LABORATORY DETERMINATION OF THE AVERAGE REFRACTIVE INDEX USING DISPERSION METHOD

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(Received April 30, 1971)

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1. Basic principle of the dispersion method

The determination of the exact atmospheric correction by electro-optical distance measurements is, in the case of long distances, a difficult problem. The dispersion method — likely to soon become the only method for determining the atmospheric correction — offers an ingenuous possibility to determine the refractive index. In the range of visible light, the refractive index of air is known not to depend on its thermodynamic characteristics (temperature, pressure) alone but also on the wavelength of light propagating in the air. Accordingly, if a distance is measured by making use of quite different optical carrier waves (e.g. blue and red) within the range of visible light, the difference of measurement results (optical path difference) permits to calculate the average value of the group refractivity for any of the two carrier wavelengths.

The familiar fundamental formula of the method will be written with some modification, namely, from the viewpoint of energy propagation it is more appropriate to determine the average refractivity for the light component of the greater wavelength:

$$n_2 = \frac{\varkappa D_2}{\varkappa D_2 - (D_1 - D_2)} \tag{1}$$

where n_2 is the average value of the group refractivity for the light component of wavelength λ_2; D_1 and D_2 are distances delivered by light components of wavelengths λ_1 and λ_2, respectively; $\varkappa = \dfrac{n_{01} - n_{02}}{n_{02} - 1}$; n_{01} and n_{02} are group refractivity values referred to standard air for wavelengths λ_1 and λ_2, respectively.

It should be noted that Eq. (1) does not involve the effect of air moisture, but the average value of the water vapour pressure may generally be identified with the value at one of the end points; this is mostly sufficient for determining the average refractive index with a rms error ± 1 ppm.

2. Accuracy possible by the dispersion method

The accuracy of the average group refractivity calculated from (1) depends on the following factors.

1. Accuracy of the dispersion formula, determining the accuracy of calculated $\varkappa$. The case where the systematic error of the formula depends on the wavelength is a particularly difficult one.

2. Accuracy of knowledge of the two carrier wavelengths selected: inaccurate values of λ_1 and λ_2 result in a contradiction between the measured optical path length and the calculated $\varkappa$ value.

3. Accuracy of determination of the optical path difference, i.e., stability of measuring frequencies, sensitivity of the phasemeter and the range of deviations of the refractive index in turbulent atmosphere.

The accuracy of dispersion formulae used for defining the refractivity of standard air cannot be analyzed here; the first partial research result by the author has been published previously [1]; research along those lines is under way.

The inaccurate knowledge of the wavelengths of the two light components used to measure the difference between optical path lengths implies that the dispersion of the real atmosphere is determined for actual but unknown wavelengths and at the same time, the dispersion of standard air is calculated for nominal (inaccurate) wavelengths. Let us examine the resulting discrepancy.

The rms error m_n of the value n calculated from the formula (1) is the function of the rms error $m_\varkappa$ resulting from the inaccurate knowledge of the wavelengths and of the rms error $m_{\varDelta D}$ arising in the measurement of the difference of the optical path lengths:

$$m_n = \frac{1}{\varkappa} \left/ \overline{(n_{02} - 1)^2 \cdot m_\varkappa^2 + \left(\frac{m_{\varDelta D}}{\varDelta D}\right)^2}\right. . \tag{2}$$

It is convenient to specify the ratio of the two components as $4 \cdot 10^{-7}$ to $9 \cdot 10^{-7}$, then $m_n \sim 1 \cdot 10^{-6}$. Accordingly

$$m_\varkappa = \frac{\varDelta n_0}{(n_{02} - 1)^2} \cdot 4 \cdot 10^{-7}. \tag{3}$$

Let us examine how the $m_\varkappa$ value depends on the wavelength incertitude values $m_{\lambda 1}$ and $m_{\lambda 2}$, then replace (3). Assuming a Cauchy-type dispersion formula, the wanted relationship will be

$$\frac{0{,}4 \cdot \varDelta n_0}{n_{02} - 1} = \left/ \overline{\left(\frac{6B}{\lambda_1^3} + \frac{20C}{\lambda_1^5}\right)^2 \cdot m_{\lambda 1}^2 + \left(\frac{6B}{\lambda_2^3} + \frac{20C}{\lambda_2^5}\right)^2 m_{\lambda 2}^2}\right. \tag{4}$$

where B and C are the known constants of the dispersion formula.

For the determination of the refractive index according to the principle of dispersion measurement, only discreet radiation light sources may be applied. From those, the radiating wavelength of lasers is exactly known from the high-pressure mercury-arc tube light combined of a few bands, the appropriate band should be separated by interference filter. In this latter case, however, from the point of view of the dispersion measurement, the effective wavelength of the interference filter will prevail; the uncertainty in the effective wavelength is the multiple of that of the laser radiation.

The uncertainty in the wavelength of the laser radiation is of the 10^{-3} Å order. According to our research results, using laser as light source provides the possibility in principle to determine the average refractivity at an accuracy of 0.001 ppm whilst mercury-arc tube as light source requires, even for an accuracy of $\pm$ 1 ppm, to know very exactly (at a rms error 1.2 to 1.5 Å) the effective wavelength of the interference filters applied.

In the special literature, similar research work by GOLUBEV [2] has been published.

In the following, let us examine the question what an uncertainty of phase measurement permits to determine the difference of optical path lengths at an accuracy sufficient to determine the average refractivity at a rms error $\pm$ 1 ppm. According to (2):

$$\Delta D = \frac{c}{4\,\pi \cdot f \cdot n}\,\Delta\varphi, \qquad (5)$$

where $c = 3 \cdot 10^5$ km/sec is the light velocity in vacuum; $f =$ modulation frequency of light; $n \sim 1 =$ approximate value of the refractive index; $\Delta\varphi =$ phase difference corresponding to the optical path length difference. Be $f = 60$ Mc/s (Zeiss EOS), then

$$m_{\Delta\varphi}^{\circ} = \frac{720° \cdot 60 \cdot 10^6 \cdot D \cdot \varkappa}{3 \cdot 10^5} \cdot 9 \cdot 10^{-7}$$

so that for a distance of 10 km, the allowable rms error of the phase measurement is:
using *violet* and *yellow* lines of the mercury-arc tube:

$$m_{\Delta\varphi}^{\circ} = \pm\ 0.072°;$$

using *blue* and *yellow* lines of the mercury-arc tube:

$$m_{\Delta\varphi}^{\circ} = \pm\ 0.054°;$$

in the given case, the two values correspond to the determination of the optical path length difference at $\pm$ 1.00 mm and $\pm$ 0.75 mm accuracy, respectively.

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Let us examine the effect of instable frequency m_f on the phase fluctuation m_φ. From the familiar formula of distance measurements we obtain by derivation:

$$m_\varphi = \frac{4\pi \cdot n \cdot D}{c} \cdot m_f \tag{6}$$

Proceeding now to the difference $(m_{\Delta\varphi} \sim \sqrt{2}m_\varphi)$ and assuming that the allowable full phase deviation may be attributed to the frequency swing, by replacing the known values, we have

$m_f = \pm\ 2.1$ Hz (violet and yellow),

$m_f = \pm\ 1.6$ Hz (blue and yellow).

The corresponding relative frequency deviations are $\pm\ 3.5 \cdot 10^{-8}$ and $\pm\ 2.7 \cdot 10^{-8}$, respectively. Since only a part of the phase deviation may result from frequency instability, for determining the refractivity index at the required rms error $\pm\ 1$ ppm, the relative variation of at least one of the measuring-frequencies has to be reduced to the order of $\pm\ 1 \cdot 10^{-8}$.

The electro-optical distance measuring instrument Zeiss-EOS needs correction for both frequency stability and accuracy of phase measuring to be suitable for determining the refractivity by dispersion method. Accordingly, a new procedure for phase measurement was wanted, likely to record during a relatively short interval as many values as possible from the population of the optical path length differences varying in every moment. Namely, during the few minutes required for distance measurement the systematic variation of the values of both the average refractivity and the modulation frequency may be considered negligible. Thus, phase deviations during measurement may be taken as of random, therefore their mean value calculated from a great number of measurements corresponds to the states of atmosphere and instrument superimposed by the effects of turbulence and frequency deviation as a "disturbing function".

3. Purpose of the laboratory experiment

Meeting the accuracy requirements of the principle of determination of refractivity by dispersion is by no means a simple problem. Perfection of the measuring instrument by dispersion needs still many tests and checkings. These latter are, however, rather cumbersome, since measurements are advisably done on a test section of at least 10 km long.

No wonder if possibility of dispersion measurements under laboratory conditions has been examined. Rather than to produce small changes in length,

to find a system developing a measurable path difference between the two modulated components of light, and a possibility to influence this difference from outside, caused problems.

To produce a measurable path difference between the two light components in the system (referred to in the followings as optical path difference simulator) the dispersion of the path difference simulator material should exceed that of the air by several orders of magnitude. The possibility to vary the path difference from outside (advisably by thermal effect) relies, however, on the thermal expansion coefficient of the basic material, not much less than that of the air. Accordingly, the basic material of the path length difference simulator had to be find among the gaseous (eventually fluid) matters of high dispersion.

To this purpose, carbon disulphide in liquid state had been selected, with a dispersion about 15 000 times that of air at room temperature, thus 1 m length of the optical path difference simulator was equivalent to an air layer 15 km thick. Although the thermal expansion coefficient is only one-tenth of that of air, due to its high dispersion value, a moderate change in temperature can change measurably the optical path difference between the two light components (i.e., blue and yellow lines of the mercury-arc tube).

Among the numerous studies on the dispersion of carbon disulphide, as well as on the relation between dispersion and temperature, excels that by FLATOFF [3] reporting on his test results. The fundamental formula of the laboratory experiment has been deduced from the reported results and the dispersion formula based on them.

The laboratory experiment has been designed taking in mind the following: the optical path difference corresponding to the phase difference of the two modulated light components, hence a measurable quantity, is proportional to the difference of group refractivities calculated for the wavelength of the two light components; the proportionality factor is the geometric length of the system. The difference of group refractivities will vary for each thermal state of the optical path difference simulator, resulting in variable optical path differences. Establishment of a relationship between the variations of group refractivity differences and temperature, and knowledge of the thermal state of the system will offer two independent ways, i.e., calculation and measurement, for determining the variation of the group refractivity.

Thus, for carrying out the laboratory experiment, refractivity variation of carbon disulphide as a function of temperature had to be known.

The value of the group refractivity can be obtained from the Ketteler—Helmholtz dispersion formula:

$$n_{gr} = \frac{1}{n_f} \cdot \left[m + \frac{m'\lambda^4}{(\lambda^2 - \lambda'^2)^2} \right] \tag{7}$$

where λ = wavelength, n_f = phase refractivity; $m, m'\ \lambda'$ are physical constants.

The values of the group refractivity have been calculated on the basis of the constants in (7) determined by Flatoff and n_f values also determined by Flatoff for three temperatures (0 °C, +20 °C, +40 °C) and eight wavelengths (0.36 μ to 0.59 μ), then the refractivity variation $\left(\dfrac{dN}{dt} = - \dfrac{\partial n_{gr}}{\partial t} \cdot 10^5 \right)$ for unit variation of temperature has been determined. It has been assumed that

$$\frac{dN}{dt} = P + \frac{Q}{\lambda^2} + \frac{R}{\lambda^4} \tag{8}$$

and that the values P, Q and R vary linearly with the temperature (e.g., $P = P_0 + P_1 \cdot (t - 20)$), leading to:

$P_0 = +\ 77.432323 \qquad Q_0 = +\ 4.688529 \qquad R_0 = +\ 0.826758$

$P_1 = -\ \ 1.730618 \qquad Q_1 = +\ 1.156158 \qquad R_1 = -\ 0.137825.$

Substituting into Eq. (8) and integrating between t_1 and t_2, the variation of the group refractivity difference of carbon disulphide for wavelengths λ_1 and λ_2 due to a temperature variation $\Delta t = t_2 - t_1$, can be calculated:

$$-10^5\ \delta\Delta n = \Delta t\ [(\sigma_1^2 - \sigma_2^2) \cdot (\overline{Q} + 2Q_1 t_m) + (\sigma_1^4 - \sigma_2^4) \cdot (\overline{R} + 2R_1 t_m)] \tag{9}$$

where $\sigma = \dfrac{1}{\lambda}$; $\overline{Q} = Q_0 - 20\,Q_1$; $\overline{R} = R_0 - 20\,R_1$; $t_m = \dfrac{1}{2}(t_1 + t_2)$.

(9) is the fundamental formula of the laboratory experiment.

Accuracy tests demonstrated that the confidence of the fundamental formula more decreased toward higher temperatures (+40 °C), than toward lower temperatures (0 °C), attributed to the very low boiling point of carbon disulphide (+46 °C). Therefore the appropriate temperature range of laboratory tests may be 0 °C to +30 °C; in this range the error of the fundamental formula will not exceed 2 to 3 per cent.

4. Test arrangement

The optical path difference simulator had to permit the modulated light to traverse the layer of carbon disulphide 1 metre thick in both directions (forth and back), and gradually to vary and check the carbon disulphide temperature. It was essential to ensure the parallelism of the beams entering and leaving the simulator. Hence, the "forward" and "backward" travel in the simulator had to be equivalent both geometrically and optically. To this

aim, the tube containing carbon disulphide had been closed with a triple prism at one end, and with a precise plano-parallel plate at the other.

For the laboratory experiment, the light source of the electro-optical distance measuring instrument EOS, an incandescent lamp, has been replaced by a mercury-arc tube.
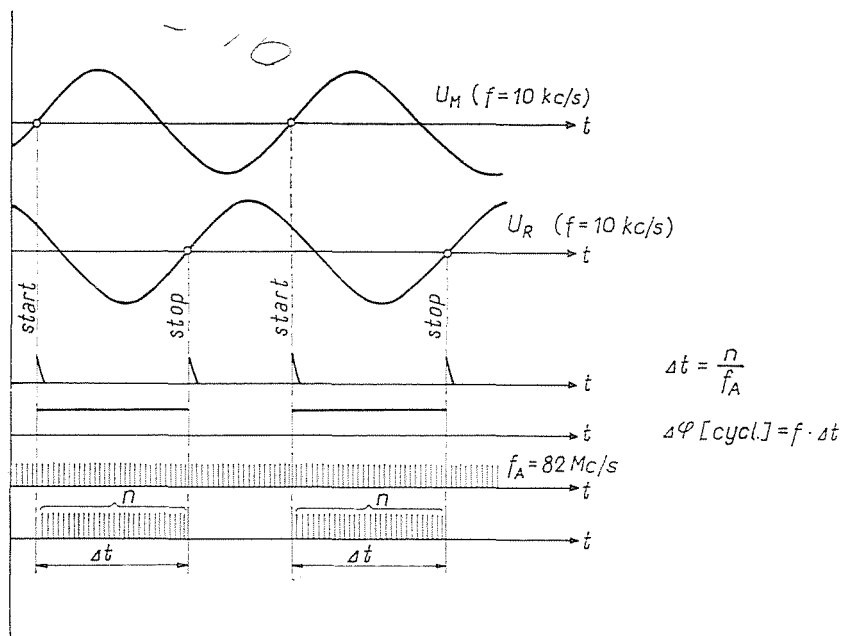

$$\Delta t = \frac{n}{f_A}$$

$$\Delta \varphi \, [cycl.] = f \cdot \Delta t$$

Fig. 1

The phase difference measurement reduced to timing had a high-precision frequency counter as basic instrument, with three modes of operation (calibration, frequency-counting, phase-measuring) to be completed by an external high-speed recorder considering that the measurement precision strongly depends on the speed of recording. The principle of phase-measuring, reduced to timing, and its block diagram are represented in Figs 1 and 2, respectively.

The frequencies f_M and f_R of the two oscillators emitting measuring and reference signals pass through the impedance transformer (1), then through the high frequency amplifier and limiter (2) to the digital frequency-meter (3). The impedance transformer minimizes the reaction of the measuring instrument, the amplifier and limiter keeps the amplitude of the signal in the specified range (0.1 to 1.0 V).

Calibration of the counter is done by means of the reference frequency 5 Mc/s, accepted by the reference frequency pick-up (4).

The digital phase-meter system works as follows: the measuring signal (U_M) and the reference signal (U_R) pass first through the narrow-band (5)

then the wide-band (6) amplifier. The timing unit of the frequency counter (3)
provides a pulse at each of the positive zero-transition points of the two sinusoi-
dal signals. The two pulses control a main gate signal equal in length to the
interval of start and stop pulses. After counting the pulses transmitted during
the gate signal, the numerical result in time units appears on the instrument
dial. (The instrument produces the timing pulses from a signal of a reference
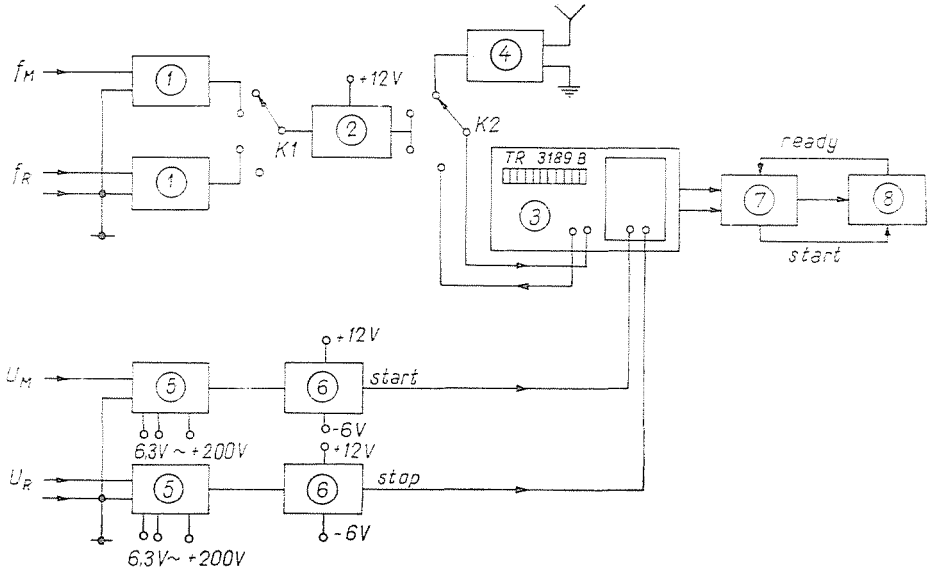oscillator.)

Fig. 2

The high operation rate of the frequency counter is of use only if the
subsequent signals are recorded at the required rate.

On the output of the universal counter — in position "out" — the codes
of output digits appear simultaneously. The parallel system of signals is
converted into a series system by a suitable transformer (7), then transmitted
to the tape puncher type PERFOMOM 30 (8). After punching the code of the
first decade digit, the tape puncher emits a "ready" signal answered by the
transformer with a "start" signal, indicating the code of the second decade
digit. After the code of the last digit is punched in, the measuring may be
continued. The "out" signal should be timed so as to permit recording of the
whole output.

Rapid measuring and recording is concomitant to processing an enormous
amount of informations, facilitated by the direct availability for data process-
ing of the punched tape. Elaboration of an appropriate program causes no
difficulties.

Summary

Dispersion of the air offers a possibility to exactly determine the atmospheric correction of electro-optical distance measurements. The basic principle of the dispersion method is described and some questions of the accuracy to be achieved are dealt with. The allowable uncertainty in the wavelength of the involved light components, the possible rate of instability of the frequency and the permissible rms error of the phase measuring are determined with the basic requirement that provided errors are superposed, the average refractivity is needed with a rms error ± 1 ppm. The laboratory experiment eliminates the familiar difficulties of field determinations. Varying the temperature of the carbon disulphide layer 1 m thick — equivalent to 15 km of atmosphere — at a known rate provides a control over the operation of the dispersion meter. The basic formula used in the laboratory experiment is a relationship between the change in temperature and in refractivity of the carbon disulphide, taking the wavelength into account. The preparation of the laboratory experiment is presented together with the principle of phase measuring reduced to timing, as well as the practical modalities.

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