ANALYSIS OF FUNCTIONS FOR THE EXTRAPOLATION OF RHEOLOGICAL PHENOMENA OF PRESTRESSING STEEL

by

Gy. Czeglédi* — A. Erdélyi

Department of Building Materials, Technical University, Budapest

(Received March 1, 1973)
Presented by Prof. Dr. J. Talabér

1. Introduction

The strain of freely extending wires loaded by tension $F =$ const. may become steady with time, hence no rupture occurs; or the strain increases and finally the wire fails at a finite or infinite time (curves *a* and *b*, resp., in Fig. 1) [11].

Applying an initial stress σ_0 to the wire to extend it by λ_0 and keeping the extension at a constant value, the stress will decrease and tend to a limit value in infinity. These phenomena are termed creep or yield, and relaxation, respectively. Fig. 2 is a diagram of the percentage relaxation function $R = \frac{\sigma_0 - \sigma}{\sigma_0} \cdot 100$. The quoted phenomena still need to be explained from metallography aspects, hence they cannot be exactly formulated so as to fit any case. Probably, however, both phenomena have identical or rather similar physical bases. Creep process of the form *b* in Fig. 1 occurs generally at higher temperature or upon a rather high tensile load. Process *a* is rather similar in form to the relaxation curve. In the construction practice, knowledge of the creep form *a* and of the relaxation process in Fig. 2 are of importance, to be analysed in the following.

The initial marked rise of the curves upon loading may be explained by the action of dislocation foci within the material, easy to initiate. Deformations cause the initial dislocation density to increase, dislocation displacements are impeded (e.g. by dispersed carbides), crossed and blocked — all being effects causing the material to strain harden; strain and relaxation rates to decrease. Normal concrete curing temperature being below that of steel recrystallization, no recovery process occurs, deformation rate decreases, both strain and relaxation tend to a finite limit value. (Cold-drawn prestressing wires are exposed to still lower temperatures to avoid risk of annealing.)

In designing prestressed concrete units it is advisable to know relaxation values, however inaccessible to direct measurement they are because of too

* Department of Engineering Mechanics, Technical University, Budapest.

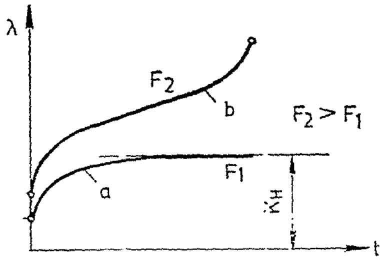

Fig. 1

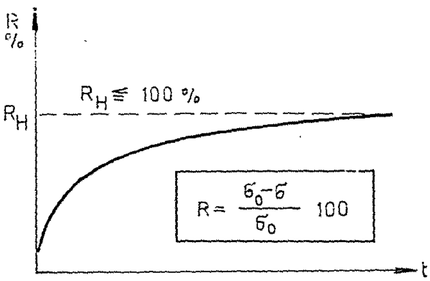

Fig. 2

long intervals, therefore results of short or long-term tests have to be extrapolated intermediating theoretical considerations.

2. Rheological models and functions

The simplest model yielding finite K_H and R_H values (see definitions in Figs 1 and 2) is the Hooke—Maxwell one involving three parameters and parallel connection. (See e.g. Fig. 2/a in [1] in this issue.)

Its behaviour is described by

$$R = R_H \left(1 - e^{-t/\tau_1}\right) \tag{1}$$

where t = time; $\tau_1 = \frac{\eta_1}{k_1}$ a constant of time unit in system "M," the so-called relaxation time. Time axis being of $\log t$ scale, the function is "S"-shaped.

Some authors state to be experimentally demonstrable that for prestressing wires exposed to rather high initial stresses σ_0 and high temperatures T, the "S" curve section about and after the inflection point can be measured, while for high-grade wires exposed to lower σ_0 and T values, only the first, concave (parabolic) section of the "S" curve can be measured for a short time [2].

In general, rheological behaviour of real materials is attempted to be simulated by means of Maxwell and Kelvin—Voigt units connected in series (Fig. 3).

Model extension being:

$$\lambda = \frac{F}{k_0} + \frac{F}{\eta_0} \cdot t + \sum_{i=1}^{n} \frac{F}{k_i} (1 - e^{-t/\tau_i}) , \qquad (2)$$

where $\tau_i = \dfrac{\eta_i}{k_i}$ $(i = 1, 2, \ldots, n)$.

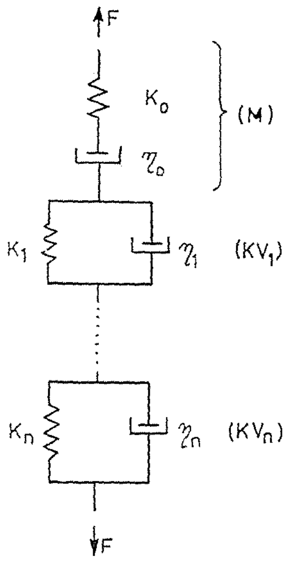

Fig. 3

Because of the second term describing the viscous flow, for $t \to \infty$, $\lambda \to \infty$, contradictory to both observations and preassessments in civil engineering. Omitting, however, the dashpot of viscosity η_0, the system had no permanent deformation. This contradiction can be lifted by an increasing function $\eta = \eta(t)$ corresponding to a time-thickening material as referred to by REINER [14].

Possible simpler forms of the second term are

$$\lambda_2 = - \frac{v}{t^b} + V \qquad (b > 0, \ t \neq 0) , \qquad (3)$$

or

$$\lambda_2 = V (1 - e^{-\nu_1 t}). \qquad (4)$$

Both have $\lambda_2 \rightarrow V$ for $t \rightarrow \infty$. Expressing the flow of increasing viscosity in Eq. (2) according to (4) (omitting the first term for elastic strain), creep or relaxation can be written as the sum of at least two functions type $(1 - e^{-x})$ — or more ones to better approach real materials.

For $\eta \neq$ const., it is justified to have $\tau_i = \eta_i(t)/k_i$ not only in the second term of (2), i.e. in the Maxwell system connected in series but in all consecutive Kelvin—Voigt units $KV_1, KV_2, \ldots, KV_n$. The increasing viscosity slows down the creep and relaxation process. This deceleration can be illustrated by replacing

$$f_i(t) = -\frac{t}{\tau_i}$$

in the exponent of (2) e.g. by

$$f_i(t) = -at^c \quad (0 < c < 1), \tag{5}$$

or

$$f_i(t) = -a \log t \tag{6}$$

(In this latter case the exponential function becomes of course a hyperbolic one.) Thereby the form $(1 - e^{-\sqrt{t}})$ proposed by PALOTÁS [3] for the concrete creep functions becomes phenomenologically justified. Logarithm base has intentionally been left undefined for the log t function, it being irrelevant because of the constant chosen arbitrarily.

In creep or relaxation tests on real materials, a non-negligible part of creep and relaxation takes part up to starting the test at time t_0, or better, already during the load application, a proportion depending on the initial stress σ_0, the loading rate and the temperature. This fact should be considered in writing the equations, to avoid important distortions especially over room temperature.

3. Functions for extrapolating stress relaxation

Functions possibly harmonizing with rheology models, fitting measurement points in the measuring range, permitting extrapolation and easily computerized are sought for. This last specification means that the equation system defining the indefinite constants of the approximating function — according to e.g. the Gaussian principle of the sum of least squares of deviation — is possibly a linear algebraic equation system.

3.1. Utilized and suggested functions

The following functions have been tested for extrapolability:

$$R = a_0 + a_1 \log t + a_2 \log^2 t + \ldots; \tag{7}$$

$$\log R = a_0 + a_1 \log t + a_2 \log^2 t + \ldots; \tag{8}$$

$$\log v_R = a_0 + a_1 \log t, \tag{9}$$

v_R being the relaxation rate;

$$R = R_H \left[1 - \exp\left(\sum_{i=0}^{k} a_i x^i\right)\right]; \tag{10}$$

$$R = R_H \left[1 - \exp\left(a f(t) + c\right)\right], \tag{11}$$

possible forms of time function $f(t)$ being e.g.:

$$f(t) = t^b, \ (0 < b < 1), \tag{12}$$

$$f(t) = \log t. \tag{13}$$

Approximate functions of forms (7), (8), (9) have often been published (e.g. [4], [5], [6], [7], [8], [10]). Their common deficiency is to have no limit value and to describe only the first, so-called parabolic section of the quoted "S"-shaped functions, at an about satisfactory accuracy.

They suit approximation, hence extrapolation in the measuring range, but no reliable result may be expected of an extrapolation if not after long-term measurements.

The higher the degree number of the approximating polynomial has been chosen for (7) and (8), the better the approximation within the measuring range. The suitable number of polynomial degree can only be found by trial, with due care to have the function monotonously increasing e.g. in the range $0 < t < 10^7$ h. Our tests showed the degree number to be $3 \div 4$ as a maximum. Omitting all but the first two terms in (8), this will be a simple parabolic approximation to linear scale, namely the coefficient a_1 is always greater than zero.

Function of form (9) approximating the relaxation rate supposes a hyperbolic rate function, relation $a_1 < 0$ being always valid. In this case, for $a_1 < -1$ the relaxation has a limit value, nevertheless the usual measurement periods exhibited $-1 < a_1 < 0$.

Developing first differences of the obtained relaxation values and dividing them by the time interval delivers an approximate value for the difference quotients, equal to the derivative at a given point of the given interval. As a first approximation, this difference quotient can be assigned to the mid-point of the interval, and to the obtained point set, an approximate function — approximate relaxation rate function — may be fitted. In its knowledge another assignment point can be chosen, yielding a closer determination of the rate function. This procedure may be continued to the desired accuracy, the relaxation function will then be obtained after integrating the rate function, by causing the relaxation function to pass through a selected point of the recorded relaxation point set.

In using functions of forms (7), (8) and (9), from computer technique aspects it is rather useful to obtain unknown coefficients a_i by solving a linear, inhomogeneous algebraic equation system.

Rheology characteristics are better met by a function type (10). By increasing the degree number of the polynomial in the exponent of the natural logarithm base, the approximation may be improved. However, difficulties mentioned for (7) and (8) subsist; in particular, if e.g. in the range $0 < t < 10^7$ h the exponent polynomial is no monotonous function, then it is unfit for extrapolation. For $k \geq 2$, this problem almost certainly occurs. Increase of the degree number and extrapolability are thus contradictory requirements.

Further computing difficulties are due to the non-linearity of the equation system defining constants R_H and a_i. Conveniently choosing R_H, this problem may, however, be linearized, and making the minimum sum of square deviations in the measuring range a requirement, the R_H value may be determined by iteration. Appropriately choosing $k = 1$, monotony can be provided for, then, however, there is a rather poor approximation, and thus, practically, no extrapolation is possible with this function type.

To now, best results have been achieved with functions type (11) and (12). Although in defining the constant k, R_{H} and b make the problem a non-linear one, this function has the advantages quoted in item 2. R_H lends a limit value to the function, b accounts for deceleration and c for delay — at a little error.

Function types (11) and (13), in fact hyperbolic due to $a < 0$, have similar properties. Function log t in the argument of the exponential function is also here for deceleration.

To illustrate suitability of functions type (7), (8) and (11), Table 1 presents recorded and calculated percentages in a relaxation test of 35,000 h. [12]. The wire ∅ 7 mm was tested as delivered, at 20 ± 1 °C and at an initial stress $\sigma_0 \approx 0.65\ \sigma_B$.

Equation system defining unknown constants of functions type (11) and (13) may be linearized by appropriately choosing R_H and b, requiring the specific minimum of the sum of squares of deviation interpreted by the relationship:

$$\Phi_{\text{spec}} = \frac{1}{n} \sum_{i=1}^{n} (R_m - R_c)^2$$

where n is the number of records, R_m and R_c are the recorded and calculated relaxations, resp. Choosing tabulated values for R_H and b, the value of function $\Phi_{\text{spec}} = \Phi_{\text{spec}}(R_H, b, a, c)$ is near the optimum (minimum), of course, however, the accuracy could still be improved by iteration. The fair agreement between recorded and calculated values proves this function type to yield a good phenomenological description of relaxation.

Table 1

Functions fitted to a data set of 35,000 hours [12]

Time h	Relaxation R %				
	Recorded R_m	Calculated R_c			
		(11)—(12) $R_H = 12$ $b = 0.314$	(11)—(13) $R_H = 100$	(7) $k = 3$	(8) $k = 2$
10	2.53	2.53	1.92	2.54	2.50
100	3.85	3.85	4.31	3.81	3.96
1 000	6.03	6.03	6.63	6.09	6.01
8 760	8.67	8.68	8.78	8.67	8.54
10,000	8.81	8.85	8.90	8.83	8.71
15,000	9.39	9.34	9.30	9.32	9.26
17,520	9.48	9.52	9.45	9.50	9.47
20,000	9.71	9.67	9.58	9.66	9.66
25,000	9.89	9.92	9.79	9.92	9.98
26.280	10.00	9.97	9.84	0.98	10.05
35,050	10.25	10.27	10.12	10.31	10.47
100,000	—	11.16	11.12	11.46	12.09
300,000	—	11.71	12.16	12.52	13.91
1,000,000	—	11.94	13.28	13.42	16.03
Φ_{spec}		0.000 864	0.094 808	0.001 792	0.010 798

In the second column of calculated values, approximation results by the functions type (11) and (13) have been compiled. Though less than the former one, this function type can also be stated to suit extrapolation. For increased R_H values, Φ_{spec} decreased gradually. The table shows calculated values belonging to an — if not optimum but physically still meaningful — $R_H = 100$ value.

Next two columns show approximations by functions (7)-lin-log and (8)-log-log. In order to improve the approximation, the degree number k of polynomials has been raised as long as monotony could still be maintained — in the range often referred to. In the records range, the calculated and Φ_{spec} values show a close approximation but yield rather deviating extrapolated results, in spite of the long test period. Choosing $k = 1$ for function (8) according to the FIP recommendation [6], 1,000,000 h (about 114 years) would exhibit a relaxation of 19.17%, a rather overestimated value. It is interesting to note that the Skandinavian formula for loss prediction [15] yields a final relaxation $R_H = 12.5\%$, near to the extrapolated values by functions (11)—(12), (11)—(13) and (7), resp.

Table 2

Functions fitted to a data set of 70,000 h [13]

Time h	Relaxation R kgf/mm²			
	Recorded R_m	Calculated R_c		
		(11)–(12) $R_H = 30.5$ $b = 0.272$	(7) $k = 4$	(8) $k = 4$
120	13.61	13.66	13.62	13.62
1 000	16.00	15.89	15.96	15.97
8 760	19.00	19.24	19.25	19.25
17,520	20.85	20.52	20.50	20.50
26,280	21.30	21.31	21.28	21.28
35,040	21.82	21.88	21.85	21.85
43,800	22.18	22.33	22.31	22.31
52,560	22.58	22.70	22.69	22.69
61,320	23.05	23.01	23.02	23.02
70,080	23.40	23.28	23.31	23.31
78,840	23.57	23.51	23.57	23.57
83,593	23.68	23.63	23.70	23.70
87,600	23.77	23.73	23.80	23.81
100,000	—	23.99	24.10	24.10
300,000	—	26.11	26.72	26.72
1,000,000	—	28.07	29.92	29.89
Φ_{spec}		0.023 49	0.022 61	0.022 60

Table 2 offers a further possibility of comparison. DUMAS pre-stretched a wire of nominal 150 kgf/mm² tensile strength by applying roughly the same stress of 150 kgf/mm², unloaded it, then measured relaxation under the same initial prestress during 10 years at 20 °C (wire No. 10, p. 14 in [13]).

Our calculations for all three types were made by fitting functions to the measured points only to 70,080 h, and the other values have been extrapolated. It is interesting to see functions type (7) and (8) to deliver almost the same result, at an approximation somewhat better than for (11) and (12), although the accuracy of these latter functions could still be improved by iteration, as already mentioned. The startling accuracy of results of functions type (7) and (8) may be attributed to the rather prolonged measurement, obviously, the log. scale of the abscissa axis "densifies" long-time values.

This table supports our statement that "lin-log" and "log-log" functions (7) and (8) proved for prolonged measurements, while being normally useless for extrapolation from short-term measurements.

3.2. *Other applicable functions*

There are still other functions likely to truly describe the quoted rheological properties of the prestressing steel, and suitable to extrapolation.

Statements on function type (11) in item 3.1 concluded it to well fit records — especially using function $f(t)$ type (12) — and to deliver reliable assessment values. Another possible choice of function $f(t)$ takes time-dependent increase of viscosity into consideration:

$$f(t) = \frac{t}{\lg (t + m)} \tag{14}$$

where m is an aptly chosen constant.

The relaxation curve can be approximated by a hyperbola such as:

$$R = R_H \left(1 - \frac{a}{(t + b)^c}\right) . \tag{15}$$

For $a > 0$ and $c > 0$, the function tends to a limit value of R_H for $t \to \infty$. This function may be considered in fact an improved variety of (11) to (13).

Still, actual computations are needed to decide utility of functions type (11) to (14) and (15). Computing "difficulties" are also present here, without causing serious trouble.

Summary

Fitness of approximation and extrapolation functions of the form usual in publications on the prediction of prestressing steel relaxation (linear or simple polynomial in lin-log or log-log scale to express relaxation or relaxation rate) has been compared to that of a function type $(1-e^{-x})$ based on a rheological model with limit values. Unavoidable modifications of this exponential function, justified from rheological and metrological aspects, have been determined. Practical use of this kind of function has been verified by fitting to measurement data sets of several thousand hours.

References

1. Erdélyi, A.: Prestressing Wire Relaxation Values Expected at 20 to 80 °C. Per. Pol. Civ. Eng. Vol. **17.** 1973, No. 3—4.
2. Papsdorf, W.—Schwier, F.: Kriechen und Spannungsverlust bei Stahldraht insbesondere bei leicht erhöhten Temperaturen. Stahl u. Eisen. H. **14.** 1958. 937—947.
3. Palotás, L.: Inherent Concrete Stresses. Per. Pol. Civ. Eng. Vol. **17.** (1973.) No. 3—4.
4. Erdélyi, A.: Extrapolability of the Relaxation of Prestressing Wires as Related to Technology Parameters.* Mélyép. Szle. January 1971. 24—31.

5. ERDÉLYI, A.: Extrapolability of the Rheological Behaviour of Prestressing Steel.* Dr. Techn. Thesis, Budapest, 1972.
6. CEB (Comité Européen du Béton)-FIP (Fédération Internationale de la Précontrainte): International Recommendations for the Design and Construction of Concrete Structures. June 1970: FIP Sixth Congress, Prague; Cem. & Concr. Ass. London, 1970.
7. CUR Report 46 (Netherlands Committee for Concrete Research): A Mathematical Analysis of Results of Relaxation Tests on Drawn Heat-Treated Prestressing Wires. Beton Vereniging, Zoetermeer, 1971.
8. KUBIK, F.: Delta 100. ein österreichischer Spannbetonstahl. FIP Report, Prague, January 1970. 47–50.
9. HAVE, R.—HALLEUX, P.: Influence de la température sur le fluage et la relaxation des fils de précontrainte. «Béton Précontraint 69» February No. 266. 1970.
10. BRANCH, G. D.: The Relaxation Properties of Prestressing Wires in the Temperature Range 20–100 °C. Estimation of Long-Term Properties. GKN Group Research Lab. Development Report No. 4. 1964.
11. GILLEMOT, L.: Material Structure and Materials Testing.* Tankönyvkiadó, Budapest, 1967.
12. ENGBERG, E.—WALLIN, L.: Long Time Creep Relaxation Tests on High-Tensile Steel Prestressing Wires; Nordisk Betong, 1966/23. 231–236.
13. Symposium International sur les Aciers de Précontrainte. FIP Madrid, juin. 1968. (Contribution française, par M. F. DUMAS) p. 11. Thème 3.
14. REINER, M.: Deformation, Strain and Flow. H. K. Lewis Co. Ltd. London 1960.
15. Rekommendationar rörande dimensionering och utforande av spännbetong, Nordisk Betong, 1963/1.

Gyula CZEGLÉDI, Research Worker
Senior Ass. Dr. Attila ERDÉLYI
1111 Budapest, Műegyetem rkp. 3
Hungary

* In Hungarian.