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An Integrated Time-frequency Framework for Cable Force Identification in Long Cables of Long-span Bridges

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Abstract

Accurate identification of cable forces is crucial for the Structural Health Monitoring (SHM) of long-span cable-stayed bridges. However, vibration-based methods face significant challenges when analyzing closely spaced high-order frequencies in long cables, including the difficulty of identifying low-order frequencies and the propensity for modal aliasing. This paper proposes a novel integrated time-frequency analysis framework to automatically identify cable frequencies from vibration data without prior information. The framework employs a hierarchical approach where the Secondary Fourier Transform (SFT) first automatically estimates the fundamental frequency difference, thereby enabling two advanced techniques: the modified short-time Fourier transform (MSTFT) for sparse modal identification and the modified Hilbert transform (MHT) for high-precision instantaneous frequency (IF) tracking. Several long cables in a certain long-span bridge were employed, and the results demonstrated that the framework successfully identifies cable frequencies from higher-order modes (10th to 15th order), effectively compensating for weak low-order signals. SFT provided a rapid and robust estimation of the average frequency difference. Building on this, MSTFT enabled the sparse frequency identification of designated modal orders, while MHT precisely captured IF fluctuations, revealing dynamic force changes that correlated with peak traffic periods. The proposed integrated framework offers a powerful and adaptable solution for cable force identification. The proposed framework automates the analysis, mitigates modal aliasing, and accommodates multi-precision requirements. This enhances traditional vibration-based monitoring, delivering a robust solution for SHM systems to better assess operational safety and optimize the maintenance of long-span bridges.

Keywords

cable force, identification, cable-stayed bridge, time-frequency analysis, cable vibrations, Hilbert transform

1 Introduction

Stay cables are critical structural components in cable-stayed bridges, responsible for transferring primary loads from the deck to the pylons [1]. These cables are subject to continuous variations in load from traffic, wind, and temperature, causing them to experience complex vibrations [2, 3]. Consequently, the accurate and continuous identification of cable forces is indispensable for assessing the operational condition and ensuring the structural health of these vital infrastructures [4, 5]. For modern Structural Health Monitoring (SHM), this requires automated and robust methods that can continuously track force variations in near real-time, providing timely insights into the bridge's performance without relying on prior design information or manual intervention.

The existing cable force monitoring techniques [6] include the pressure gauge measuring method [7], the magnetic flux method [8], the frequency method [1, 9, 10] and the computer-vision method [11, 12]. However, the frequency method is still the most widely used identification method in the operation life cycle [13]. The frequency method refers to obtaining the natural frequency by acquiring and analyzing the vibration acceleration signal of the cable, and constructing the relationship between cable force and natural frequency by numerical calculation or empirical fitting, to realize the identification of cable force by identifying the natural frequency of the cable. Despite its widespread use, the frequency method faces two primary challenges:

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1. Establishing the corresponding relationship between the cable force and its natural frequency. Extensive research has been conducted on the influence of bending stiffness, sag and different boundary conditions of the cable [14–17], and numerous calculation methods have been proposed under different assumptions. Kim et al. [18] proposed a comprehensive method to estimate cable forces by measuring its natural frequency, which can identify the tension, bending stiffness and axial stiffness of the cable system at the same time.
2. Identifying the natural frequency of the cable through the vibration signal [19]. Hou et al. [20] adopted variant mode decomposition based time-varying force identification, the maximum error of cable force identification under laboratory environment is 8.4%. Zhang et al. [21] put forward a time-varying identification algorithm of cable force based on synchronous compressed short-time Fourier transform, and analyzed the dynamic response of cable force using the time-frequency spectrum. Bao et al. [22] proposed a sparse time-frequency analysis method (TFM) to estimate the instantaneous frequency (IF). Li et al. [7] proposed the extended Kalman filter method to identify the time-varying cable force through the superposition of a few modes of the cables. Since only a few modal frequencies were considered, there is a certain error in the process of identification. Dan et al. [23] proposed a combined method of cable modal frequency identification based on band-pass filter and Hilbert transform. However, prior estimation of cable force may make this method difficult to apply to long cables with complex changes. To address the non-stationary nature of vibration signals under operational loads, TFMs have been extensively investigated to track time-varying cable forces [24], such as those based on variational mode decomposition, synchrosqueezed transforms, and sparse time-frequency analysis. However, these methods can have limitations; for instance, some techniques rely on a limited number of modes or require prior estimations of cable properties, which complicates their application to cables without detailed design information.

These challenges are particularly acute for the long cables characteristic of modern long-span bridges [25]. First, vibration sensors are typically installed near the cable ends for accessibility, a location where the signal energy of the fundamental and low-order modes is minimal. This makes robust identification of these frequencies difficult. Second, the natural frequencies of long cables are very closely spaced, especially at higher orders. Traditional TFMs are prone to modal aliasing in this scenario, where energy from adjacent modes leaks and contaminates the identification of a target frequency, leading to inaccurate results.

To address these challenges, this paper proposes an integrated, hierarchical framework for the automated identification of time-varying cable forces. The framework is designed to operate without prior assumptions, effectively utilize higher-order modal information, and robustly mitigate modal aliasing. It sequentially integrates the Secondary Fourier Transform (SFT) for automated frequency difference estimation, the modified short-time Fourier transform (MSTFT) for sparse identification of designated modal orders, and the modified Hilbert transform (MHT) for high-precision IF tracking. The vibration characteristics of long cables from a case-study bridge are analyzed, followed by the application and validation of the proposed methods. This tiered approach is designed to progressively refine frequency information, thereby overcoming the limitations of conventional methods in monitoring long cables.

2 Theoretical fundamentals

2.1 Calculation models

In the simplest idealization, a cable is modeled as a theoretically tensioned string, ignoring the effects of bending stiffness, sag, and boundary conditions. Based on the theory of string vibration [26], the cable force T is related by:

$$T = 4mL^2\left(\frac{f_n}{n}\right)^2, \quad (1)$$

where m is the mass per unit length, L is the cable length, and f_n is the n-th natural frequency.

A key characteristic of this model is that the natural frequencies are integer multiples of the fundamental frequency f_1, resulting in a constant frequency difference (Δf) between adjacent modes:

$$\frac{f_1}{1} = \frac{f_2}{2} = \frac{f_3}{3} = \cdots \frac{f_n}{n} = f_n - f_{n-1} = \Delta f. \quad (2)$$

If f_1 can be clearly identified, the cable force is easily calculated. However, the fundamental frequency vibration is often weak and obscured by noise in field measurements. In such cases, the frequency difference method can be used if any two higher-order frequencies, f_r and f_s, are known:

$$f_r - f_s = \Delta f_{rs} = (r-s)\frac{1}{2L}\sqrt{\frac{T}{m}} = (r-s) f_1. \quad (3)$$

2.2 Frequency identification

2.2.1 Secondary frequency transform

A unified computational framework is advanced for the autonomous identification of spectral components and the delivery of controllable precision. The architecture is anchored in the SFT, which automates the frequency difference method. The procedure initiates with the classical Fourier transform of the vibration signal $x(t)$:

$$F(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt. \quad (4)$$

According to the cable vibration model, the natural frequencies f_n are approximately integer multiples of the fundamental frequency f_1 (i.e., $f_n \approx n \cdot f_1$). This creates a distinct periodic pattern in the frequency spectrum $F(f)$, where the period is the fundamental frequency difference, $\Delta f \approx f_1$. The SFT is designed to identify this periodicity. By applying a second Fourier transform to the magnitude of the spectrum, the dominant period Δf is revealed as a prominent peak in the secondary spectrum. The SFT, which yields the secondary spectrum $G(\xi)$, is therefore defined by the following integral:

$$\begin{aligned} G(\xi) &= \int_{-\infty}^{\infty} F(f) e^{-2\pi i \xi f} df \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} e^{-2\pi i \xi f} df dt, \end{aligned} \quad (5)$$

where f is the frequency variable, and the abscissa ξ represents the reciprocal of frequency. A prominent peak at ξ_0 directly corresponds to the periodicity in $F(f)$. Therefore, the frequency difference Δf can be automatically and robustly estimated as $\Delta f = 1/\xi_0$. This concept can be extended to the time domain by applying the SFT process to each time slice of a spectrogram, $F(f,\tau)$. The spectrogram is generated using the Short-Time Fourier Transform (STFT), which analyzes the signal $x(t)$ using a sliding window function $\omega(t)$ centered at time τ. The secondary spectrum $G(\xi,\tau)$ is then computed by applying a second Fourier transform to the spectrogram:

$$\begin{aligned} G(\xi,\tau) &= \int_{-\infty}^{\infty} F(f,\tau) e^{-2\pi i \xi f} df \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t)\omega(t-\tau) e^{-2\pi i f t} e^{-2\pi i \xi f} df dt. \end{aligned} \quad (6)$$

If the maximum value ξ_0 of the secondary spectrum is obtained at a certain time τ, $\xi_0(\tau) = \text{argmax}(G(\xi,\tau))$, then time-varying FD can be expressed as:

$$\Delta f(\tau) = \frac{1}{\arg\max\left(G(\xi,\tau)\right)}, \quad (7)$$

where argmax( ) are the points of some function at which the function values are maximized. The FD calculated in Eq. (7) can be called as the SFT, which is adaptive and fully automatic in the time domain without manual intervention. The cable force can be calculated by substituting $\Delta f(\tau)$ into the cable force calculation model (taking Eq. (1) as an example):

$$T_1(\tau) = T(\Delta f(\tau)) = 4mL^2 \Delta f^2(\tau). \quad (8)$$

2.2.2 Modified short-time Fourier transform

Building upon the foundational Δf estimated by SFT, the MSTFT procedure is employed to identify the frequency of a specific modal order with enhanced accuracy. This targeted approach is designed for applications where the behavior of a particular mode is of interest.

The MSTFT procedure leverages the STFT spectrogram $F(f,\tau)$. Instead of analyzing the entire spectrum at each time step, it uses the SFT-derived Δf to define a narrow search band centered around the expected frequency of the target mode. The i-th natural frequency $f_2(i,\tau)$ is then identified as the frequency corresponding to the maximum spectral amplitude exclusively within this predefined band:

$$f_2(i,\tau) = \underset{f \in \left[\left(i-\frac{1}{2}\right)\Delta f, \left(i+\frac{1}{2}\right)\Delta f\right]}{\arg\max} \left(F(f,\tau)\right). \quad (9)$$

This technique achieves sparse frequency identification by focusing the analysis only on the modal component of interest, thereby discarding irrelevant spectral information and significantly reducing interference from adjacent modes and noise. Once the modal frequency trajectory $f_2(i,\tau)$ is extracted, a more refined time-varying cable force $T_2(i,\tau)$ can be computed.

Unlike traditional STFT which analyzes the entire spectrum, MSTFT in Eq. (9) disregards the spectral amplitude information and records only the frequency corresponding to the peak within a predefined band. Cable force corresponding to the i-th frequency of the cable vibration can be obtained as:

$$T_2(i,\tau) = T(f_2(i,\tau)) = 4mL^2\left(\frac{f_2(i,\tau)}{i}\right)^2. \quad (10)$$

The window function of STFT must be selected according to the cable vibration characteristics. If the window is too small, it has good time resolution, but the input signal is insufficient, and the frequency resolution is lost; Conversely, the window is too large and has high frequency resolution, but the averaging in time dimension results in the inability to capture the cable force changes in detail. For MSTFT, similar principles apply regarding window selection. A Hanning window [18] is employed here to mitigate spectral leakage and maintain a balance between time and frequency resolution. The selection of window length involves a critical trade-off; therefore, the goal is not to find a single optimal value but a suitable one for the specific application. As demonstrated in the validation Section 4.1, a comparative analysis was performed to select a window length that ensures stable frequency difference estimates while being computationally practical.

2.2.3 Modified Hilbert transform

It is crucial to note that the Hilbert transform (HT) is theoretically meaningful for identifying IF only on mono-component signals. A raw cable vibration signal is multi-component, containing numerous modal frequencies. Therefore, applying HT directly to the raw signal would yield meaningless results. The MHT procedure addresses this by first using a dynamic band-pass filter, guided by the Δf from SFT, to isolate a single modal component. This creates a quasi-mono-component signal, for which the HT can then be validly applied to extract its IF with high precision. The HT of a time series $x(t)$ is defined by the convolution integral:

$$\hat{x}(t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(t-\tau)}{\tau} d\tau, \quad (11)$$

where the symbol "*" denotes convolution operation, and τ is the integration variable. An analytic signal is constructed:

$$z(t) = x(t) + i\hat{x}(t) = A(t)e^{i\varphi(t)}, \quad (12)$$

$$A(t) = \sqrt{x^2 + \hat{x}^2}, \varphi(t) = \arctan\left(\frac{\hat{x}}{x}\right), \quad (13)$$

where $A(t)$ and $\varphi(t)$ are the instantaneous amplitude and phase, respectively. The IF is the time derivative of the phase:

$$f(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt}. \quad (14)$$

The MHT procedure applies this principle in a targeted, two-step process. First, using the Δf from SFT, a dynamic band-pass filter is applied to the original signal to isolate the i-th modal component, creating a nearly mono-component signal. Second, the HT is applied to this filtered, quasi-mono-component signal to compute its high-precision IF, $f_3(i,t)$. The Hilbert Spectrum, $H(f,t)$, derived from the Hilbert-Huang transform [23], is a tool that visualizes a signal's time-frequency energy distribution. Equation (15) conceptually identifies the peak energy trajectory of the now-isolated modal component within its designated frequency band, and consequently determines its IF:

$$f_3(i,t) = \underset{f\in\left[\left(i-\frac{1}{2}\right)\Delta f, \left(i+\frac{1}{2}\right)\Delta f\right]}{\arg\max} \left(H(f,t)\right), \quad (15)$$

where the argmax operation is performed over the frequency variable f within the defined band. By first isolating the target modal component with a dynamic band-pass filter, the MHT procedure excels at tracking subtle, IF variations while effectively avoiding modal aliasing:

$$T_3(i,t) = T\left(f_3(i,t)\right) = 4mL^2\left(\frac{f_3(i,t)}{i}\right)^2. \quad (16)$$

2.3 Identification framework

The proposed integrated identification framework is illustrated in Fig. 1. It employs a hierarchical approach that progresses from a rapid, general estimation to a highly refined, time-varying analysis. The framework is structured in a progressive relationship, where the SFT serves as the foundational algorithm for two subsequent, more specialized techniques: the MSTFT and the MHT.

The framework provides three distinct levels of frequency identification, each with a different degree of temporal and modal precision:

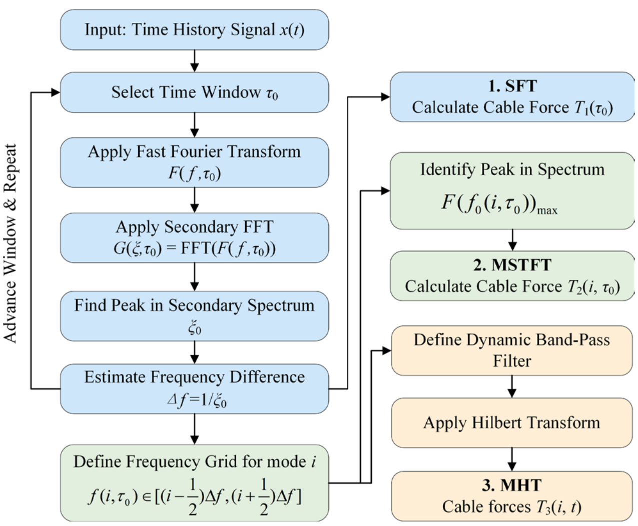

Fig. 1 The framework of frequency identification methods

1. Window-averaged frequency difference (SFT): the SFT operates on the entire selected time window (τ_0) to produce a single, average fundamental frequency difference, $\Delta f(\tau_0)$. This yields a single cable force value $T_1(\tau_0)$, representing a coarse but rapid assessment for the entire window. It offers the highest computational efficiency but the lowest temporal resolution.
2. Mode-specific, window-averaged frequencies (MSTFT): building on the SFT result, the MSTFT identifies the frequency of i specific modes, $f_2(i,\tau_0)$, which are still averaged over the same time window τ_0. This results in a vector of mode-specific cable forces $T_2(i,\tau_0)$. This step refines the analysis by providing modal detail without a significant increase in computational cost, though its temporal accuracy remains limited by the window size.
3. Mode-specific, instantaneous frequencies (MHT): the MHT achieves the highest level of detail. For each targeted mode i, it calculates the high-resolution IF, $f_3(i,t)$, for every time point t within the window. This produces a detailed time-history matrix of cable forces $T_3(i,t)$, capturing true dynamic variations. Consequently, while MHT delivers the highest accuracy, it is also the most computationally intensive.

This multi-precision structure allows users to select the appropriate analysis depth based on their needs, from quick overall checks (SFT) to detailed dynamic investigations (MHT). To illustrate the integrated nature of the framework, the detailed calculation process for obtaining the most complex output, the time-varying cable force $T_3(i,t)$, is described below:

1. Window selection: select an appropriate time window length (width) and window step (τ_0) for the analysis of the time history signal $x(t)$.
2. SFT application: within the current time window, apply the Fast Fourier Transform (FFT) to obtain the frequency spectrum $F(f,\tau_0)$. Then, apply a second FFT to this spectrum to generate the secondary spectrum $G(\xi,\tau_0)$.
3. Frequency difference estimation: identify the peak location ξ_0 in the secondary spectrum. Calculate the average frequency difference (Δf) for the current window as the reciprocal of this peak location.
4. Dynamic band-pass filtering: for a selected target modal order i, define a dynamic band-pass filter using the range $[(i - 0.5)\Delta f(\tau), (i + 0.5)\Delta f(\tau)]$. Apply this filter to the original time series $x(t)$ within the window to isolate the signal component $z(i,\tau,t)$ corresponding to that order.
5. Instantaneous frequency calculation: apply the HT to the filtered signal $z(i,\tau,t)$ to compute the i-th order IF, $f_3(i,t)$, for the current window.
6. Time iteration: advance the time window by one step (τ_0) and repeat steps 2. through 5. for the subsequent data segment, creating a series of IF results.
7. Data fusion: concatenate the IF results from all time windows. For overlapping segments between consecutive windows, average the frequency values to ensure a smooth and continuous time history.
8. Cable force calculation: substitute the final, continuous IF time history into the appropriate cable force vibration model to obtain the time-varying cable force.

3 Engineering background

A long-span steel box girder cable-stayed bridge, with a main span of 1088 m, is employed as the engineering background. Its significant main span makes its long cables particularly susceptible to the monitoring challenges of weak low-order signals and modal aliasing, serving as an ideal case for validating the proposed framework.

This paper mainly studies two long cables at the north tower and upstream, named A18 and J34, and their arrangement in the overall structure is shown in Fig. 2. Two acceleration sensors were installed on each cable to monitor the vertical in-plane vibration acceleration of cables and transverse out-plane vibration acceleration, with a sampling frequency of 20 Hz. The parameters of the vibration acceleration sensors and the corresponding stay cables are shown in Table 1. The layout of a representative cable vibration acceleration sensor on the upstream side of A34 is shown in Fig. 3.

The vibration acceleration of four channels from 00:00 to 01:00 on January 1, 2019 was selected, and the time history of acceleration was also transformed in frequency domain, as shown in Fig. 4.

The average amplitude of in-plane vibration acceleration of the two cables is about 0.02 m/s^2, and the average amplitude of out-plane vibration acceleration is about 0.01 m/s^2, which is only half of the in-plane. Moreover,

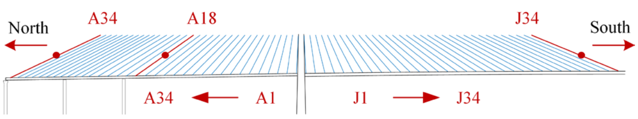

Fig. 2 Number and position of monitoring cable

Table 1 Parameter of four vibration acceleration sensor channels

Name	Description	Cable type	Unit mass (kg/m)	Diameter (m)	Length (m)	Design cable force (kN)
A18-in	No. A18 (in-plane)	PESM7-223	71.7	0.118	336.811	4721
A18-out	No. A18 (out-plane)					
J34-in	No. J34 (in-plane)	PESM7-313	100.8	0.158	576.751	6810
J34-out	No. J34 (out-plane)					

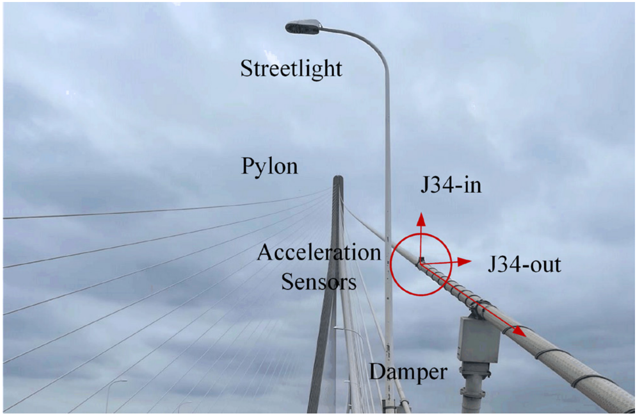

Fig. 3 Installation of vibration acceleration sensor for measuring cable force

the correlation of the in-plane and out-plane acceleration amplitudes on the time domain is low visually.

The energy of each natural frequency of out-plane vibration is relatively concentrated, and the spacing between each frequency is very clear. The maximum energy basically appears in the 2~4 Hz range. For long cables in the bridge, the acceleration sensor is generally installed at a height of 3.5 m from the bridge deck. Considering the total lengths of cables A18 and J34, this position corresponds to merely 1.0% of the cable lengths from the anchorage. Physically, the anchorage acts as a node (zero displacement) for all vibration modes. A sensor placed at just 1.0% of the cable's length is therefore extremely close to this nodal point. As visualized by the low-order mode shapes in Fig. 5, displacement and acceleration are minimal in this region. This suboptimal placement makes the fundamental and low-order frequency components in the vibration signal extremely weak and difficult to detect, as they are easily obscured by noise.

While optimal sensor placement closer to the cable's center would enhance the detection of low-order frequencies, this is often impractical for long-term monitoring. This study's proposed methods are specifically designed to overcome this common limitation. Instead of relying on the weak low-order signals, the framework, particularly through MSTFT and MHT, incorporates algorithmic strategies to leverage the more energetic higher-order modes found in the 2–4 Hz band. The dynamic band-pass filtering in MHT and the targeted windowing and peak-picking mechanisms in MSTFT are designed to enhance the signal-to-noise ratio and extract this relevant modal information, thereby effectively compensating for the suboptimal sensor locations.

In-plane vibration involves vehicle loads, so the performance of cable vibration in frequency domain is more complex and the influence of noise is greater. Notably, an obvious peak value appeared at f = 0.1847 Hz of the in-plane acceleration vibration spectrum of the cable J34. However, the cable force corresponding to this peak value is far from the designed cable force or the frequency difference of out-plane cable vibration. A small peak value also appears in front of the fundamental frequency of the cable A18, so this peak value is not the fundamental frequency of the cable. This suggests that the in-plane vibration acceleration sensor is vulnerable to some vertical interference. Both the fundamental frequency method and the frequency difference method for measuring cable force by in-plane vibration are easily interfered by environmental conditions, and the out-plane vibration acceleration is more stable and suitable for measuring cable force by the frequency method than that in-plane vibration.

4 Methods verification

To validate the proposed hierarchical framework, vibration acceleration data from the background cable-stayed bridge were analyzed. As established in Section 3, the out-plane vibration data from J34-out is more stable and less

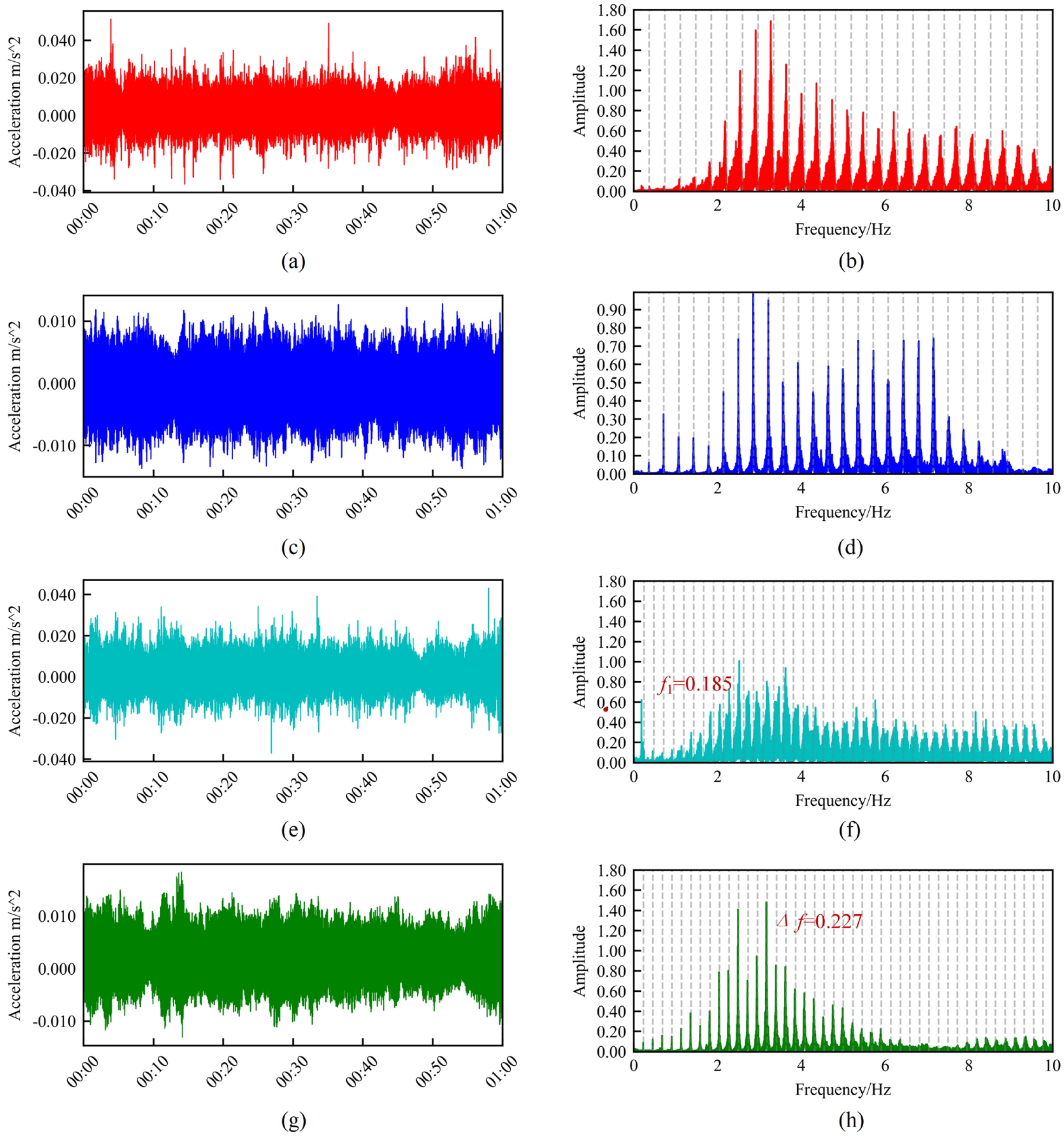

Fig. 4 Four accelerations and their spectrums from 00:00 to 01:00 on 2019-01-01: (a) Time history of A18-in; (b) Spectrogram of A18-in; (c) Time history of A18-out; (d) Spectrogram of A18-out; (e) Time history of J34-in; (f) Spectrogram of J34-in; (g) Time history of J34-out; (h) Spectrogram of J34-out

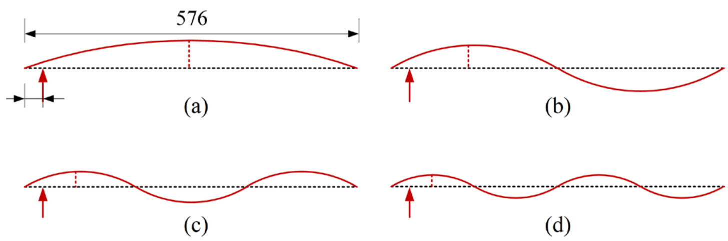

Fig. 5 Diagram of the first four vibration modes of the cable J34: (a) First-order mode; (b) Second-order mode; (c) Third-order mode; (d) Fourth order mode

susceptible to noise, making it suitable for this verification. The analysis sequentially demonstrates the efficacy of each component of the framework: SFT for automated frequency difference estimation, MSTFT for sparse modal identification, and MHT for high-precision IF tracking.

4.1 Secondary frequency transform

The primary function of SFT is to automatically and robustly estimate the fundamental frequency difference Δf from the vibration signal, which is a critical prerequisite for subsequent high-resolution analyses. To assess its performance and determine an optimal time window, the analysis was conducted using window lengths of 0.5, 1, 2, and 5 min. This range was selected to systematically investigate the fundamental trade-off in time-frequency analysis: shorter windows (e.g., 0.5 min) offer better temporal resolution to track rapid changes but may yield unstable frequency estimates due to insufficient data, while longer windows (e.g., 5 min) improve frequency resolution and stability at the cost of averaging out short-term dynamic effects.

Fig. 6 (b)–(e) show the amplitude of the power spectral density (PSD) obtained *via* Fast Fourier Transform (FFT) alongside the FD estimated using SFT for each time window. The results demonstrate that SFT can derive a stable and accurate FD estimate even with a short 0.5 min window, where the estimated FD (0.2258 Hz) closely aligns with the clear periodicity observed in the spectrum. As the window length increases, the FD estimate converges; the FD obtained from the 2 min window (0.2269 Hz) is nearly identical to that from the 5 min window (0.2271 Hz). Considering the trade-off between computational efficiency and estimation accuracy, a 2 min time window is selected as the optimal balance for the subsequent steps of the analysis. This demonstrates SFT's capability to automate the frequency difference method reliably without requiring manual peak-picking or prior information.

4.2 Modified short-time Fourier transform

Building upon the FD estimated by SFT, the MSTFT is employed to overcome the limitations of traditional time-frequency analysis and achieve sparse identification of specific modal orders. Fig. 7 contrasts the results of a conven-

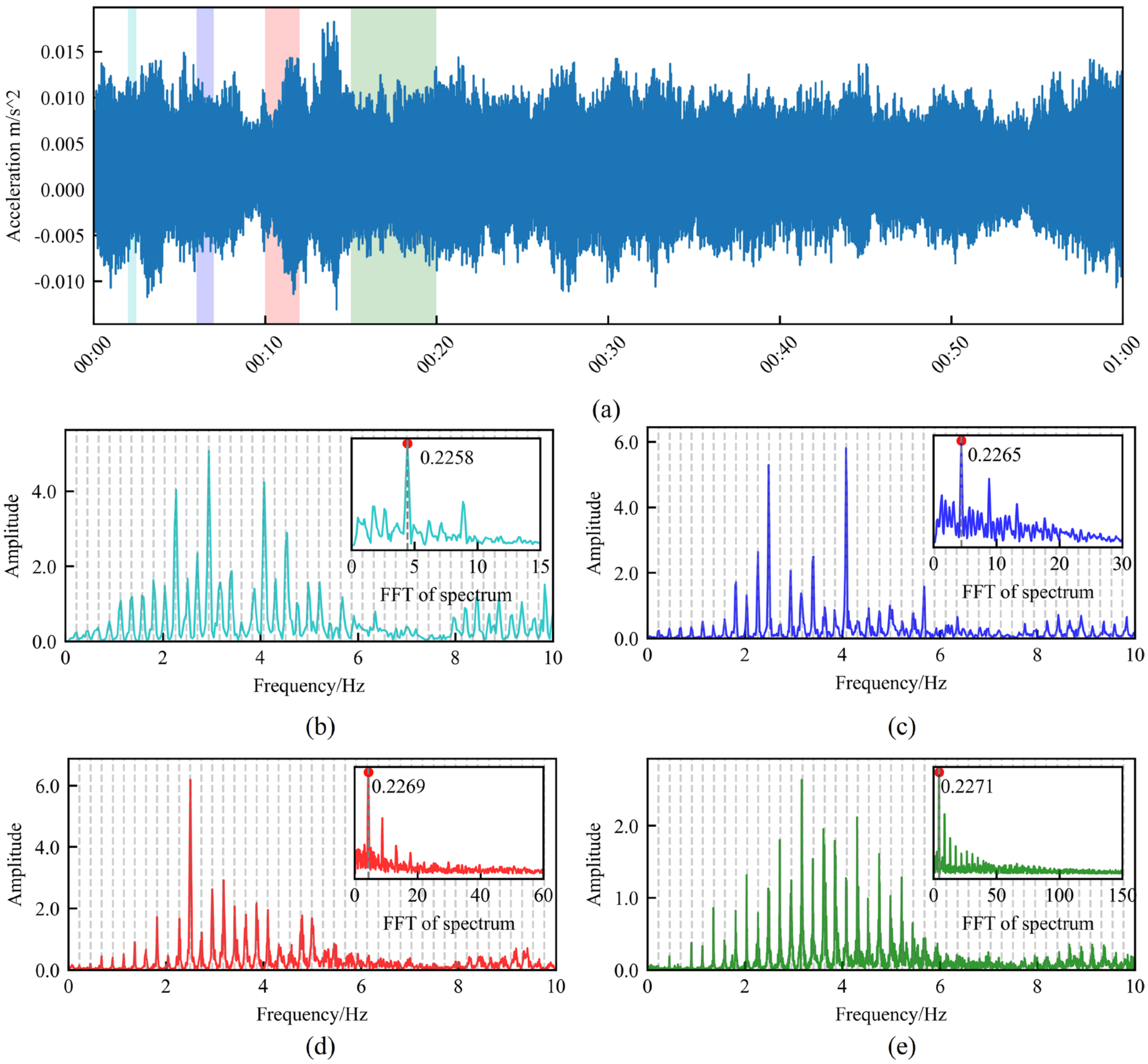

Fig. 6 FFT and SFT of sensor J34-out in 4 different time windows: (a) Time series of J34-out from: 2019-01-01-00:00 to 2019-01-01-01:00; (b) FFT and SFT in 0.5 min; (c) FFT and SFT in 1 min; (d) FFT and SFT in 2 min; (e) FFT and SFT in 5 min (NOTE: the vertical lines in the spectrum diagram is drawn with spacing Δf)

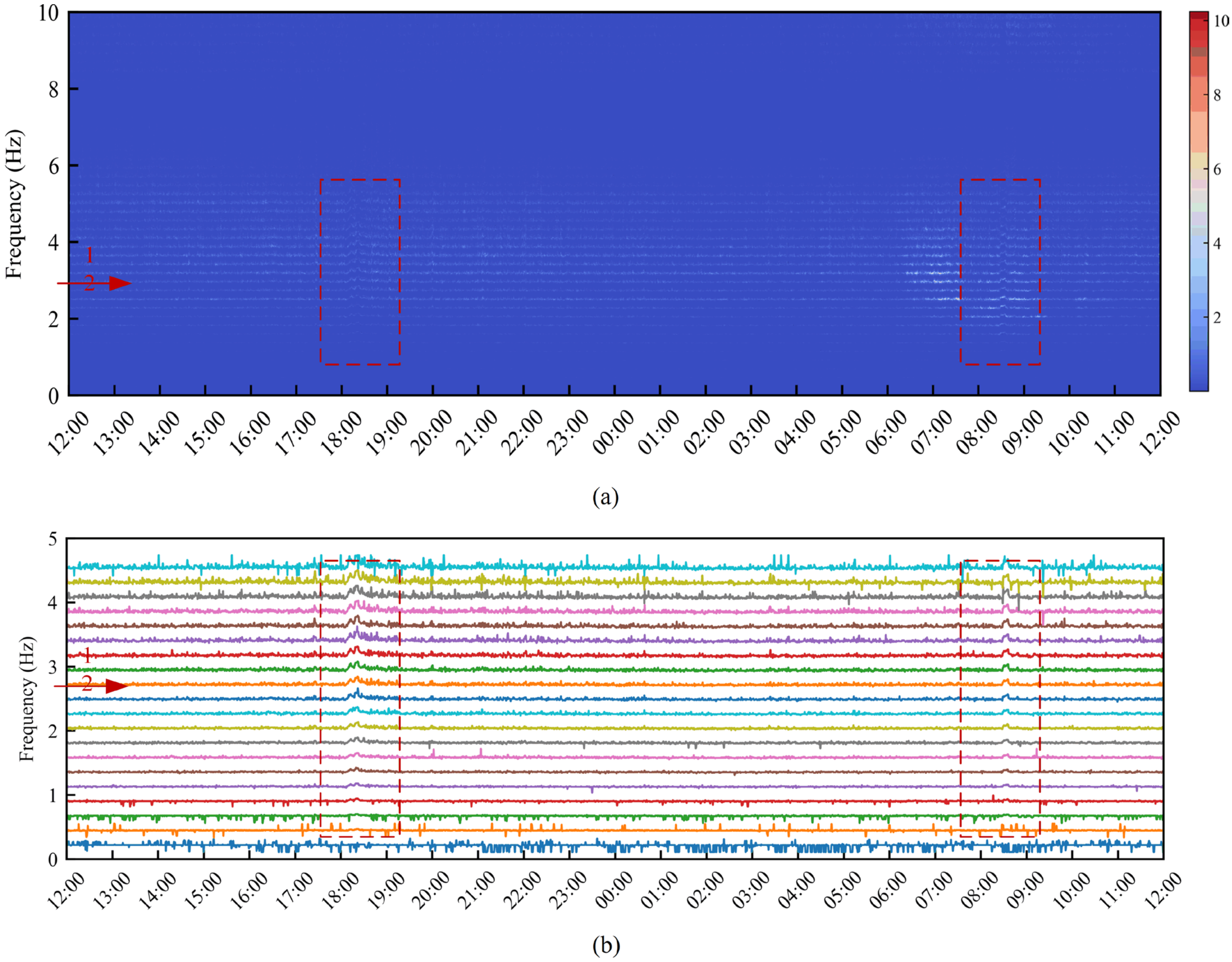

Fig. 7 IF calculated of sensor J34-out using STFT and MSTFT: (a) Traditional STFT of J34-out from 2019-01-03 to 2019-01-04; (b) MSTFT of J34-out from 2019-01-03 to 2019-01-04

tional STFT with those of the proposed MSTFT for a 24 h monitoring period, using a 2 min window and a 0.5 min step.

The spectrogram from the traditional STFT, shown in Fig. 7 (a), highlights several challenges. In the low-frequency range of 0–2 Hz, the spectral energy is minimal and heavily contaminated by environmental noise, making the IF effectively unmeasurable. Conversely, while frequency peaks are visible above 4 Hz, their energy is weaker and the noise impact is more significant, leading to violent fluctuations. According to Eq. (1), the influence of cable force changes is amplified at higher frequencies, making the IF more sensitive but also more susceptible to noise. The most reliable and energetic spectral information is concentrated in the 2–4 Hz band. However, even in this range, the low resolution of STFT can cause peak jumping, where the identified maximum energy peak erroneously shifts between adjacent modes, making it impossible to track a single modal order consistently over time.

In contrast, Fig. 7 (b) demonstrates the efficacy of MSTFT. By leveraging the FD from SFT to pre-define frequency bands for each modal order, MSTFT successfully isolates and tracks individual modes. The resulting time-frequency plot is clean and stable, with clearly discernible modal frequencies that exhibit high similarity in their trends. This sparse identification effectively mitigates the issue of peak jumping. The analysis clearly reveals two periods of abrupt, parallel frequency increases: 18:00–19:00 on January 3 and 8:00–9:00 on January 4. These events are attributed to changes in cable tension likely caused by morning and evening peak traffic congestion. This direct correlation between identified frequency shifts and expected traffic patterns is a critical validation, demonstrating the framework's capability to capture physically meaningful load-induced dynamic changes. This ability to discern operational load effects is a key requirement for an effective SHM system. The targeted approach also confirms that the 2–4 Hz frequency band, where higher-order modes reside, provides the most robust data for subsequent cable force calculations.

4.3 Modified Hilbert transform

To achieve a more refined time history analysis of cable force, the MHT is applied as the final step. This analysis focuses on periods of interest identified by MSTFT to extract high-resolution, IF information. We examine two 1 h segments: the dynamic period of 18:00–19:00, which exhibited sudden changes, and a relatively stationary period from 00:00–01:00 on the following day for comparison.

Fig. 8 displays the instantaneous frequencies of the 12^{th} and 15^{th} modes identified using MHT. The gray dotted lines represent the upper and lower bounds of the dynamic band-pass filter, which are derived from the SFT-estimated FD. Most of the IF values obtained by MHT fall within these bounds. A small portion of the IF exceeds the limits because the bounds are constrained based on the average FD over the time interval, whereas MHT captures true instantaneous variations. During the dynamic period from 18:00 to 19:00, the trend of the IF is generally consistent with the mean SFT result, but MHT reveals more fine-scale fluctuations in frequency. This highlights the ability of MHT, through its use of dynamic band-pass filtering, to effectively isolate and amplify specific frequency bands, which is crucial for accurate time-varying identification, particularly when the original signal is weak due to suboptimal sensor placement.

Fig. 9 provides a comprehensive comparison of cable force identification results obtained from the SFT, MSTFT, and MHT methods over a 24 h period. The blue line represents the force calculated by SFT, which offers a rapid, time-averaged estimation, suitable for a coarse, initial check. Its value is relatively stable but lacks the precision to capture dynamic fluctuations. The orange line shows the results from MSTFT, which builds upon the SFT's frequency difference to provide a more refined force time-history. It exhibits a higher level of detail and accuracy compared to SFT, effectively tracking the general trend of cable force changes. The magnified view at the bottom shows the high-precision results from MHT, with separate lines for modes 12^{th} through 15^{th}. The MHT lines show the most detailed and instantaneous fluctuations in cable force, accurately capturing the fine-scale variations that correlate with dynamic events such as peak traffic periods. The strong consistency and parallel trends among the different modal orders (12^{th} to 15^{th}) identified by MHT serve as a powerful validation of the framework's accuracy. This comparison visually demonstrates the hierarchical nature of the

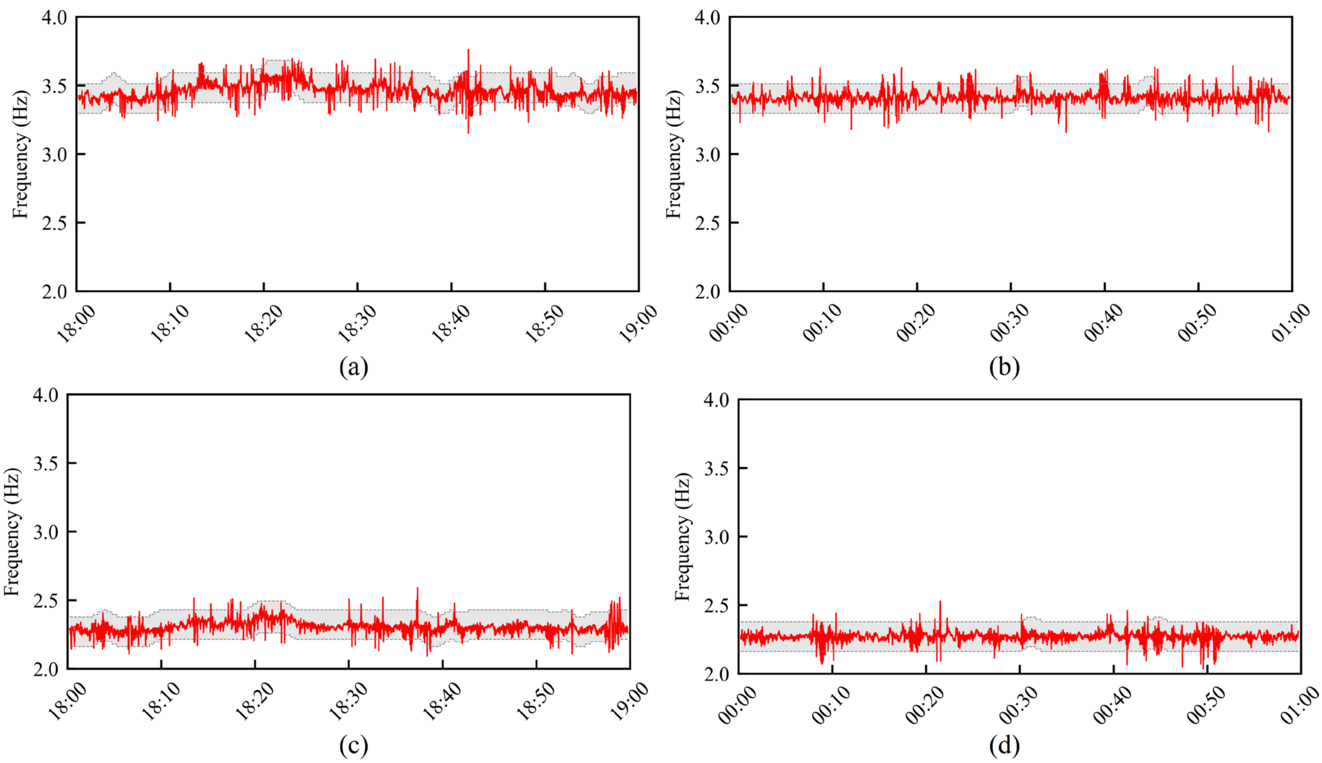

Fig. 8 IF calculated of sensor J34-out using MHT: (a) 15^{th} frequency from 18:00 to 19:00; (b) 15^{th} frequency from 00:00 to 01:00; (c) 12^{th} frequency from 18:00 to 19:00; (d) 12^{th} frequency from 00:00 to 01:00

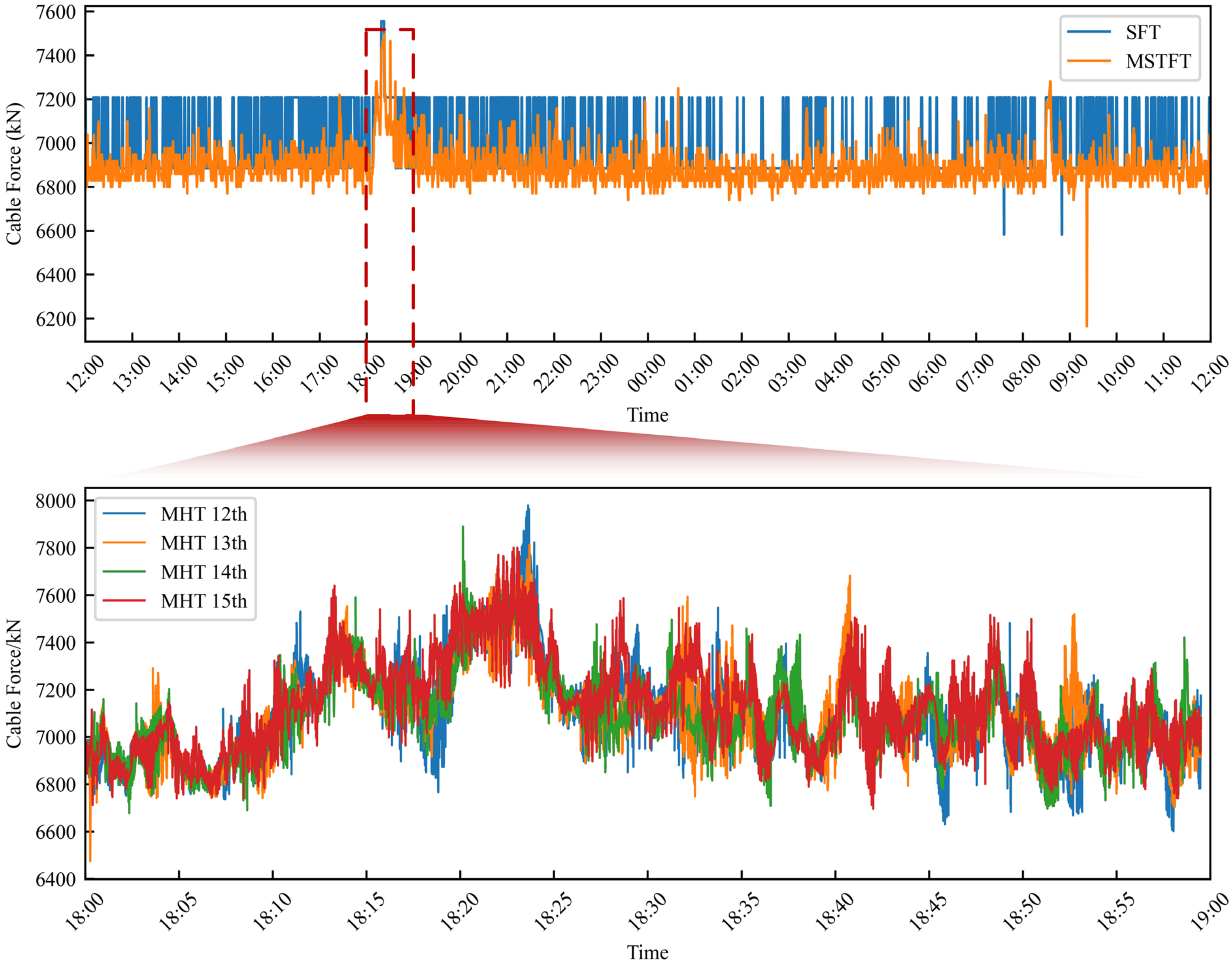

Fig. 9 Comparison of cable forces calculated by SFT, MSTFT and MHT for sensor J34-out

proposed framework, where SFT provides a foundational estimation, MSTFT offers a refined time-history, and MHT delivers the highest level of detail for in-depth analysis.

While the theoretical cable force should be identical regardless of the modal order used for calculation, the specific force values obtained from different modes (e.g., the 12^{th} vs. the 15^{th}) may show slight variations. This is primarily attributed to two factors: the influence of bending stiffness, which becomes more pronounced at higher modal orders, and the unique vibration characteristics and signal-to-noise ratio of each individual mode. However, despite these minor differences in absolute value, the observed trends in force evolution across all orders are highly consistent, confirming that the proposed method accurately captures real-time changes in cable tension. This demonstrates that each modal frequency responds not only to the overall cable force but also to its own unique vibration characteristics.

5 Conclusions

This paper introduced a novel, integrated framework to automatically identify the natural frequency of cables in real bridges without requiring prior information. The framework is built upon a hierarchical approach, where the SFT provides a foundational frequency difference estimation. This, in turn, enables more advanced methods like the MSTFT for targeted modal analysis and the MHT for high-precision time-varying frequency identification. The resulting workflow provides a versatile, multi-precision solution for practical cable force monitoring. The primary contributions and findings of this study are summarized as follows:

1. The framework successfully automates cable force identification by using SFT to determine the fundamental frequency difference directly from vibration data. This eliminates the need for prior assumptions

and establishes a robust basis for subsequent multi-precision analysis with MSTFT and MHT.

2. The methodology effectively addresses the challenge of weak low-order signals by reliably identifying cable frequencies from higher-order modes (e.g., 10^{th} to 15^{th} order in the 2–4 Hz range). This capability compensates for suboptimal sensor placement, a common issue in field monitoring of long cables.
3. The key strategy of using SFT to establish a dynamic band-pass for subsequent filtering and analysis with MHT proves highly effective in mitigating modal aliasing. This is a critical advantage for analyzing long cables, which characteristically exhibit small frequency differences and are prone to such aliasing with traditional methods.

Future work could focus on implementing this framework into an online, real-time SHM platform for long-span bridges, paving the way for more proactive and data-driven maintenance strategies.

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