AN ALGORITHM FOR ROUTING THE MAIN CONDUIT OF IRRIGATION NETWORKS BY DYNAMIC PROGRAMMING

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For the solution of the research, planning and operation problems of sprinkler irrigation, at the Institute of Water Management and Hydraulic Engineering of the Technical University, Budapest, recently a program package has been developed in ALGOL language. One of the largest programs of the package was developed in 1970 for determining the optimum route of the main conduit of irrigation systems by dynamic programming [2, 3]. Under extreme conditions (for example, too high or too low pumping lift, significant differences in elevation) the program could not be used for solving the problem. In this paper an improved algorithm is presented, as basis for a program eliminating the deficiency mentioned above.

Stating the problem

Given is the route of the subsoil branching pipes of a sprinkling irrigation plant (Fig. 1) and the sites where the pumping station might be located. The optimum route of the main conduit connecting the pump station with the branches, as well as the optimum lift of the pumping station should be determined.

Branches is the name for conduits containing tapping points — so-called hydrants. To the tapping points overground, portable irrigation systems are connected. The branches are, in general, parallel to each other, their spacings being defined by the type of the irrigation system. Routes of the mains are subject to less restriction, permitting great many route varieties.

The points where the branching pipes can be connected to the main conduit (in the following "branching points") are marked on the branches.

The pipe network can be considered as a directed physical system in the xy plane and divided into m steps, its condition being characterized by the motion of point S of the main conduit; and the optimum policy of this point should be found.

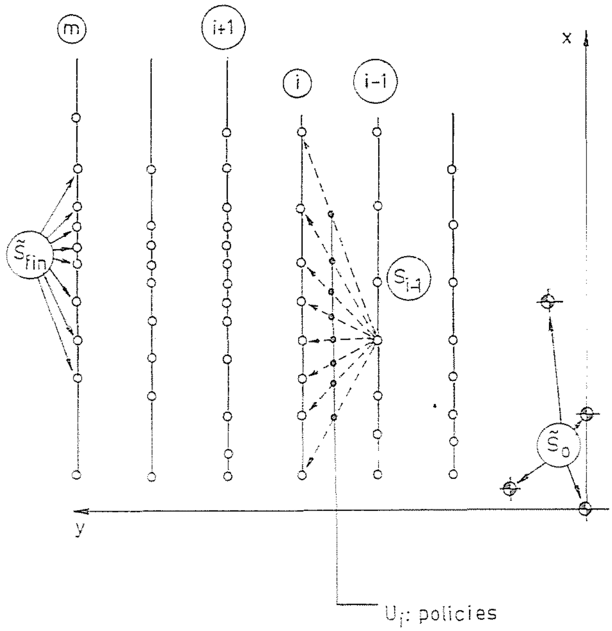

Fig. 1

According to the theory of dynamic programming, the problem may be defined as follows: from the possible set of policies U a policy U^* should be found, likely to shift point S of the phase space from the initial state of $S_0 \in \tilde{S}_0$ to the final state $S_{fin} \in \tilde{S}_{fin}$ so that the optimality criterion $W(U)$ assumes a minimum value, i.e.,

$$W^* = \frac{\min}{U} \{W(U)\} , \tag{1}$$

where the term $\frac{\min}{U}$ means the minimum with respect to U.

In this example, the *optimality criterion W is the construction cost of the pipe network*, or better, the total of the construction and water delivery costs. The *set of policies U designates the possible routes of the main conduit*. The problem is to determine the vector of the optimum policy:

$$U^* = (U_1^*, U_2^*, \ldots, U_m^*) ,$$

where $U_1^*, U_2^*, \ldots, U_m^*$ are the sections of the optimum route of the main conduit between the branches.

From the range of the initial state $\tilde{S}_0$, under the effect of policies $U_1^*, U_2^*, \ldots, U_m^*$, the system should get into the range of the final state $\tilde{S}_{fin}$. The range of the initial state S_0 involves the possible sites of the pumping station, and that of the final state $\tilde{S}_{fin}$ comprises the branching points of the farthest branch.

Such an initial state $S_0^* \in \tilde{S}_0$ (i.e., such a siting for the pumping station), and at every step such $U_1^*, U_2^*, \ldots, U_m^*$ policies should be found that cause after m steps the system to get into the range $\tilde{S}_{fin}$, and the optimality criterion W to reach a minimum.

Essentials of the solution method

The method of dynamic programming is characterized by dividing the progress of point S from $\tilde{S}_0$ to $\tilde{S}_{fin}$ into successive steps. This means in this instance that the determination of the optimum route will proceed from one branch to the other and from one branching point to the other.

In practice, the dynamic programming tends always from the state $\tilde{S}_{fin}$ toward the initial state $\tilde{S}_0$, thus, in this case, from the farthest branching pipe toward the pumping station.

Optimization of the i-th step

The algorithm will be illustrated on the optimization of the i-th step.

The optimization of the i-th step means seeking for the optimum routes of the main conduit connecting the i-th and $(i-1)$-st branches. Thus, the possible outputs S_i of the i-th step are the branching points where the main conduit may be connected to the i-th branching pipe. Provided the $(i+1)$-st step is already optimized with respect to any output of the i-th step, that is, the optimum policies leading from each branching point of the i-th branch to the $(i+1)$-st branch (the $U_{i+1}^* S_i$) are known and so is the corresponding optimality criterion $W_{i+1, \ldots, m}^*(S_i)$, the assumed optimization of the i-th step can be expressed as:

$$W_{i,i+1,\ldots m}^* (S_{i-1}) = \frac{\min}{U_i} \{W_{i,i+1,\ldots,m}^+ (S_{i-1}, U_i)\} \tag{2}$$

wherein

$$W_{i,i+1,\ldots,m}^+ (S_{i-1}, U_i) = w_i (S_{i-1}, U_i) + W_{i+1,\ldots,m}^* \big(S_i(S_{i-1}, U_i)\big).$$

Definitions are:

$W_{i,i+1,\ldots,m}^*(S_{i-1})$ — construction cost of optimum route — policy $U_i^* (S_{i-1})$ — leading from one of the branching points S_{i-1} of the i-th branch to the m-th branch.

$W_i(S_{i-1}, U_i)$ — construction cost of the main conduit section of policy U_i leading from branching point S_{i-1} of $(i-1)$-st branch to point S_i of the i-th branch.

$W^*_{i+1,\ldots,m}(S_i(S_{i-1}, U_i))$ — optimum construction cost of the pipe network with optimum-route main conduit led by policy U_i from branching point S_{i-1} on the $(i-1)$-st branch to branching point S_i on the i-th branch and to the m-th branch.

$W^+_{i,i+1,\ldots,m}(S_{i-1}, U_i)$ — optimum construction cost of a pipe network of main conduit route led by a possible policy U_i from branching point S_{i-1} of $(i-1)$-st branch (sign $+$ refers to that this is an optimum value only from a certain point of view; from the possible policies U_i the optimum should be selected).

The algorithm of the dynamic programming is based upon the above relations. According to the symbolic formulation the algorithm seems to be simple, however, the practical realization is far from easy.

The optimum route of the main conduit is sought for in case of a given lift, i.e., often of several given lifts, or also the optimum pumping lift is wanted. Thus, the range of initial state $\tilde{S}_0$ represents the possible pumping-station sites and the given lifts. In selecting from among the initial states the service costs of water lifting, the construction costs of the pumping station and of the channel leading to this latter should be taken into account.

The optimization proceeds from one branch to the other; it is not known in advance how parts of the pipe network share the permissible head loss. Therefore, the optimality criterion W means the optimum cost of construction for all of the possible pressure levels: the polygon of the construction cost minima, rather than separate data.

The polygons of the minimum costs of construction may intersect, thus, the different routes of the main conduits might be associated with different pressure levels.

In realizing the dynamic programming, the fact that optimum policies U_i^* are also functions of the pressure level H_i should be remembered, and so is the optimality criterion:

$$W^*_{i+1,\ldots,m}(S_i\, H_i) \sim U^*_{i+1,\ldots,m}\,(S_i\, H_i)\;.$$

This can be taken into account by considering the policies U_i for the different pressure levels.

The optimum route of the main conduit is determined by pointing out the branching points where it can be connected to the branching pipes. Theoretically, also a "continuous" dynamic programming could be realized by reducing the point intervals to zero, the investigations indicated, however, to be useless to densify the points beyond a limit. The construction cost does not decrease to a degree to justify the computer time excess upon increasing the number of the branching points.

Major parts of the algorithm

The routes examined as those possible for the main conduit are evaluated by the construction costs, involving LABYE's discontinuous method.

Computing the polygon of minimum building cost of a pipe section

In applying the discontinuous method, one proceeds from section to section and the minimum construction cost polygon should be produced for each section giving the optimum cost of construction for any given head loss corresponding to the pipe sizes possible in view of the velocity range.

Given is a 1 m long pipe section with a discharge q and the series of commercial pipe sizes ($d_{max} > \ldots d_i > d_j \ldots > d_{min}$) likely of discharge q, because of too low or too high velocities, pipe sizes over or under the d_{min} to d_{max} range cannot be applied.

Let us introduce the symbols:

ε_i — head loss along a pipe of unit length, diameter d_i and discharge q',
b_i — construction cost of conduit of unit length and diameter d_i;
$B_i = l \cdot b_i$ — construction cost of conduit of length l and diameter d_i;
$h_i = l \cdot \varepsilon_i$ — head loss in a pipe of length l, diameter d_i and discharge q.

Compute the conjugate values h_i and B_i for the given pipe sizes and plot the pairs of obtained values in a system of co-ordinates. Connecting these points by straight lines yields the polygon of straight lines in Fig. 2, i.e. polygon of the minimum cost of construction of the conduit section.

If the permissible head loss in the conduit equals any of the h_i values, then the whole conduit length l has to be built of corresponding pipe sizes, and the assigned B_i value will be the construction cost.

If, however, the given head loss is an intermediate value, then the conduit will be composed of different pipe sizes, and if the problem is to be solved at the minimum cost of construction, then, in general, pipe sizes belonging to head losses intercepting the permissible values are needed.

Connecting in series the minimum cost polygons

From the minimum construction cost polygon of the pipe sections forming the pipe network, the minimum construction cost polygon of the pipe network can be produced.

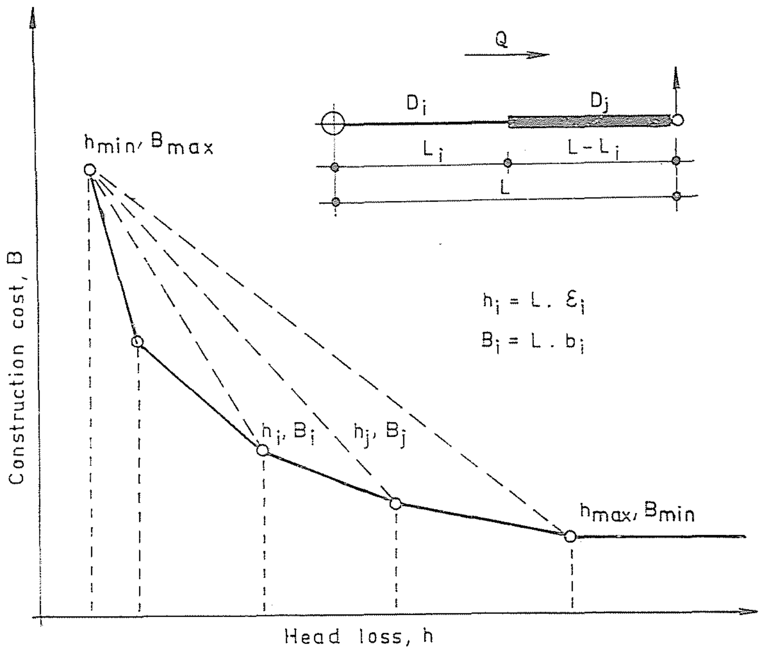

Fig. 2

For example, the minimum cost polygon of the conduit $A-C$ shown in Fig. 3 may be produced by connecting the straight sections of the polygons of sections $A-B$ and $B-C$ in series starting at the point of co-ordinates $h_{13}+h_{23}$, $B_{13}+B_{23}$ in the order of succession of their angles of inclination as seen in the figure. This polygon arrives at the point of co-ordinates $h_{11}+h_{21}$, $B_{11}+B_{21}$, this being the other extreme position (maximum head loss, minimum construction cost).

Following the process of the solution of the problem, it can be understood that within each section of the final minimum cost polygon a single pipe section may change in size.

When the conduit consists of an arbitrary number of, rather than two, sections, and on each section any, rather than three pipe sizes are allowed, then the progress of solution of the problem is the same as shown above. The straight sections of the minimum construction cost polygons of each section should be connected in series, starting at the point corresponding to the highest or lowest head loss and to the pertaining construction cost, in the sequence of the absolute value of their slopes. It follows that the resulting minimum construction cost polygon for the whole conduit will consist of straight sections.

The numerical solution of the algorithm of connecting into series may readily be followed in the ALGOL procedure in Fig. 4.

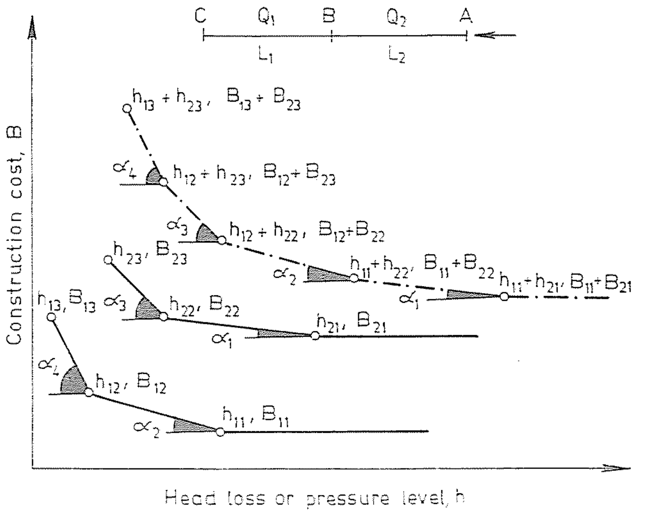

Fig. 3

```
procedure sorf(nl, kl, hl, betl, nk, kk, hk, betk, atm, nlk, klk, hlk, belk, at):
  value nl, nk;
  integer nl, nk, nlk;
  real kl, kk, klk;
  array hl, betl, hk, betk, hlk, belk;
  integer array atm, at;
  begin
    integer nn, ii, jj;
    nn: = 0;
    ii: = jj: = 1;
    klk: = kl + kk;
    betl [nl + 1]: = betk [nk + 1]: = 10000000;
    belk [1]: = 0
    for nn: = nn + 1 while ii lt nl + 1 and jj lt nk + 1 do
      begin
        at [nn]: = atm [jj];
        hlk [nn]: = hl [ii] + hk [jj];
        if betl [ii + 1] lt betk [jj + 1]
          then
          begin
            ii: = ii + 1;
            belk [nn + 1]: = betl [ii]
          end
          else
          begin
            jj: = jj + 1;
            belk [nn + 1]: = betk [jj]
          end;
        nlk: = nn
      end
  end;
```

Fig. 4

The formal parameters of the procedure are as follows:

nl and *nk* — number of corners of minimum construction cost polygons to be connected in series;
kl and *kk* — minimum construction cost co-ordinates of polygons to be connected in series;
hl and *hk* — pressure level co-ordinate of polygons to be connected in series;
betl and *betk* — absolute values of slopes of sides of polygons to be connected in series;
nlk, *klk*, *hlk*, *belk* — data of the resulting minimum construction cost polygon in the former sequence.

Summing up the minimum construction cost polygons

Determine the minimum cost polygons of the two branching conduits shown in Fig. 5.

First of all, the minimum construction cost polygons of each of the pipe sections should be determined by proceeding as described above. Subsequently the minimum cost polygon of the two conduits is obtained by summing as shown in the figure, the construction cost values for the same pressure levels of the cost polygons of the two sections.

The resulting polygon also consists of straight lines and its break points coincide with those of the component curves.

The algorithm of the numerical solution of the summing is represented by the ALGOL procedure in Fig. 6.

The formal parameters of the procedure are as follows:

ni and *nj* — number of corners of minimum cost polygons to be summed;
ki and *kj* — minimum construction cost co-ordinates of polygons to be summed;
hi and *hj* — pressure level co-ordinates of polygons to be summed;
beti and *betj* — absolute value of slopes of polygon sides to be summed;
nij, *kij*, *hij*, *bij* — outputs from minimum cost polygons in the former sequence.

Program flow chart for solving the problem

The presented algorithm steps lend themselves to produce the minimum construction cost polygon of either a part or the whole of the network. This permits to compare the possible main conduit routes.

The program flow chart is shown in Fig. 7.

The minimum construction cost polygon of the branches above and below the *i*-th branching point may be constructed by connecting in series the minimum construction cost polygon or the branch sections. One must be

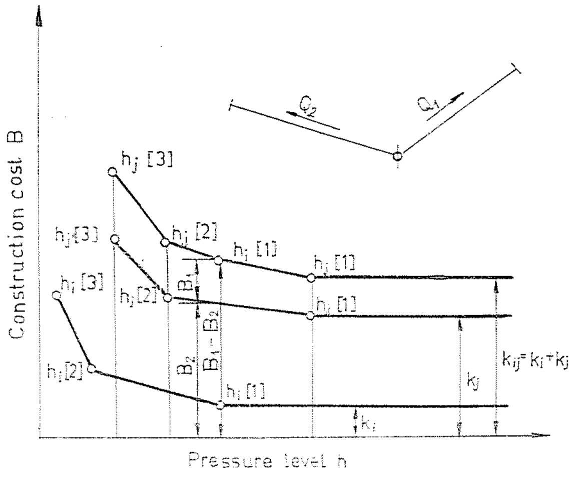

Fig. 5

```
procedure suma (ni, ki, hi, beti, nk, kj, hj, betj, nij, kij, hij, bij);
value ni, nj;
integer ni, nj, nij;
real ki, kj, kij;
array hi, beti, hj, betj, hij, bij;
begin
  integer ii, jj, nn;
  real vv;
  kij: = ki + kj;
  ii: = jj: = 1;
  nn: = 0;
  for nn: = nn + 1 while ii lt ni + 1 and jj lt nj + 1 do
    begin
      bij [nn]: = beti [ii] + betj [jj];
      vv: = hi [ii] — hj [jj];
      if vv gt 0.001
        then
        begin
          hij [nn]: = hi [ii];
          ii: = ii + 1;
          go to agi
        end
        else
          if vv lt —0.001
            then
            begin
              hij [nn]: = hj [jj];
              jj: = jj + 1;
              go to agi
            end;
      hij [nn]: = bi [ii];
      ii: = ii + 1;
      jj: = jj + 1;
agi: nij: = nn
    end
  end;
```

Fig. 6

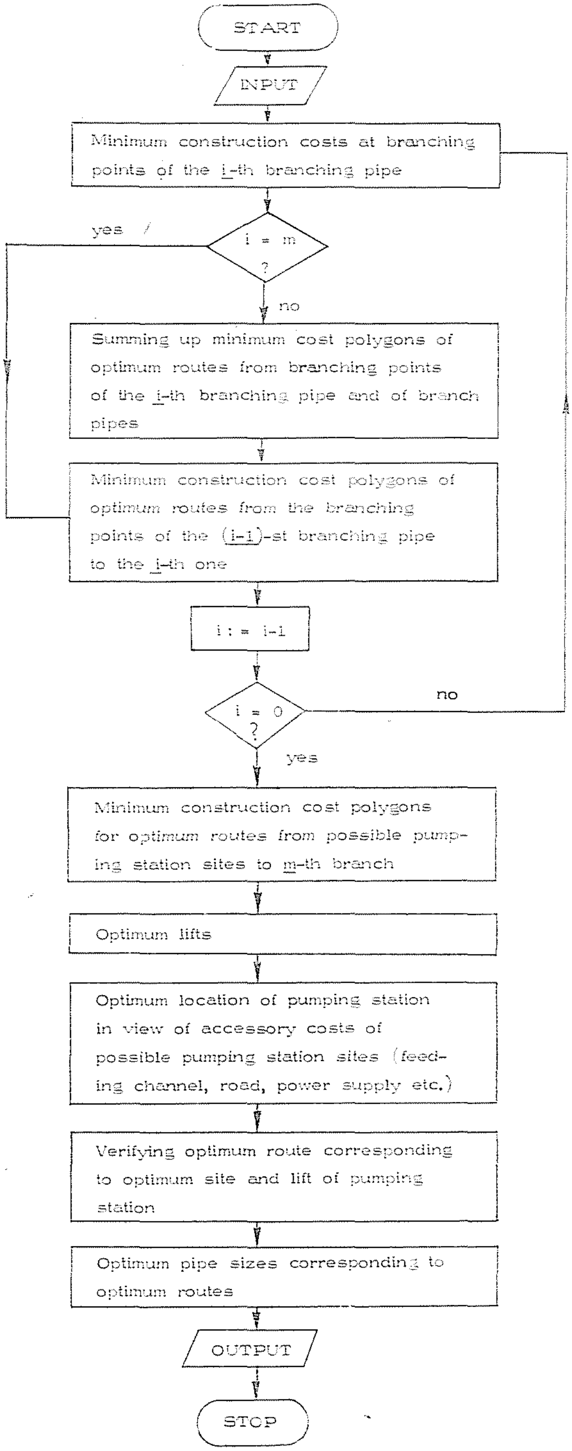
START
INPUT
Minimum construction costs at branching points of the i-th branching pipe
i = m ?
yes
no
Summing up minimum cost polygons of optimum routes from branching points of the i-th branching pipe and of branch pipes
Minimum construction cost polygons of optimum routes from the branching points of the (i-1)-st branching pipe to the i-th one
i : = i-1
i = 0 ?
no
yes
Minimum construction cost polygons for optimum routes from possible pumping station sites to m-th branch
Optimum lifts
Optimum location of pumping station in view of accessory costs of possible pumping station sites (feeding channel, road, power supply etc.)
Verifying optimum route corresponding to optimum site and lift of pumping station
Optimum pipe sizes corresponding to optimum routes
OUTPUT
STOP

Fig. 7

aware that the pipe strengths and the intermediate elevations define pressure levels h_{max} and h_{min}, respectively, setting out the validity range of the computed minimum construction cost polygons (Fig. 8).

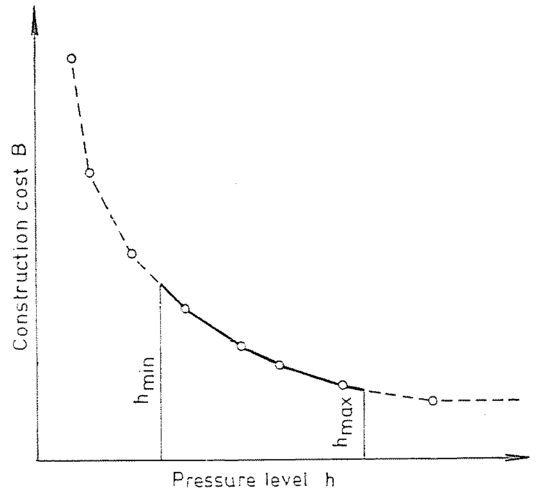

Fig. 8

Confrontation of the routes of the main conduits

The minimum construction cost polygons of the routes of the main conduit leading from the i-th branching point in Fig. 9 to the j-th and $(j + 1)$-st branching points, as well as of the corresponding parts of the pipe network may intersect. In this case it depends on the development of optimality criteria for different pressure levels $W(U_{i,j})$ and $W(U_{i,j+1})$, i.e., those of the minimum construction cost polygons whether $U^*_{i,j+1}$ or $U^*_{i,j}$ will be the optimum policy.

Examining the route of the main conduit from point i to all branching points of the subsequent branch, the corresponding polygons of minimum construction cost may intersect at several points, thus, the optimum route may often change, depending on the pressure level. Therefore, also the optimum routes belonging to the co-ordinates of the minimum construction cost polygon should be stored in the computer.

Determination of the optimum pumping lift of the pumping station

Fig. 10 represents the principle of determining the optimum pumping lift (pressure level).

The optimum lift having been established, the program determines the optimum route by proceeding from one branch to the other (away from the pumping station).

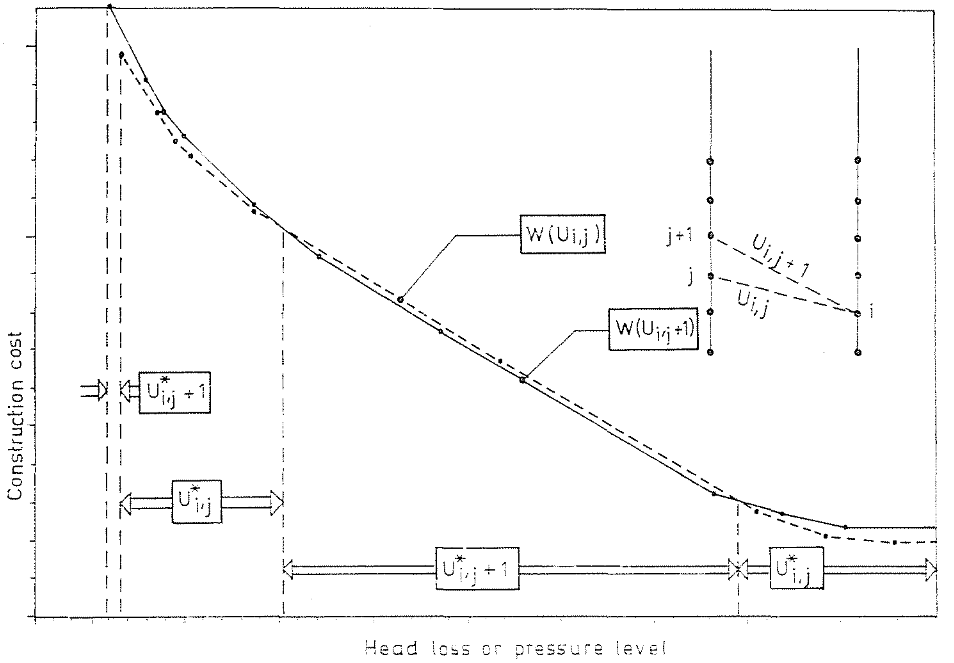
Construction cost
W(Ui,j)
W(Ui,j+1)
j+1
j
i
Ui,j+1
Ui,j
U*i,j +1
U*i,j
U*i,j +1
U*i,j
Head loss or pressure level

Fig. 9

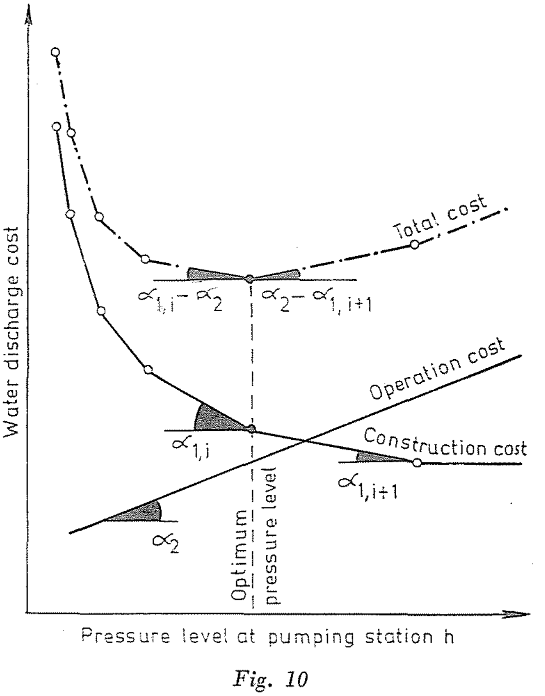
Water discharge cost
Total cost
α1,i − α2
α2 − α1, i+1
Operation cost
Construction cost
α1,j
α1,i+1
α2
Optimum
pressure level
Pressure level at pumping station h

Fig. 10

Summary

The branches of the irrigation pipe networks are, in general, parallel, their spacing ranges being rather closely defined by the type of irrigation equipment, therefore only a few varieties of routes are realizable.

On the contrary, the route of the main conduit feeding the branches is less restricted, several tracings are possible.

A dynamic programming algorithm is presented for the determination of the optimum route of the main conduit, of the site and optimum lift of the pumping station, as well as of the optimum pipe sizes. The algorithm takes into account that also the optimum route depends on the lift of the pumping station.

Programs to determine the optimum route of the main conduit have been written in ALGOL language for the Soviet computer RASDAN—3 and the Polish computer ODRA—1204.

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