INHERENT CONCRETE STRESSES

by

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1. General: fundamental relationships; notations

Let us first recapitulate concrete stresses due to temperature variations, shrinkage and creep.

Normal concrete can be considered as a two-phase solid composed of elasto-viscous cement stone and elastic aggregates. Since in inhomogeneous, anisotropic solids, positive interface connections prevent deformations due to physical influences between phases with different properties (temperature variations, shrinkage, mechanical effects) from freely developing, the mutually inhibited deformations give rise to internal forces within the material phases, so that the multiphase material gets into an *inherent stress state*. For instance, thermal or humidity variations or permanent load cause creep and concomitant mechanical deformations. Let me make some simplified mechanical assumptions in answering informatively the questions, of what degree the deformation inhibitions might be; what internal forces may arise due to inner mechanical constraints; might unfavourable ultimate conditions develop upon purely physical effects, without outer mechanical stresses, or not, based on the knowledge of resultant deformations and on inherent free phase deformations?

Some mechanical simplifying assumptions — computation base models — for estimating inherent stresses have been presented in [1] through [4]. Now, the simplest of them, a simple, mutually intercrossing plane disc model will be started from, assuming that in the concrete the hardened cement (cement stone) and the particles consist of perpendicular, parallel layers (linear disc model).

As concerns the volume distribution of hardened cement and aggregate in the concrete — also by elements — the absolute volume of hardened cement paste (cement, water and voids) and aggregates v_s and v_a, resp., contained in unit volume ($v_c = 1$) is considered to be characteristic. Hence:

$$v_c = v_s + v_a = 1. \tag{1}$$

(For sake of simplicity, Poisson's ratio of cement and aggregate is considered to be zero.)

Again, cement and aggregate phases in the concrete are assumed to adhere slip-free — strongly — and to meet the compatibility principle. Thus, at any point of phase interfaces, the forced deformation of hardened cement and aggregate particles in a given direction ($\Delta\ \varepsilon_s$, $\Delta\ \varepsilon_a$) equal the inhibited deformation in that direction.

For a uniform treatment, the inhibited concrete deformation ε_c will be assumed to follow the relationship between uninhibited deformations of cement and aggregate ε_s and ε_a of the form:

$$\varepsilon_c = c\ \varepsilon_s + (1 - c)\ \varepsilon_a \tag{2}$$

Here the c value depends on the specific absolute volume of the cement in the concrete:

$$c = v_s^{3/2} \tag{3}$$

in a fair agreement with test results [4]. Stresses will be expressed by a relationship of similar form:

$$\sigma_c = c\ \ \sigma_s + (1 - c)\ \sigma_a \tag{4}$$

σ_s, σ_a and σ_c being stresses in cement, aggregate and concrete.

Strains of concrete and its phases being identical,

$$\sigma_c = \varepsilon\ E_c;\ \sigma_s = \varepsilon\ E_s;\ \sigma_a = \varepsilon\ E_a \tag{5}$$

leading to

$$E_c = c\ E_s + (1 - c)\ E_a = E_s\,[c + (1 - c)\ n] = v_i\ E_s \tag{6}$$

and

$$E_s = E_c/v_i$$

where

$$n = \frac{E_a}{E_s}\ \text{ i. e. }\quad E_a = n\,E_s \tag{7}$$

and

$$v_i = c + (1 - c)\ n \tag{8}$$

E_c, E_s and E_a being Young's moduli of concrete, cement and aggregate.

The n value is determined from the deformational and statical equilibrium conditions. It is advisable to start from experimentally probabilizable thermal deformations.

Young's modulus for concrete at 28 days can be written as a function of the coefficient

$$\varrho = f_c/(200 + f_c) \tag{9}$$

characterizing the concrete cube strength f_c.

$$E_c = \nu\ E_0 \tag{10}$$

For compression and tension $E_0 = 550.000\ \varrho$ (11)

$$\nu = \nu_0/(1 + \varphi) \tag{12}$$

$$\nu_0 = \frac{1}{2}\left[1 + \left(1 - \frac{\sigma}{(\sigma)}\right)^{1/2}\right] \tag{13}$$

σ being the concrete stress, (σ) the compressive strength σ_p under compression, and the tensile strength σ_t under tension; φ means the value of the creep.

For the concrete compressive and tensile strength at 28 days, again as a function of the ϱ coefficient — provided f_c ranges from 150 to 500 — the relationships

$$\sigma_p = 700\ \varrho^2 \tag{14}$$

$$\sigma_t = 40\ \varrho^{3/2} \tag{15}$$

and for its creep

$$\varphi = k_0\, k_r \delta\, \varphi_n \tag{16}$$

can be suggested. k_0 in (16) is the so-called ageing coefficient expressing the time of permanent loading τ. The substituting function of the ageing coefficient referred to 20 °C curing temperature can be expressed for slow and rapid hardening cement, resp., by the formulae

$$k_0 = 0.25 + 2.7\ e^{-0,55 \sqrt[4]{\tau}} \tag{17}$$

$$k_0 = 0.15 + 3.3\ e^{-0,75 \sqrt[4]{\tau}}\ . \tag{18}$$

The ageing coefficient of a concrete cured at other than 20 °C has to be determined with respect to the curing temperature R from the formula recommended by DIN 1045—72:

$$R = \Sigma\, \tau\, (T + 10\ ^\circ\mathrm{C}) \tag{19}$$

where T is the daily mean temperature in °C; $\Sigma\, \tau$ the number of days of mean temperature T;

k_r is the coefficient of ambient humidity, with the suggested expression:

$$k_r = \frac{115 - n_r}{100 - 0.7\, n_r} \tag{20}$$

n_r being the percentage of relative atmospheric humidity; δ is the process function of creep, depending on loading time t, on concrete composition, and on the drying thickness of the loaded unit.

The dry thickness is expressed by

$$d_i = 2\ \mathrm{V}/A \tag{21}$$

where V is the drying volume and A the drying surface.
Relationship, analysed in detail in [4] and [5],

$$\delta = 1 - e^{-0.1\, t^{1/2}} \tag{22}$$

yields an average, with t in days.

φ_n is the final creep value of concrete under mixed storage conditions ($n_r = 40-45\%$) loaded at 28 days; in lack of test data or recommendations, it can be expressed as a function of concrete strength as

$$\varphi_n = \varrho^{-5/4} \tag{23}$$

φ_n values for 28-day concrete strength values of 140, 200, 280, 400, 560 kp/cm^2 being 3.03, 2.38, 1.96, 1.66, 1.47, respectively.

2. Basic relationships to yield inherent stresses

Based on a linear disc model, inherent stresses of concrete components meeting deformation and statical conditions can be written as follows:

in the cement stone:

$$\sigma_s = \Delta\, \varepsilon_s\, E_s = (\varepsilon_c - \varepsilon_s)\, E_s \tag{24}$$

in the aggregate:

$$\sigma_a = \Delta\, \varepsilon_a\, E_a = (\varepsilon_c - \varepsilon_a)\, E_a \tag{25}$$

where ε_c is the resultant deformation of the concrete composed of deformations of cement and aggregate ε_s and ε_a, respectively; E_s and E_a are delivered by Eqs (6) and (7), resp.

2.1 *In case of temperature variation* ($\Delta\, T$) the effect of creep is negligible, hence $\varphi = 0$, i.e.:

$$\varepsilon_c - \varepsilon_s = (\alpha_c - \alpha_s)\, \Delta\, T \tag{26}$$

$$\varepsilon_c - \varepsilon_a = (\alpha_c - \alpha_a)\, \Delta\, T$$

$$E_s = \nu_0\, E_{s0} \tag{27}$$

where α_c, α_s, α_a are thermal expansion coefficients of concrete, cement and aggregate, respectively. According to [1] and [6], thermal expansion coefficient of the concrete may be assumed as

$$\alpha_c = c\, \alpha_s + (1 - c)\, \alpha_a \tag{28}$$

c being delivered by Eq. (3).

In Fig. 1, variation of the absolute volume v_c of cement stone vs. w/c ratio x and cement dosage C, as well as of the ratio of deformation moduli of aggregate to cement, i.e. of the $n = E_a/E_s$ value, deduced from deformation and statical conditions have been plotted, indicating the respective formulae. γ_s values of 3.15 and 3.05 are densities of Portland cements grade 600 and 500, resp.

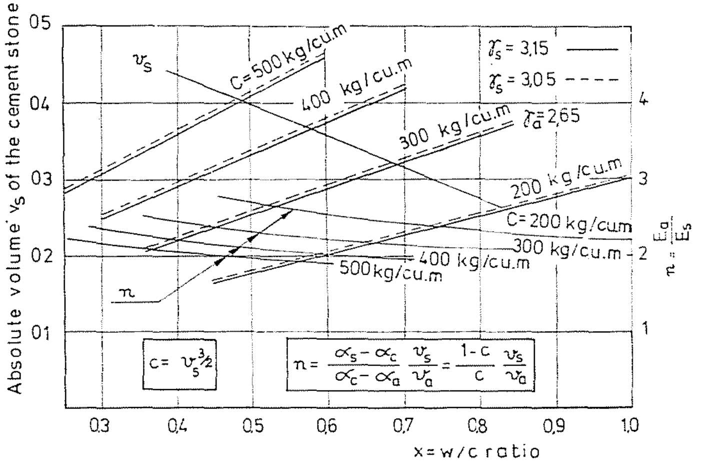

Fig. 1

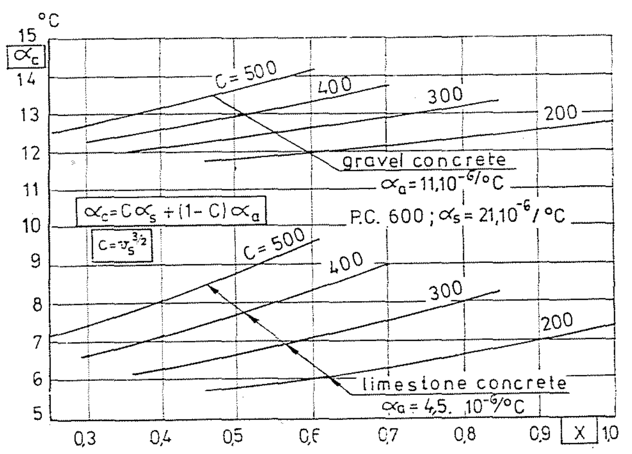

Fig. 2

α_c values for different concrete types have been plotted in Fig. 2 vs. w/c ratio x and cement dosage C. Diagram values have been calculated with α_s values of $21 \cdot 10^{-6}$/°C.

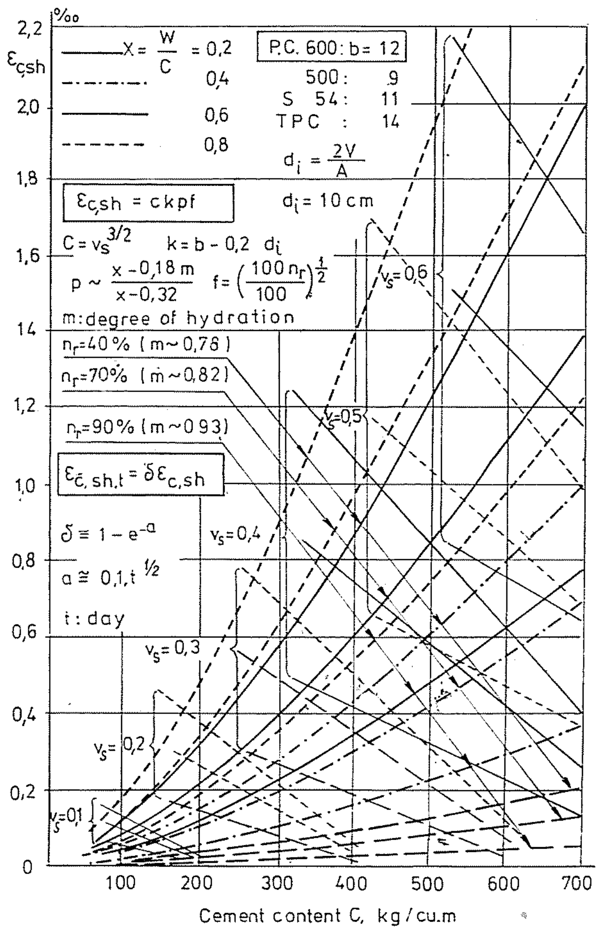

Fig. 3

2.2 *In case of shrinkage* ε_{sh}, $\varphi \neq 0$, and since in general, the shrinkage of the aggregate is negligible,

$$\varepsilon_c - \varepsilon_s = \varepsilon_{c,sh,t}\frac{1-c}{c}$$

$$\varepsilon_c - \varepsilon_a = \varepsilon_{c,sh,t} \tag{29}$$

where $\varepsilon_{c,sh,t}$ is concrete shrinkage during time t.

Concrete shrinkage $\varepsilon_{s,sh,t}$ can be obtained by multiplying the final concrete shrinkage value $\varepsilon_{c,sh}$ by the shrinkage process function:

$$\varepsilon_{c,sh,t} = \delta\varepsilon_{c,sh} \tag{30}$$

being governed by the creep process function.

Final concrete shrinkages $\varepsilon_{c,sh}$ of informative value, for different Portland cement dosages C vs. relative humidity n_r and w/c ratio x have been compiled in Fig. 3 for a dry thickness $d_i = 10$ cm. If final shrinkages of cement and aggregate $\varepsilon_{s,sh}$ and $\varepsilon_{a,sh}$ resp., are known, the concrete's final shrinkage may be assessed from the relationship

$$\varepsilon_{c,sh} = c\,\varepsilon_{s,sh} + (1 - c)\,\varepsilon_{a,sh}\,. \tag{31}$$

Shrinkage of the most common heavy aggregates (quartz, basalt, limestone etc.) is generally negligible, thus, Eq. (31) simplifies into:

$$\varepsilon_{c,sh} = c\,\varepsilon_{s,sh}\,. \tag{32}$$

For the values of $\varepsilon_{s,sh}$ let us refer to [4]. As an information, the final shrinkage of high-strength hardened Portland cement made up with a w/c ratio of 0.4 to 0.6 and of 10 cm dry thickness can be estimated at 1.0 to 1.5, 1.9 to 2.7 and 2.8 to 3.9 per mille for relative humidities of 90, 70 and 40 per cent, respectively.

Shrinkage causes tension in cement stone and compression in the aggregate. In view of Eq. (29), inherent shrinkage stresses in cement and aggregate σ_s, σ_a are:

$$\sigma_s = \varepsilon_{c,sh,t}\,\frac{1 - c}{c}\,E_{st} \tag{33}$$

$$\sigma_a = -\,\varepsilon_{c,sh,t}\,E_{at}\,.$$

In determining the E_{at} and E_{ct} values, remind, however, of the moderating effect of creep on shrinkage stresses. Thus, E_c in the numerator of Eq. (7) has to be replaced by a time-dependent creep modulus E_t:

$$E_{st} = \frac{E_t}{c + (1 - c)\,n} = \nu \cdot E_{s0} \quad \text{and} \quad E_{at} = n\,E_{st} \qquad (34) \quad (35)$$

with

$$E_t = \nu \cdot E_0 = \frac{\nu_0}{1 + \varphi} \cdot E_0 \tag{36}$$

n being delivered by the formula in Fig. 1:

$$n = \frac{1 - c}{c}\,\frac{v_s}{v_a}\,. \tag{37}$$

2.3 *In estimating the redistribution of creep stresses*, for sake of simplicity, rock type differences will be ignored and their creep omitted. It should be emphasized, however, that recent test results clearly point out the effect of the rock type on the development of shrinkage and creep, as concerns both order of magnitude and influence.

Permanent loads cause initial stresses in the elements of concrete cross-section: cement stone and aggregate, already at the instant of loading. After a while, stress values alter alongside with the increase of deformations, namely initial stresses in elements higher in creep, lower in elasticity (cement stone) decrease and in elements of higher elasticity (aggregate skeleton) increase, while in the cross-section the stress variations add up to zero, i.e. there is evolving an *equilibrium stress condition.*

At time $t = 0$ i.e. at loading, compression P causes stresses σ_{s0} and σ_{a0} in cement and aggregate, respectively. Initial compressions P_{s0} and P_{a0} in cement stone and aggregate — under conditions outlined in item 1 -- as well as initial stresses are delivered by

$$P_{s0} = c\,P \qquad P_{a0} = (1 - c)\,P \tag{38}$$

and

$$\sigma_{s0} = \frac{c\,P}{A_s} \qquad \sigma_{a0} = \frac{(1-c)\,P}{A_a} \tag{39}$$

respectively, where

$$A_s = v_s\,A_c \quad A_a = v_a\,A_c \tag{40}$$

A_c being the cross-sectional area of the compressed concrete bar.

Upon creep, initial strains ε_0 increase as a function of time, the strain increment, the creep being

$$\varepsilon_l = \varepsilon_0\,\varphi\,. \tag{41}$$

Because of creep, initial compressive stresses σ_{s0} in the cement stone undergo first slight (probably due to self-compaction), then more intense, and subsequently again slow variation that can be attributed to tensile stresses $\sigma_{s\varphi}$, while aggregate compressive stresses σ_{a0} increase by a compressive stress increment $\sigma_{a\varphi}$. During this stress redistribution, the cross-section is in an equilibrium stress state. Creep ε_l is a reduced value of the cement creep ε_{lc} because of the mechanical resistance of the aggregate.

At time t, because of the inherent stress condition:

$$P_\varphi = \sigma_{a\varphi}\,A_a = -\,\sigma_{s\varphi}\,A_s \tag{42}$$

i.e., knowing $\sigma_{a\varphi}$ and introducing $\mu_a = \dfrac{v_a}{v_s}$:

$$\sigma_{s\varphi} = -\,\sigma_{a\varphi}\,\mu_a \tag{43}$$

where $\sigma_{a\varphi}$ = compressive stress (a negative value) and $\sigma_{s\varphi}$ = tensile stress.

For estimating the compressive stress increment in the aggregate, for the sake of simplicity, introduction of a *reducing factor* φ_c based on the *Dischin-*

ger principle seems to be advisable. Taking Eqs (38) into consideration and after due transformations,

$$\varphi_c = \frac{c}{1-c}\left[1 - e^{-(1-c)\varphi}\right]. \tag{44}$$

The free deformation of cement stone, as basic characteristic of cement creep is obtained, according to Eq. (2), from:

$$\varepsilon_s = \frac{\varepsilon_c}{c} = \frac{\varepsilon_0}{c}. \tag{45}$$

The reduced concrete creep, typical for developing stresses:

$$\varepsilon_{c,c} = \varepsilon_s \frac{\varphi_c}{\varphi_n}. \tag{46}$$

Thus, aggregate and cement stresses due to creep are:

$$\sigma_{a\varphi} = \varepsilon_{c.c} \cdot E_{at}$$

$$\sigma_{s\varphi} = -\sigma_{a\varphi}\mu_a = -\varepsilon_{c,c}\mu_a n_a E_{st} = -\varepsilon_{cc}\frac{1-c}{c}E_{st}. \tag{47}$$

Summary

Concrete stresses due to thermal variations, shrinkage and creep (inherent stresses) have been investigated to establish approximate results and general guidelines. A simple, so-called linear disc model is started from, assuming the cement stone and aggregate elements of the two-phase concrete to be perpendicular and parallel layered, and the Poisson's ratio to be negligible. Again, cement stone and aggregate phases are assumed to adhere strongly, without slipping, i.e., at any interface point, the forced deformations of cement and aggregate elements in a given direction equal the restrained deformation in that direction. Effective cement stone and aggregate deformation moduli typical for the concrete composition are determined from the known concrete deformation modulus, taking statical and deformation conditions valid to the inherent stress state into consideration.

References

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