LEVELLING METHODS IN THE INVESTIGATION OF VERTICAL CRUSTAL MOVEMENTS

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Reliability of the results of the investigation of vertical crustal movements by repeated precise levelling much depends on the measurement accuracy. Thus, an important factor of planning the crustal movement investigation — especially in areas featuring low-rate crustal movements — is the selection of an appropriate levelling method.

In what follows, an outline will be given — omitting known details — of some fundamentals of the selection of the levelling method, alongside with some recent research results. Possible measurement methods will be recapitulated and their utility assessed. Investigations of vertical crustal movements are assumed to be conveniently carried out in areas featured by low-speed movements, and the measurement will in any case be done by respecting general rules of precise levelling.

1. Number of measurements

In course of high-precision levelling of a country, all height differences are measured twice, as a rule, (i.e. fore and back), partly to eliminate blunders and partly to increase measurement precision. This was the case for both last high-precision levellings also in Hungary, namely for networks 1921 to 44, and 1948 to 59.

Though these levelling networks served only to meet bench mark needs of a general technical activity, their data were later applied to investigate vertical crustal movements in this country during the past 20 to 25 years. These investigations resulted in several geokinetic maps using the same basic data but partly different principles [1, 2].

These geokinetic maps, however, significantly differ by several details. Lack of coincidence may be attributed to the unreliability of the mentioned levelling networks -- otherwise convenient for their intended use — for demonstrating low-rate crustal movements typical for the territory of this country. Rather than to the layout of the network or to the applied bench marks, this can be reduced to the accuracy of levellings, insufficient for crustal movement investigations, in spite of the application of up-to-date instruments and the respect of quite rigorous specifications.

Thus, to increase accuracy, either another measuring method has to be chosen or the number of repeated measurements has to be increased. (Of course, relative reliability of the investigations may be increased by protracting time intervals between each two measurements in the network. Though, too long intervals are inadvisable because of the possibility for bench marks to perish, change of gravity, international aspects of these investigations etc.) Thus, multiple height difference determinations, as certain means to increase accuracy, may be advisable.

Of course, a too great number of repetitions is not possible, since it would involve unduly much labour, cost and time requirement for levelling the investigation network. Therefore only the possibility and accuracy-increasing effect of doubling the usual number of repetitions will be spoken of.

Doubling the repetition number is known to *theoretically* increase accuracy by $\sqrt{2}$. It was examined whether making, instead of a single, *double* independent fore and back levellings for each height difference increases the accuracy as expected.

A possibility was offered by test results published in [3]. In course of these tests levelling sections 1—2, 2—3 and 3—1 (combining into a closed circuit) were levelled throughout by two teams, 44 times in all. Averages from 44-fold measurements closely approached true values of height differences within the quoted sections. Deviation of each result from the calculated true value can thus be considered a true error.

44 results were combined in time sequence to fore and back levellings (done in opposite daytimes) and their true errors established. Mean values S of absolute values of true errors are compiled separately for each levelling section and team, in the first row of Table 1 (team AB indicating teams A and B levelling fore and back, respectively).

Average true errors divided by $\sqrt{2}$ deliver *theoretical* average true errors of *double* fore and back levellings $S/\sqrt{2}$ (second row in Table 1).

Table 1

	Average true error (mm)								
	Section 1–2			Section 2–3			Section 3–1		
Team	A	B	AB	A	B	AB	A	B	AB
S	0.16	0.34	0.22	0.17	0.19	0.22	0.29	0.39	0.49
$S/\sqrt{2}$	0.11	0.24	0.16	0.12	0.13	0.16	0.21	0.28	0.35
D	0.13	0.26	0.15	0.12	0.13	0.11	0.08	0.32	0.27

Also double fore and back levellings could be composed (in time sequence) of test results by both teams and true errors of these results established in the described manner. Obtained *empirical* average true errors D are shown in the bottom row of Table 1. Confronting the $S/\sqrt{2}$ and D values shows a fair coincidence between theoretical and empirical accuracy increases, thus, in fact, doubling the number of repetitions brings about an accuracy increase by $\sqrt{2}$. A somehow greater accuracy increase than expected appears where, instead of the results of one team, those of both teams are involved into the average. This can be attributed partly to subjective errors, and partly to the increasingly random character of microclimatic effects upon repeating the measurements.

Hence, over areas exhibiting low-rate crustal movements, double fore and back levellings of the network are advisable for the sake of a more realistic geokinetic map. In view of the rapid development of automatic levels, labour excess will probably not involve prohibitive time and cost requirements.

In conformity with double levellings, accuracy requirements may be stressed. For the first order levellings in this country, permitted difference for a *single* fore and back levelling in a section had been:

$$d \leqq 1{,}2\sqrt{t}\ \text{mm}, \tag{1}$$

permitted *a posteriori mean square error* of the lines:

$$\mu_{\text{km}} = \sqrt{\frac{1}{4\,n}\left[\frac{d^2}{t}\right]} \leqq \pm\ 0.5\ \text{mm} \tag{2}$$

with permitted *relative error:*

$$a = \frac{[d]}{[t]} \leqq \pm\ 0.4\ \text{mm}. \tag{3}$$

In case of *double* fore and back levelling, the former may be replaced by accuracy requirements:

$$d' = \frac{d}{\sqrt{2}} \leqq 0.8\sqrt{t}\ \text{mm}, \tag{4}$$

$$\mu'_{\text{km}} = \sqrt{\frac{1}{4\,n}\left[\frac{d'^2}{t}\right]} \leqq \pm\ 0.35\ \text{mm} \tag{5}$$

$$a' = \frac{[d']}{[t]} \leqq \pm\ 0.3\ \text{mm} \tag{6}$$

(where n = number of levelling sections, and t = section length in km).

It can be assumed that closing errors of levelling circuits, as well as the a posteriori mean square error of the net would decrease in a corresponding proportion.

As concerns the presented criteria of reliability, notice, however, that they are unlike to be absolute for levelling accuracy if not over great lengths since their d values (or relationships (1) and (4) themselves) are not just convenient as reliability criteria for the accuracy of each levelling section (height difference between adjacent bench marks).

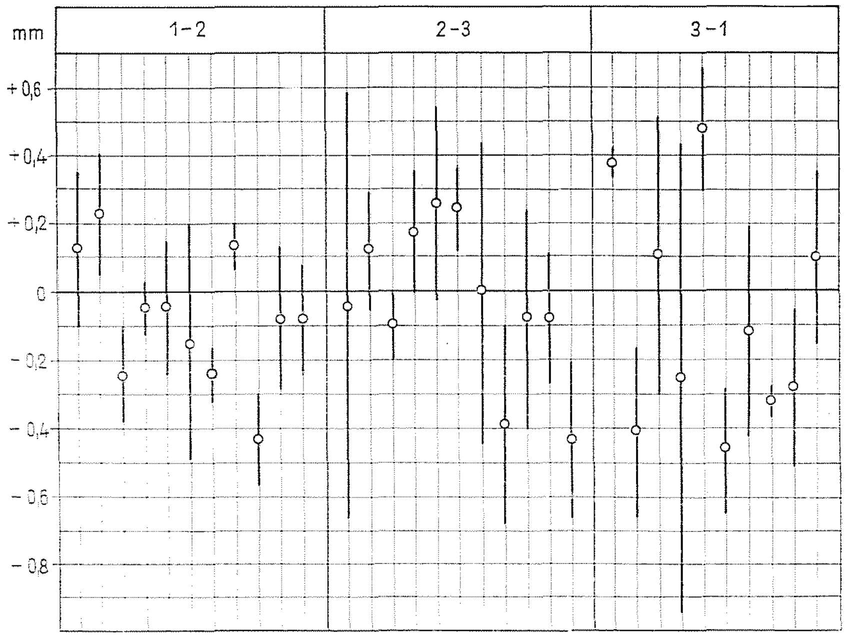

Fig. 1

This fact appears from Fig. 1 where results by team A are shown for all three test sections, as an example. Horizontal thick line indicates a mean value obtained from 44-fold test measurements on the sections, and is to be considered as the true value; each small circle represents a single fore and back levelling result; thick vertical lines joining the small circles show magnitudes of differences d. Scatter of results is seen to be in no relation to the pertaining intervals d.

2. Significance of levelling at different daytimes

An old rule of high-precision levelling is to do back levelling of any section in an opposite daytime than its fore levelling. Namely, at one and the same spot, microclimatic effects differ from morning to evening, these being the two daytimes convenient for levelling. Thus, if fore and back levelling are done in different daytimes, opposite sign errors due to opposite effects offset each other in the mean value from the two measurements. According to recent research, however, sign of *temperature gradients* during levelling, rather than the different daytimes, is of importance. For the reliability of the obtained height difference, it is favourable to have average values of temperature gradients of opposite sign or about zero for fore and back levelling. [4]

Also, since the instant of *isothermy* (when temperature gradient changes its sign) coincides with the period convenient for levelling, it follows that it is

Table 2

	Average true error (mm)								
	Section 1–2			Section 2–3			Section 3–1		
Team	A	B	AB	A	B	AB	A	B	AB
S	0.14	0.26	0.19	0.24	0.23	0.22	0.21	0.40	0.30
$S/\sqrt{2}$	0.10	0.18	0.13	0.17	0.17	0.16	0.15	0.28	0.21
D	0.14	0.22	0.14	0.19	0.11	0.13	0.08	0.39	0.25

useless to do fore and back levelling of the same section in opposite daytimes, especially if (favourably selecting beginning and ending times) it is endeavoured to have the instant of isothermy at the mid-time of levelling interval.

To verify the above, quoted test results have been compiled — keeping time orders — so that both single and double fore and back levellings include those done in *identical* daytimes. These were processed into data of Table 2 by the same method as for Table 1.

Comparison of both tables shows no important deviation to occur between them. (Average values of S being 0.27 and 0.24 mm and of D being 0.17 and 0.18 mm in Tables 1 and 2, respectively.)

Thus, results obtained in identical daytimes are equivalent to those for different ones. Of course, this is not to imply that the usual levelling practice must be altered; it is, however, by no means a source of error to do levellings in identical daytimes.

Also, the attitude towards the so-called *Cholesky* levelling method ought to be revised. A principal objection to this method — simultaneously delivering fore and back levelling results by a single proceeding over a double row of spikes — is not to allow levelling in different daytimes. (Notice that this method had been successfully applied — in a slightly modified form — for some circuits of the Hungarian network of 1948 to 59.)

3. Independence between fore and back levelling

An important rule in precise levelling instructions is to do fore and back levelling quite independently of each other. This principle is absolutely correct from several aspects, even, a proposal by CSATKAI [5] aims at an increased independence, namely to do fore and back levelling by different teams.

No doubt, such a levelling would increase reliability by making both subjective errors and those due to instrument peculiarities more of random character, besides of increasing the care in levelling, it being connected to a continuous checking for either team.

This proposal is thus worth of consideration, even if a higher percentage of measurements will need repetition because of insufficient correspondence between independent measurement results.

Another major objection to the quoted *Cholesky* method was that fore and back levellings were not independent. This really serious drawback can be eliminated by *double* fore and back levellings as discussed in item 1. Namely, each of the simultaneous fore and back levellings can be done quite independently, or even these may be done in different daytimes, by different teams.

Accordingly, the variety of the *Cholesky* method as applied for the Hungarian net may be applied for crustal movement investigations. Beside of automatic levels, this highly productive method would help to eliminate labour and time excess involved in the absolutely advisable double fore and back levellings.

4. Order of staff readings

Up-to-date precise levelling is done by means of invar staffs with double graduation, by independently reading off both staff graduations.

This is done the most simply by aiming back and reading off left, and then right graduations of the staff; thereafter aiming fore and reading off left and then right graduations. Reading order is thus: back left, back right, fore left, fore right, or abbreviated: *BBFF*.

Publications, however, disapprove the *BBFF* order, essentially because of its insufficient accuracy related to the variation with time of the atmospheric refraction. Therefore the levellings of the Hungarian net of 1948—59 have been done in the order *BFFB* considered as more favourable.

CSATKAI suggested a third variety [3], namely *BBFFFFBB*, in fact, a combination from both.

Several investigations have been done [6] to establish a reading order preferable for crustal movement determinations. The findings can be recapitulated in the followings:

a) Errors due to disturbing phenomena varying with time, provided they act *identically* both in fore and in back levelling, are eliminated by any of the reading orders from the final results of each instrument station. (Disturbing phenomena include atmospheric refraction, other microclimatic effects, displacements of instrument and staff. Because of the similar effect of its variation, also levelling rate is classified as such.)

b) If these phenomena act *differently* during fore and back levelling, then their effect can best be reduced by the *BFFB* order, less by the *BBFFFF BB* order and the least by the *BBFF* order.

c) Levelling in the *BBFF* order is advantageous by requiring the least time from the three (see Table 3).

Table 3

Order	Staff distance (m)	Time demand (min)
BFFB	35	3.8
BFFB	40	4.0
BBFF	40	3.8
BBFFFFBB	40	4.7

d) Advantage of the *BBFFFFBB* order is to raise the reliability of *reading* possible by the two other orders, to the $\sqrt{2}$-fold.

In the *Cholesky* method this reading order is not applicable, because it would inhibitively increase the time demand for levelling sections.

5. Recapitulation and evaluation of levelling alternatives for crustal movements

To our present knowledge, levelling alternatives include:

1. Single, separate fore and back levellings, in
   a) *BBFF* order
   b) *BFFB* order
   c) *BBFFFFBB* order.

2. Single, simultaneous fore and back levelling (*Cholesky* method) in
   a) *BBFF* order
   b) *BFFB* order.

3. Double, separate fore and back levellings, in
   a) *BBFF* order
   b) *BFFB* order
   c) *BBFFFFBB* order.

4. Double, simultaneous fore and back levellings (*Cholesky* method) in
   a) *BBFF* order
   b) *BFFB* order.
5. Alternatives 1, 3 and 4 may be done
   a) by team *A* for both fore and back levelling,
   b) by teams *A* and *B* for fore and back levelling, respectively.

Of these alternatives it can be stated that: During quoted investigations in Hungary, alternatives 1 and 2 proved to be unsatisfactory to achieve the required accuracy for areas characterized by low-rate vertical crustal movements, lest a time interval of half century or so is left between subsequent levellings. (This is not much helped by item *c* of alternative 1 either.)

The greatest accuracy is no doubt possible with alternative 3/*c* but satisfactory results are likely to be those of alternatives 3/*b* or 4/*b*. In any case, however, engagement of two independent teams is to be considered.

Summary

Accuracy of the measurement of height differences is a primordial problem of the vertical crustal movement investigations by repeated precise levelling. Tests in Hungary show the usual method of precise levelling to be insufficient for areas exhibiting low-rate crustal movements. It is recommended therefore to double the measurements in every levelling of the net. Also empirical data show this method to increase accuracy by $\sqrt{2}$ times.

Recent research work provided new aspects of the significance of levelling in different daytimes, of the independence of fore and back levelling, as well as of the optimum order of staff readings.

A short outline of these items permits to recapitulate and evaluate levelling methods proper to vertical crustal movement investigations.

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* In Hungarian.

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