_0} \tag{2}$$

representing sinks of a yield q (m^3/sec.m) distributed uniformly along the y-axis at intervals $2h_0$, between $y = 0$ and $y = \pm \infty$ (Fig. 2).

In order to calculate pressure distribution along the sluice gate, first of all, the potential and stream functions of the mapping function (2) have to be derived. For the sake of simplicity, the notation $c = \pi/2h_0$ will be intro-

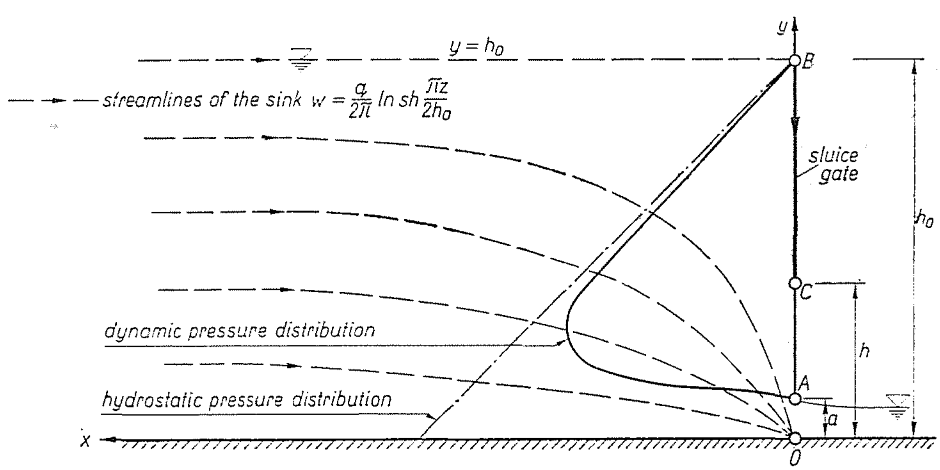

Fig. 2

duced provisionally. Thus we have

$$w = -\frac{q}{2\pi} \ln(\sinh cx \cos cy + i \cosh cx \sin cy) =$$

$$= -\frac{q}{2\pi} \ln(s + it) = \frac{-q}{2\pi} \ln u \tag{3}$$

with u being obviously a complex number. According to well-known relationships,

$$\ln u = \ln |u| + \operatorname{arc} u = \frac{2\pi}{q} \tag{4}$$

where:

$$|u| = \sqrt{s^2 + t^2} \qquad \text{and arc } u = \tan^{-1} \frac{t}{s}.$$

Hence:

$$w = \varphi + i\psi = \frac{-q}{2\pi} \ln u = \frac{-q}{2\pi} \left(\ln \sqrt{s^2 + t^2} + i \tan^{-1} \frac{t}{s} \right). \tag{5}$$

When returning to the original notations and separating real and imaginary terms on the left-hand and right-hand sides alike, one obtains

$$\varphi = -\frac{q}{2\pi} \left(\sinh^2 \frac{\pi x}{2h_0} \cos^2 \frac{\pi x}{2h_0} + \cosh^2 \frac{\pi x}{2h_0} \sin^2 \frac{\pi y}{2h_0} \right)^{1/2} \tag{6}$$

and

$$\psi = -\frac{q}{2\pi} \tan^{-1} \left(\tan \frac{\pi y}{2h_0} \Big/ \tan \frac{\pi x}{2h_0} \right) . \tag{7}$$

Through a substitution into Eq. (7) one may easily find out that the x-axis corresponds to streamline $\psi = 0$ and that both $y = h_0$ and $x = 0$ are parts of the streamline $\psi = q/4$, or in other words, the boundary conditions assumed at the beginning are really satisfied.

The velocity components of the flow are:

$$v_x = \frac{\partial \varphi}{\partial x} = -\frac{q}{4h_0} \frac{\sinh \frac{\pi x}{2h_0} \cosh \frac{\pi x}{2h_0}}{\sinh^2 \frac{\pi x}{2h_0} \cos^2 \frac{\pi y}{2h_0} + \cosh^2 \frac{\pi x}{2h_0} \sin^2 \frac{\pi y}{2h_0}} \tag{8}$$

$$v_y = \frac{\partial \varphi}{\partial y} = -\frac{q}{4h_0} \frac{\sin \frac{\pi y}{2h_0} \cos \frac{\pi y}{2h_0}}{\sinh^2 \frac{\pi x}{2h_0} \cos^2 \frac{\pi y}{2h_0} + \cosh^2 \frac{\pi x}{2h_0} \sin^2 \frac{\pi y}{2h_0}} . \tag{9}$$

Through an analysis of Eq. (9) the common-sense fact becomes verified that there is a stagnation point with zero velocity at $B(0, h_0)$. With respect to the solution of the given problem, the velocity component v_y is of primary interest. In conformity with the notations of Fig. 1, here too, heights above the channel bottom will be denoted by h instead of y. Thus, along the y-axis one will have:

$$v_y = v = -\frac{q}{4h_0} \cot \frac{\pi h}{2h_0}, \tag{10}$$

The pressure distribution will be determined, like according to the Kulka method, by using the Bernoulli theorem. The energy equation between points B and A will be

$$h_0 + \frac{p_0}{\gamma} + 0 = a + \frac{p_0}{\gamma} + \frac{v_A^2}{2g} \tag{11.a}$$

whence:

$$\frac{v_A^2}{2g} = h_0 - a\,. \tag{11.b}$$

The energy equation between point B and an arbitrary point C will yield, after the elimination of the atmospheric pressure p_0 on both sides:

$$h_0 + 0 + 0 = h + \frac{p}{\gamma} + \frac{v^2}{2g}\,. \tag{12}$$

By substituting the value of v from Eq. (10) into Eq. (12) and after rearranging, one arrives to:

$$\frac{p}{\gamma} = h_0 - h - \frac{v^2}{2g} = h_0 - h - \frac{q^2}{32h_0^2 g} \cot^2 \frac{\pi h}{2h_0}\,. \tag{13}$$

In order to eliminate q, Eq. (13) should be rewritten by substituting $a = h$. Since in this case $p = 0$, therefore

$$q^2 = (h_0 - a)\frac{32h_0^2 g}{\cot^2 \dfrac{\pi a}{2h_0}}\,. \tag{14}$$

After inserting this value into Eq. (13), the wanted expression of pressure distribution will be obtained as

$$\frac{p}{\gamma} = h_0 - h - (h_0 - a)\frac{\cot^2 \dfrac{\pi h}{2h_0}}{\cot^2 \dfrac{\pi a}{2h_0}} = h_0 - h\,(h_0 - a)\frac{\tan^2 \dfrac{\pi a}{2h_0}}{\tan^2 \dfrac{\pi h}{2h_0}}\,. \tag{15}$$

If $a = 0$ (i. e., the gate is closed), Eq. (15) will be transformed into the simple expression of hydrostatic pressure distribution $p/\gamma = h_0 - h$.

The dynamic thrust acting upon a vertical strip of 1 m width of the sluice gate may be expressed as

$$P = \int_a^{h_0} p dh \tag{16}$$

which, omitting intermediate steps of integration, will lead to:

$$P = \gamma h_0 \left\{ \frac{h_0}{2} - a - \frac{a^2}{2h_0} + \frac{2}{\pi} \frac{h_0 - a}{\cot^2 \frac{\pi a}{2h_0}} \left[\frac{\pi}{2} \left(1 - \frac{a}{h_0} \right) + \cot \frac{\pi a}{2h_0} \right] \right\}. \tag{17}$$

In case of $a = 0$, this equation too will transform into hydrostatic thrust $P = \gamma h_0^2/2$.

In Table 1 the dimensionless values of pressure $p\gamma/h_0$ against dimensionless depth h/h_0 have been tabulated for the case $a/h_0 = 0.1$, as obtained both

Table 1

Comparison of pressure distributions obtained from approximate and exact method, along a slightly lifted sluice gate, for $a/h_0 = 0.1$

Relative height h/h_0 above bottom	Dimensionless value of the pressure head $p/\gamma h_0$ according to	
	Eq. (1) [KULKA]	Eq. (15) [SZALAY]
0.10	0.000	0.000
0.11	0.144	0.179
0.12	0.256	0.262
0.13	0.331	0.340
0.14	0.404	0.404
0.15	0.453	0.457
0.20	0.582	0.586
0.25	0.614	0.619
0.30	0.608	0.613
0.35	0.585	0.590
0.40	0.552	0.557
0.50	0.473	0.477
0.60	0.384	0.388
0.70	0.290	0.294
0.80	0.195	0.198
0.90	0.098	0.100
1.00	0.000	0.000

by the Kulka method and the method proposed by the author. Although this latter method is based upon much more correct boundary conditions than that of Kulka, one will find that the difference between values calculated from the one or the other method rarely exceeds $5^0/_{00}$ of the total water depth h_0. *Thus, one may draw the conclusion that in certain cases even wrong assumptions are apt to lead to results of acceptable accuracy.* But these latter should be accepted only after being either verified by experiments or checked through more exact theoretical procedures.

Flow above a sluice gate

The mapping function now applied is suited not only to the investigation of pressure distribution along a slightly lifted sluice gate but also to describe the pressure distribution from an overfall on a sluice gate resting on the bottom, or for that matter, the overfall on any weir having a vertical upstream face.

Summary

In the literature, the method of *Kulka* to calculate dynamic pressure upon slightly opened sluice gates is often encountered. The paper proves first the untenability of Kulka's basic assumptions. Actual boundary conditions are, however, fulfilled by another well-known mapping function. If using this function as a point of departure, but following otherwise the procedure proposed by *Kulka*, another equation for the pressure distribution may be derived.

The pressure distribution calculated by the author's method shows, in a rather surprising manner, numerical results very close to those of *Kulka*. Thus, the conclusion may be drawn that, although the *Kulka* method is theoretically highly objectionable, the numerical results obtained through it are practically acceptable.

References

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