COMPUTER ANALYSIS OF ORTHOTROPIC SHALLOW SHELLS*

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1. Introduction

The widespread use of computers has initiated a fast development also in the analysis of shell structures. Approximation by trigonometric equations written for a few points, hardly manageable by desk calculators, has been replaced by the late fifties by applying the finite difference method on fine meshes, and by the finite element method.

In this paper the differential equations of orthotropic shallow shells are derived and deflection—load as well as stress function—load relationships are established by means of eighth-order differential equations. The eighth-order differential equations have been solved by a method based on the spectral decomposition of the second-order difference-operator matrix suggested by EGERVÁRY [1] and applied by SZABÓ and co-workers on a wide range of problems. The advantages of the algorithm will be discussed later.

2. Differential equations of the orthotropic shallow shell

In addition to the basic assumptions usual in the theory of thin shells (homogeneous, ideally elastic material, small displacements etc.) a further assumption usual for shallow shells, is that the second powers and the products of the first derivatives of the mid-surface $z = z(x,y)$ with respect to x and y can be neglected:

$$\left(\frac{\partial z}{\partial x}\right)^2 \approx 0 \; ; \quad \frac{\partial z}{\partial x}\frac{\partial z}{\partial y} \approx 0 \; ; \quad \left(\frac{\partial z}{\partial y}\right)^2 \approx 0 \; . \tag{1}$$

Considering an elementary part of the shell (Fig. 1) the internal forces and their projections on the axes x,y,z (denoted by asterisks) can be related as

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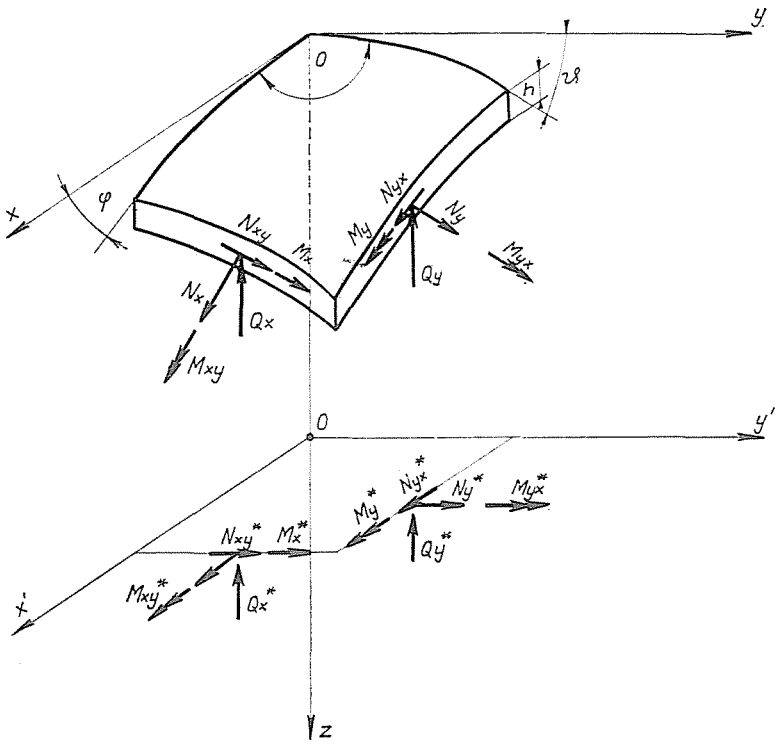

Fig. 1

follows:

$$\left.\begin{aligned} N_x^* &\approx N_x & N_y^* &\approx N_y \\ N_{xy}^* &\approx N_{xy} & N_{yx}^* &\approx N_{yx} = N_{xy} \\ Q_x^* &\approx N_x \operatorname{tg}\varphi + N_{xy} \operatorname{tg}\psi + Q_x & Q_y^* &\approx N_y \operatorname{tg}\psi + N_{xy} \operatorname{tg}\varphi + Q_y \\ M_x^* &\approx M_x & M_y^* &\approx M_y \\ M_{xy}^* &\approx M_{xy} & M_{yx}^* &\approx M_{yx} = M_{xy} \end{aligned}\right\} \quad (2)$$

where

$$\operatorname{tg}\varphi = \frac{\partial z}{\partial x} \quad \text{and} \quad \operatorname{tg}\psi = \frac{\partial z}{\partial y}.$$

The equilibrium equations of the shell element (omitting loads in x and y directions) are:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad (3a)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \quad (3b)$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial}{\partial x}(N_x \operatorname{tg}\varphi + N_{xy}\operatorname{tg}\psi) + \frac{\partial}{\partial y}(N_{xy}\operatorname{tg}\varphi + N_y \operatorname{tg}\varphi) + P = 0 \tag{3c}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \tag{3d}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0\,. \tag{3e}$$

Replacing the tangents in Eq. (3c) by the corresponding derivatives and deriving, after the necessary substitutions, the equilibrium conditions of shallow shells can be reduced into a single equation:

$$\frac{\partial^2 z}{\partial x^2} N_x + 2\frac{\partial^2 z}{\partial x\,\partial y} N_{xy} + \frac{\partial^2 z}{\partial y^2} N_y + \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -P. \tag{4}$$

The geometrical equations for shallow shells are:

$$\varepsilon_x \approx \frac{\partial u}{\partial x} - \frac{\partial^2 z}{\partial x^2} w \tag{5a}$$

$$\varepsilon_y \approx \frac{\partial v}{\partial y} - \frac{\partial^2 z}{\partial y^2} w \tag{5b}$$

$$\gamma_{xy} \approx \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2\frac{\partial^2 z}{\partial x \partial y} w\,. \tag{5c}$$

After the appropriate derivations and reductions, the compatibility equations of shallow shells are:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial_x \partial_y} = -\left(\frac{\partial^2 z}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 z}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right). \tag{6}$$

For the case of orthogonal orthotropy and two-dimensional domain, the generalized Hooke's law can be written as:

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\epsilon}\,, \quad \text{that is} \begin{bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}E_x & E' & 0 \\ E' & E_y & 0 \\ 0 & 0 & G\end{bmatrix}\begin{bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{bmatrix} \tag{7}$$

and the corresponding inverse relationship

$$\boldsymbol{\epsilon} = \mathbf{D}^{-1}\boldsymbol{\sigma}, \quad \text{where} \quad \mathbf{D}^{-1} = \begin{bmatrix} A_x & A' & 0 \\ A' & A_y & 0 \\ 0 & 0 & 1/G \end{bmatrix} \tag{8}$$

where

$$\left.\begin{aligned} A_x &= \frac{E_y}{E_x E_y - E'^2} \\ A_y &= \frac{E_x}{E_x E_y - E'^2} \\ A' &= \frac{-E'}{E_x E_y - E'^2} \end{aligned}\right\}. \tag{9}$$

Let the moments be expressed in the form usual in the theory of orthotropic plates:

$$\left.\begin{aligned} M_x &= -\left(D_x \frac{\partial^2 w}{\partial x^2} + D' \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\left(D' \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2}\right) \\ M_{xy} &= 2 D_{xy} \frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{10}$$

where

$$D_x = \frac{E_x h^3}{12}; \quad D_y = \frac{E_y h^3}{12}; \quad D' = \frac{E' h^3}{12}; \quad D_{xy} = \frac{G h^3}{12}.$$

Introducing the stress function:

$$N_x = \frac{\partial^2 F}{\partial y^2}, \quad N_y = \frac{\partial^2 F}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y}. \tag{11}$$

Substituting (10) and (11) into Eq. (4) delivers the first differential equation of orthotropic shallow shells:

$$\left(\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - 2\frac{\partial^2 z}{\partial x \partial y}\frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}\frac{\partial^2 F}{\partial x^2}\right) - \left(D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}\right) = -P. \tag{12a}$$

Substituting Hooke's law into Eq. (6) the second equilibrium equation of orthotropic shallow shells is:

$$\frac{1}{h}\left(A_y\frac{\partial^4 F}{\partial x^4}+2A\frac{\partial^4 F}{\partial x^2\partial y^2}+A_x\frac{\partial^4 F}{\partial y^4}\right)+ \\ +\left(\frac{\partial^2 z}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 z}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y}+\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right)=0 \tag{12b}$$

where

$$H=2(D'-2D_{xy}) \quad \text{and} \quad A=A'+\frac{1}{G}\,.$$

To formulate the problem in terms of differential operators, let us have:

$$\Delta_p=\frac{\partial^2 z}{\partial y^2}\frac{\partial^2}{\partial x^2}-2\frac{\partial^2 z}{\partial x\,\partial y}\frac{\partial^2}{\partial x\,\partial y}+\frac{\partial^2 z}{\partial x^2}\frac{\partial^2}{\partial y^2} \quad \text{("Pucher"'s operator)} \tag{13a}$$

$$\Delta_1=D_x\frac{\partial^4}{\partial x^4}+2H\frac{\partial^4}{\partial x^2\,\partial y^2}+D_y\frac{\partial^4}{\partial y^4} \quad \text{("plate" operator)} \tag{13b}$$

$$\Delta_2=A_y\frac{\partial^4}{\partial x^4}+2A\frac{\partial^4}{\partial x^2\,\partial y^2}+A_x\frac{\partial^4}{\partial y^4} \quad \text{("membrane" operator)} \tag{13c}$$

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \quad \text{("Laplace" 's operator)} \tag{13d}$$

Thus, the differential equations of an orthotropic shallow shell are:

$$\boxed{\begin{aligned}\Delta_p F-\Delta_1 w &= -P\\ \Delta_2 F+h\Delta_p w &= 0\end{aligned}} \tag{14}$$

The same for the case of isotropic shallow shells (Wlassow-Marguerre equations):

$$\boxed{\begin{aligned}\Delta_p F-D\Delta\Delta w &= -P\\ \Delta\Delta F+Eh\Delta_p w &= 0\end{aligned}} \tag{15}$$

3. Direct expressions for the stress function and the deflection

Since the operators defined by Eqs (13) are linear, their successive application permits the order of succession to be interchanged.

Letting Δ_p and Δ_1 operate on Eqs (14a) and (14b), respectively, dividing the latter by h, and then reducing:

$$\boxed{\Delta_p \Delta_p F + \frac{1}{h} \Delta_1 \Delta_2 F = -\Delta_p P} \tag{16a}$$

i.e. the eighth-order differential equation of the orthotropic shallow shell in terms of the stress function. Applying the operators Δ_2 and Δ_p to Eqs (14a) and (14b), respectively, after subtraction we obtain:

$$\boxed{h\Delta_p \Delta_p w + \Delta_1 \Delta_2 w = \Delta_2 P} \tag{16b}$$

i.e. the eighth-order differential equation of the orthotropic shallow shell in terms of the deflections. For the special case of isotropy the equations become:

$$\Delta_p \Delta_p F + \frac{D}{Eh} \Delta\Delta\Delta\Delta F = -\Delta_p P \tag{17a}$$

$$Eh\Delta_p \Delta_p w + D\Delta\Delta\Delta\Delta w = \Delta\Delta P\,. \tag{17b}$$

4. The solution of the differential equation

The procedure applied here is a special case of the finite difference method (restricted to rectangular domain and homogeneous boundary condition) and based on the fact that the spectral decomposition of the second-order partial difference-operator matrix (Fig. 2)

$$\left[\frac{\partial^2}{\partial x^2}\right] \approx \mathbf{C}_m = \frac{1}{a^2} \begin{array}{c} \\ 1 \\ 2 \\ \\ m \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & & & & m \end{array} \\ \begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ & & & 1 & -2 & \end{bmatrix} \end{array} \tag{18}$$

is known in a closed form.

$$\mathbf{C}_m = \mathbf{U}_m \mathbf{L}_m \mathbf{U}_m \tag{19}$$

$$[u_{jk}] = \left[\sqrt{\frac{2}{m+1}} \sin \frac{jk\pi}{m+1} \right] \tag{20}$$

$$\langle \lambda_k \rangle = \left\langle 2 \left(1 - \cos \frac{k\pi}{m+1} \right) \right\rangle . \tag{21}$$

For the sake of comprehensibility, the further derivations will be restricted to the case of elliptic-paraboloidal isotropic shells. Let us take e.g. Eq. (17a):

$$\Delta_p \Delta_p F + \frac{D}{Eh} \Delta\Delta\Delta\Delta F = -\Delta_p P .$$

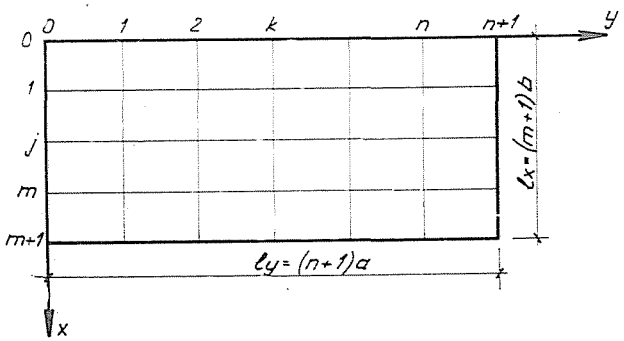

Fig. 2

Operators in full form are:

$$\begin{aligned} &\left(\frac{\partial^2 z}{\partial x^2} \frac{\partial^2}{\partial y^2} - 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial^2}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} \frac{\partial^2 x}{\partial x^2} \right) \left(\frac{\partial^2 z}{\partial x^2} \frac{\partial^2 F}{\partial y^2} - \right. \\ &\left. - 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} \frac{\partial^2 F}{\partial x^2} \right) + \frac{D}{Eh} \left(\frac{\partial^2}{\partial x^2} + \right. \\ &\left. + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right) = \\ &= - \left(\frac{\partial^2 z}{\partial x^2} \frac{\partial^2 P}{\partial y^2} - 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial^2 P}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} \frac{\partial^2 P}{\partial x^2} \right) \end{aligned} \tag{22}$$

one gets for the elliptic-paraboloid (Fig. 3)

$$z = 4 f_x \left(\frac{x}{l_x} - \frac{x^2}{l_x^2} \right) + 4 f_y \left(\frac{y}{l_y} - \frac{y^2}{l_y^2} \right) \tag{23}$$

$$\frac{\partial^2 z}{\partial x^2} = k_2 = -8 \frac{f_x}{l_x^2} ; \quad \frac{\partial^2 z}{\partial y^2} = k^2 = -8 \frac{f_y}{l_y^2} ; \quad \frac{\partial^2 z}{\partial x \partial y} \equiv 0 . \tag{24}$$

Expanding (22):

$$\left(k_1^2\frac{\partial^4 F}{\partial x^4}+2k_1k_2\frac{\partial^4 F}{\partial x^4\partial y^4}+k_2^2\frac{\partial^4 F}{\partial y^4}\right)+\frac{D}{Eh}\left(\frac{\partial^8 F}{\partial x^8}+4\frac{\partial^8 F}{\partial x^6\partial y^2}+\right.$$
$$\left.+6\frac{\partial^8 F}{\partial x^4\partial y^4}+4\frac{\partial^8 F}{\partial x^2\partial y^6}+\frac{\partial^8 F}{\partial y^8}\right)=-\left(k_1\frac{\partial^2 P}{\partial x^2}+k_2\frac{\partial^2 P}{\partial y^2}\right). \tag{25}$$

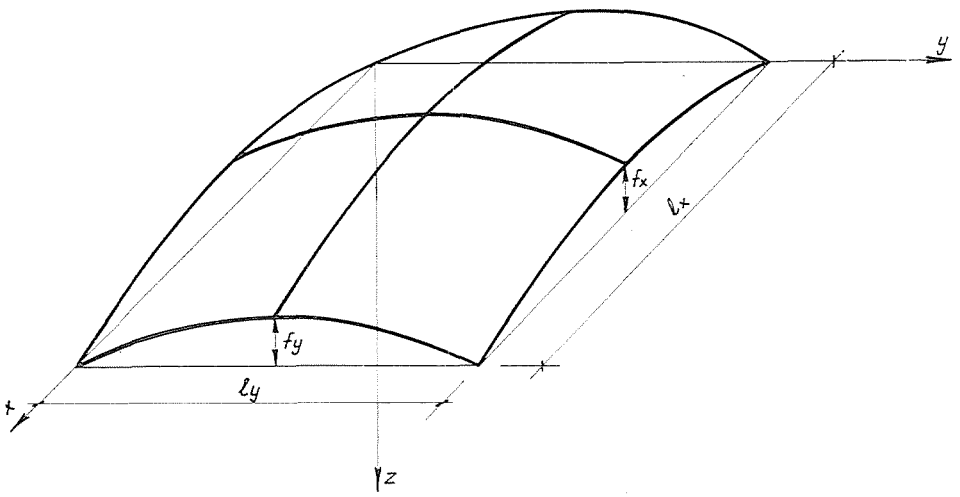

Fig. 3.

$$Z=4fx\left(\frac{x}{lx}-\frac{x^2}{lx^2}\right)+4fy\left(\frac{y}{ly}-\frac{y^2}{ly^2}\right)$$

Let matrices $\mathbf{F}$ and $\mathbf{P}$ include discreet nodal values of the stress function, $\overset{(m,n)}{\mathbf{F}}$ and the load values $\overset{(m,n)}{\mathbf{P}}$ in the same nodal points, resp., and approximating the differential operators by the matrix form of the corresponding difference operators:

$$\left(\frac{k_1^2}{a^4}\mathbf{C}_m^2\,\mathbf{F}+\frac{2k_1k_2}{a^2b^2}\mathbf{C}_m\,\mathbf{F}\,\mathbf{C}_n+\frac{k_2^2}{b^4}\mathbf{F}\,\mathbf{C}_n^2\right)+\frac{D}{Eh}\left(\frac{1}{a^8}\mathbf{C}_m^4\mathbf{F}+\right.$$
$$+\frac{4}{a^6b^2}\mathbf{C}_m^3\mathbf{F}\,\mathbf{C}_n+\frac{6}{a^4b^4}\mathbf{C}_m^2\mathbf{F}\,\mathbf{C}_n^2+\frac{4}{a^2b^6}\mathbf{C}_m\mathbf{F}\,\mathbf{C}_n^3+ \tag{26}$$
$$\left.+\frac{1}{b^8}\mathbf{F}\,\mathbf{C}_n^4\right)=-\left(\frac{k_1}{a^2}\mathbf{C}_m\mathbf{P}+\frac{k_2}{b^2}\mathbf{P}\,\mathbf{C}_n\right).$$

Substituting the spectral form of $\mathbf{C}_m$ and $\mathbf{C}_n$:

$$\left(\frac{k_1^2}{a^4}\,\mathbf{U}_m\mathbf{L}_m^2\,\mathbf{U}_m\mathbf{F}+\frac{2k_1k_2}{a^2b^2}\,\mathbf{U}_m\mathbf{L}_m\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n\,\mathbf{U}_n+\frac{k_2^2}{b^4}\,\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^2\,\mathbf{U}_n\right)+$$
$$+\frac{D}{Eh}\left(\frac{1}{a^8}\,\mathbf{U}_m\mathbf{L}_m^4\,\mathbf{U}_m\mathbf{F}+\frac{4}{a^6b^2}\,\mathbf{U}_m\mathbf{L}_m^3\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n\,\mathbf{U}_n+\right.$$
$$+\frac{6}{a^4b^4}\,\mathbf{U}_m\mathbf{L}_m^2\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^2\,\mathbf{U}_n+\frac{4}{a^2b^6}\,\mathbf{U}_m\mathbf{L}_m\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^3\,\mathbf{U}_n+ \tag{27}$$
$$\left.+\frac{1}{b^8}\,\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^4\,\mathbf{U}_n\right)=-\left(\frac{k_1}{a^2}\,\mathbf{U}_m\mathbf{L}_m\,\mathbf{U}_m\mathbf{P}+\frac{k_2}{b^2}\,\mathbf{P}\,\mathbf{U}_n\mathbf{L}_n\,\mathbf{U}_n\right).$$

Multiplying the equation by $\mathbf{U}_m$ from the left and by $\mathbf{U}_n$ from the right, $\mathbf{U}_m$ and $\mathbf{U}_n$ being considered orthonormal:

$$\left(\frac{k_1^2}{a^4}\mathbf{L}_m^2\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n+2\,\frac{k_1k_2}{a^2b^2}\mathbf{L}_m\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n+\frac{k_2^2}{b^4}\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^2\right)+$$
$$+\frac{D}{Eh}\left(\frac{1}{a^8}\mathbf{L}_m^4\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n+\frac{4}{a^6b^2}\mathbf{L}_m^3\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n+\frac{6}{a^4b^4}\mathbf{L}_m^2\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^2+\right.$$
$$\left.+\frac{4}{a^2b^6}\mathbf{L}_m\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^3+\frac{1}{b^8}\,\mathbf{U}_m\mathbf{F}\,\mathbf{U}_n\mathbf{L}_n^4\right)= \tag{28}$$
$$=-\left(\frac{k_1}{a^2}\mathbf{L}_m\,\mathbf{U}_m\mathbf{P}\,\mathbf{U}_n+\frac{k_2}{b^2}\,\mathbf{U}_m\,\mathbf{P}\,\mathbf{U}_n\,\mathbf{L}_n\right).$$

Introducing the symbol $\mathcal{M}$ of logic multiplication defined as:

$$\mathbf{A}\mathcal{M}\mathbf{B}=\mathbf{C}\,,\quad\text{if}\quad[a_{ij}\cdot b_{ij}]\;=\;[c_{ij}]$$

then

$$\mathbf{U}_m\,\mathbf{F}\,\mathbf{U}_n\;=\;\mathbf{M}\mathcal{M}(\mathbf{U}_m\,\mathbf{P}\,\mathbf{U}_n)\,. \tag{29}$$

This multiplied again by $\mathbf{U}_m$ and $\mathbf{U}_n$ gives the result

$$\mathbf{F}=\mathbf{U}_m\,\{\mathbf{M}\mathcal{M}(\mathbf{U}_m\,\mathbf{P}\mathbf{U}_n)\} \tag{30}$$

where

$$[m_{j,k}]=\left[\frac{-\left(\frac{k_1}{a^2}\,\lambda_j+\frac{k_2}{b^2}\,\lambda_k\right)}{\left(\frac{k_1}{a^2}\,\lambda_j+\frac{k_2}{b^2}\,\lambda_k\right)^2+\frac{D}{Eh}\left(\frac{\lambda_j}{a^2}+\frac{\lambda_k}{b^2}\right)^4}\right] \tag{31}$$

— λ_j being the j-th eigenvalue of matrix $\mathbf{C}_m$; and
— λ_k the k-th eigenvalue of matrix $\mathbf{C}_n$.

Solving Eq. (17b), in a similar way:

$$\mathbf{W} = \mathbf{U}_m \{\mathbf{N}\Delta(\mathbf{U}_m \mathbf{P} \mathbf{U}_n)\}\mathbf{U}_n \tag{32}$$

where

$$[n_{j,k}] = \frac{\left(\frac{\lambda_j}{a^2} + \frac{\lambda_k}{b^2}\right)^2}{Eh\left(\frac{k_1}{a^2}\lambda_j + \frac{k_2}{b^2}\lambda_k\right)^2 + D\left(\frac{\lambda_j}{a^2} + \frac{\lambda_k}{b^2}\right)^4} . \tag{33}$$

Solutions to the differential equations (16) of the orthotropic shell are similar in form, only the elements of matrices $\mathbf{M}$ and Δ are more complicated to express. Theoretically it can be proved that any problem describable by partial differential equations of arbitrary even order has its solution in the same form, except that the components of the matrix of modification will change. [5, 3]

5. Appreciation of the method and its results

The outlined method is directly valid to shallow shells where $\frac{\partial^2 z}{\partial x \partial y} = 0$, but with certain smaller modifications it can also be used for the case of hypar shells.

(Namely here $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} = 0$ and $\frac{\partial^2 z}{\partial x^2 \partial y^2} = \text{constant}$.) Using again the Pucher's operator, $\Delta_p \Delta_p = 4k^2 \frac{\partial^4}{\partial x^2 \partial y^2}$, which means again the application of a difference operator of even order. Two computer programs have been tested on actual problems, the first for elliptic paraboloid shells and their varieties (elliptic vault), the other for hypar shells. The results were checked by re-substituting into the basic equations and by comparing them with published examples.

The advantages of the algorithm are:

— the equation system with $2 \times m \times n$ unknowns is practically decomposed into $2 \times m \times n$ one-unknown equations;
— the storage space needed is very small, about four times the number of the points (an array each of the matrices, $\mathbf{P}$, $\mathbf{W}$, $\mathbf{F}$ and $\mathbf{U}_m$, $\mathbf{U}_n$ for diagonals $\mathbf{L}_m$ and $\mathbf{L}_n$);
— many operations are repeated, e. g. three multiplication by $\mathbf{U}_m$, $\mathbf{U}_n$, thus the program is a relatively short one;
— it is enough to calculate $\mathbf{U}_m \mathbf{P} \mathbf{U}_n$ only once and here the program can be branched to calculate $\mathbf{F}$ and $\mathbf{W}$.

For information it has to be mentioned that with the small-category second-generation computer ODRA—1204 and a program written in ALGOL—

1204, using only the main storage unit, solution can be obtained for about 1600 points.

There are some problems to be mentioned. The homogeneous boundary condition has a physical meaning for the simultaneous equations of fourth order, since for the deflections $W = 0$; $\frac{\partial^2 w}{\partial n^2} = Mn = 0$ and for the stress function:

$$F = 0\,; \quad \frac{\partial^2 F}{\partial n^2} = 0$$

thus the homogeneous boundary condition corresponds to the problem of the simply supported shell with no lateral pressure. For the differential equations of eighth order, there are four additional boundary conditions — referring to the fourth and sixth normal derivatives of the deflection and the stress function — lacking physical interpretation so far. Nevertheless the result obtained by solving the differential equations of eighth order agrees with that of the simply supported shell with no lateral pressure.

The developed method permits to analyze any arbitrary translational shell by iteration.

For inhomogeneous boundary conditions or a shell over a non-rectangular domain the method of singular solutions seems to be effective.

6. Further research trends

The research may be continued in two directions. The one is to insert arbitrary boundary conditions into the program so that the effect of the free edge or of the elastic edge beam can be analyzed also for elliptic-paraboloid shells. The other direction is the investigation of geometric non-linearity effects on elliptic paraboloid shells. In this connection there are already some results available, to be published shortly.

Summary

Differential equations of orthotropic shallow shells are derived and reduced into eighth-order partial differential equations to express deflection and stress function, both as a function of vertical load.

A version of the finite difference method, based on the known spectral decomposition of the second-order difference-operator matrix is suggested for solving the differential equation, a rather economic method for the computer analysis of the deflections and stress functions of elliptic paraboloid shells over rectangular domain. in case of homogeneous boundary conditions.

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