# NUMERICAL ANALYSIS OF DOUBLE-LAYER SPACE TRUSSES BY THE CONTINUUM METHOD

by

P. TOMKA
Department of Steel Structures, Technical University, Budapest

(Received November 2nd, 1972)

Presented by Prof. Dr. O. HALÁSZ

Recently, the use of double-layer space trusses for covering industrial halls has spread due to their relatively low cost and simple assembly. The computation of bar forces has mostly been based upon convential, somewhat tedious methods. The so-called continuum analysis is likely to offer greater ease, however, two questions emerge:

— Is this approximate method accurate enough for practical design purposes?

— What are its advantages compared to other possible ways of calculation?

Basic concept of the method will be illustrated on the structure seen in Fig. 1, a space truss for roofs of two-nave halls. In direction $x$ it can be as long as needed. The interim supports are under the bottom nodes while the supports along the edges join the top nodes.

The bars in both the upper and lower chords are arranged in regular square mesh so that every bottom node is exactly below the centre of the top square mesh. The diagonals between the two chords outline pyramids. Structures of this kind may be replaced by a continuum [1].

The torsional rigidity of the chords being zero, the differential equation of the corresponding continuum is:

$$w^{\mathrm{IV}} + w^{::} = \frac{p}{B} \tag{1}$$

where $p$ — intensity of the uniform load.

The bending rigidity:

$$B = h^2 \frac{A + A_a}{A \cdot A_a} \tag{2}$$

where $A$, $A_a$ — axial rigidity of the bars in the upper and the lower chord, resp.;

$h$ — depth of the structure (i. e. distance between the two chords), (missing subscript means upper chord, subscript $a$ means lower chord).

The axial rigidity of bars e. g. in the upper chord:

$$A = \frac{E \cdot F}{c} \tag{3}$$

where $E$ — modulus of elasticity,
$F$ — bar cross-sectional area,
$c$ — parallel bar spacing.

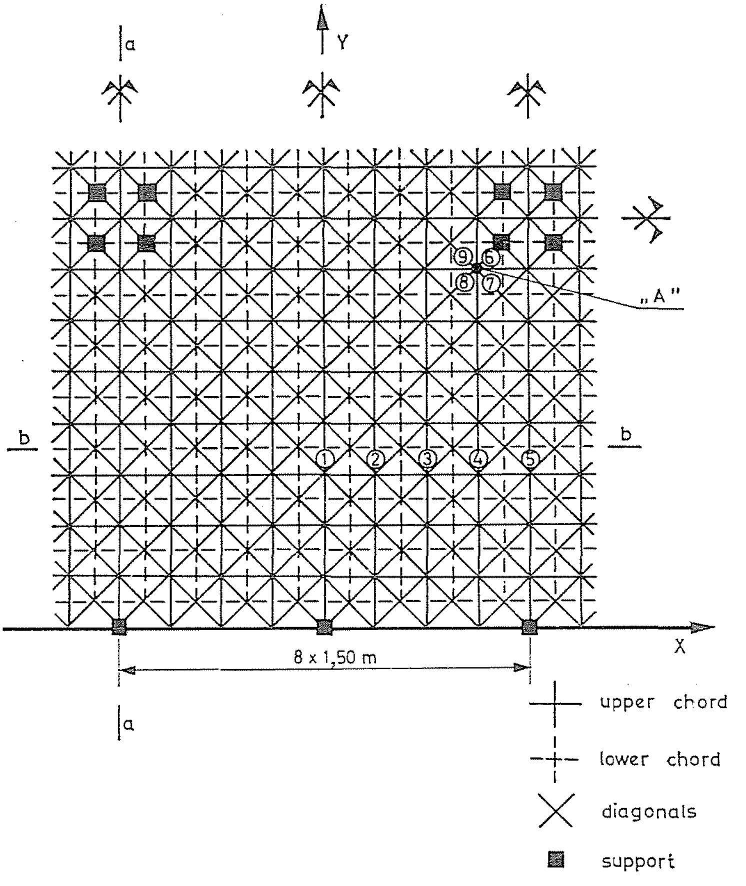


*Fig. 1*

The double-layer truss is converted into a corresponding continuum (plate without torsional rigidity) by means of Eqs. (2) and (3). The shear rigidity due to the diagonals is ignored. The effect of this simplification will be discussed later.

The boundary conditions are

a) for the free edge ($\nu = 0$):

$$w^{\cdot\cdot} = 0 \tag{4}$$

and

$$w^{\cdot\cdot\cdot} = 0, \tag{5}$$

i. e. the moment and the vertical shear force perpendicular to the edge are zero;

b) for the supported joints:

$$w = 0. \tag{6}$$

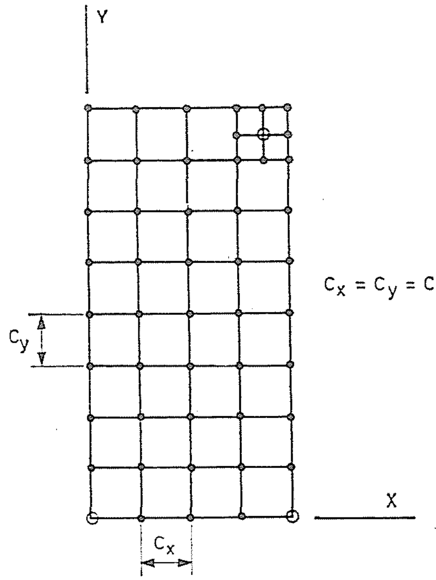


*Fig. 2*

The structure is symmetrical about, and is assumed to be infinitely long, in direction $x$. The latter requirement can be met by providing symmetry conditions about the axes in direction $y$, Fig. 1.

To practically test this method of analysis, it has been applied to the erected structure schematically shown in Fig. 1. The numerical solution was based on the finite-difference method. The substituting network is shown in Fig. 2. Its nodes coincide with those in the upper chord of the space truss and for this reason the mesh is denser near the interim supports.

The numerical operators representing the fourth derivatives must be chosen so that the solution is as accurate as possible even near the edges. Hermite's method was found likely to help meeting boundary conditions [2]. Hermitian formulae for the different nodes of the network have been obtained according to [2]. These differential operators satisfy all the boundary conditions and — where it is possible — consist of two adjacent values of $w$. For example, the operators belonging to the patterns in Fig. 3 are:

$$w_1^{IV} = \frac{1}{c^4}(1{,}71\, w_1 - 3{,}43\, w_2 + 1{,}71\, w_3) - \frac{1{,}71}{c^2} w_1'' + \frac{1{,}71}{c} w_1^{III} \tag{7a}$$

$$w_2^{IV} = \frac{1}{c^4}(-1{,}65\, w_1 + 4{,}24\, w_2 - 3{,}5\, w_3 + 0{,}94\, w_4) - \frac{0{,}71}{c^2} w_1'' + \frac{0{,}24}{c} w_1^{III}. \tag{7b}$$

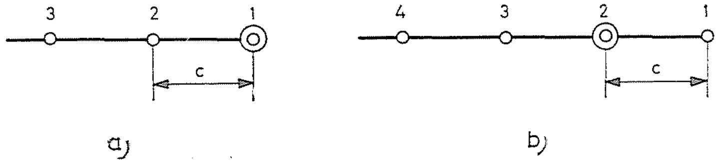


*Fig. 3*

Or generally, with a simplified notation:

$$w^{IV} = \frac{[w^{IV}]}{c^4} \tag{7c}$$

For Eq. (7b) a fifth-term Hermitian interpolation was needed.

All necessary operators being obtained, the unknown deflections can be computed by solving the system of finite-difference equations constructed from equations of the type (1) in the form of

$$\mathbf{A} \cdot w = b. \tag{8}$$

The coefficient matrix **A** is constructed by means of a special subroutine. The serial number of the unknowns and the type of the needed differential operators in directions $x$ and $y$ are computed from the co-ordinates of mesh nodes. The relative co-ordinates and the constants of the operators are stored in a vector. This way, after proper organizing steps, **A** can be automatically constructed.

The right-hand vector can be written in the form:

$$b = p' + q' \tag{9}$$

$p'$ being the vector of the reduced mean values of the distributed load. In case of uniform load, the elements of $p'$ are: for interim points 1, along the edges 0,5, for corners 0,25 and for unloaded points 0.

Furthermore, there is a possibility to involve the moments and the vertical shear forces acting along the edges because the second and the third derivatives are proportional to these loads. The elements of the vector $q'$

represents the reduced values of this effect. In our example each element is zero.

According to the previously mentioned reduction, the deflections are obtained in dimensionless form ($w_{\mathrm{red}}$). Their effective values $w_{\mathrm{eff}}$ can be computed from the acting load and the actual dimensions of the structure:

$$w_{\mathrm{eff}} = w_{\mathrm{red}} \cdot \frac{p \cdot c^4}{B} . \tag{10}$$

The bar forces can be computed from the results of the continuum analysis, e. g. upper chord forces ($S_t$) in direction $x$ are:

$$S_t = \frac{c \cdot m_x}{h} = - \frac{p \cdot c^3}{h} [w_{\mathrm{red}}]'' \tag{11}$$

where $m_x$ — bending moment per unit length on the substituting continuum;
$[w_{\mathrm{red}}]''$ — quantity proportional to the second derivative, computed from reduced values of deflections, see Eq. (7c).

Forces in the lower chord can be computed by interpolation (bars in the lower chord being shifted against the top nodes).

Forces in the diagonals ($S_d$) are:

$$S_d = c \cdot \frac{\sqrt{\left(\frac{c}{2}\right)^2 + h^2}}{h} \frac{\cos \gamma}{2} \pm (t_x \pm t_y) \tag{11a}$$

where $\gamma$ — angle between two adjacent diagonals;
$t_x$, $t_y$ — vertical shear forces of the continuum in directions $x$ and $y$, resp. Namely the forces in diagonals meeting at the same top node are obtained by projecting and combining the vertical shear forces. Here again the operators derived from the reduced deflections are of use:

$$S_d = p \cdot c^2 \frac{\sqrt{\left(\frac{c}{2}\right)^2 + h^2}}{h} \frac{\cos \gamma}{2} (\pm [w_{\mathrm{red}}]^{|||} \pm [w_{\mathrm{red}}]^{\cdots}) . \tag{12b}$$

The reaction forces of the supports can be computed from the corresponding right- and left-hand vertical shear forces.

In constructing the computer program, needlessness of any but certain outputs (e. g. max. bar forces, deflections) was kept in mind. So — according to requirements — only certain predetermined groups of data might be printed out, at a substantial economy of running time.

In the following, some of the results of continuum analysis will be confronted to those of exact computation.

The deflections of the nodes along section $a-a$ (Fig. 1) are shown in Fig. 4. The marked difference can be attributed to the shearing deformations neglected in continuum analysis. Although this effect may also be taken into consideration, measurements on completed structures show actual deflections resulting from the relatively great displacements of the bolted connections to greatly exceed even those computed by the exact method. It is a matter of consideration to choose a more complex conversion, deemed unnecessary in our case.

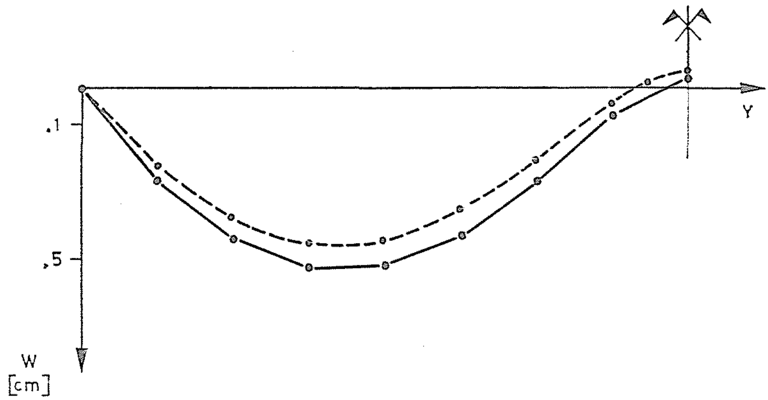


*Fig. 4*

Forces in the upper chord bars perpendicular to section $b-b$ have been compiled in Table 1. Deviations between the two kinds of results are due to the greater cross-sectional areas of a few bars (e. g. that of the bar 1). In our computation, for the sake of simplicity, bending rigidity $B$ was taken uniform throughout the space truss. The sums of the forces (column 6) show, however, a close agreement (i. e. condition of moment equilibrium is met).

**Table I**

Upper chord bar forces [Mp]

| Bar | 1 | 2 | 3 | 4 | 5 | Sum of bar forces |
|---|---|---|---|---|---|---|
| Continuum analysis | −1.78 | −1.69 | −1.46 | −1.25 | −1.16 | −5.87 |
| Exact values | −2.34 | −1.53 | −1.25 | −1.12 | −1.22 | −5.68 |

Forces in diagonals meeting at the top node $A$ are given in Table II. The deviation from the accurate values is considerably larger than in the preceding case. It should be noted that the computation of diagonal forces from

**Table II**

Diagonal forces [Mp]

| Bar | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| Continuum analysis | −2.70 | −0.03 | 2.70 | 0.03 |
| Exact value | −2.94 | 0.61 | 1.39 | 0.62 |

the results of the continuum is the most difficult part of our method. It is remarkable, however, that the maximum forces can be obtained at a satisfactory accuracy.

The above comparisons lead to the conclusion that the continuum analysis meets all requirements for an approximate method.

Finally let us mention a possible further improvement of this method. Remind that bar forces are to be determined from the derivatives of the $w$ function. Consequently, the nodes of the network of the finite difference method need not coincide with those of the structure. It is sufficient to use a relatively coarse network and to compute the corresponding derivatives at the nodes. This reduction of the number of unknowns may also involve a substantial economy of running time. Different shapes of structures may be treated by different mesh widths in the two directions ($c_x$, $c_y$).

This method, applied to the rather complex structure discussed in [3] proved to give satisfactory results.

## Summary

An approximate computation method has been applied to one type of double-layer space trusses as practical verification of the so-called continuum analysis. The numerical results show that

— the accuracy of this method is sufficient for practical purposes,

— the method is flexible enough to treat relatively complex cases of loading and structural forms,

— a substantial economy of running time, thus, of cost may be achieved.

## References

1. Kollár, L.: Continuum Method of Analysis for Double Layer Space Trusses with Upper and Lower Chord Planes of Different Rigidities. Acta Techn. Hung. 76 (1973), 1–2
2. Holnapy, D.: A New Computer Method of Numerical Analysis of Shell Structures.* Candidate's Thesis, Budapest, 1972
3. Kollár, L.: Analysis of Double-Layer Space Trusses with Diagonally Square Mesh by the Continuum Method. Acta Techn. Hung. 74 (1973), 1–2

Research Assistant Pál Tomka, 1111 Budapest, Műegyetem rkp. 3. Hungary.

* In Hungarian.