FOUNDATIONS FOR TOWER-SHAPED CONSTRUCTIONS

by

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Tower building looks back to about the same age as architecture itself. Recently, towers, lighthouses etc. are paralleled by an increasing variety of tall, even very high tower-like structures built for various purposes. The design and construction of these is a complex problem, not only due to architectural problems related to their function but also, to a not lesser degree, to the fact that, because of their peculiar dimensions, wind loads and seismic forces, i.e. horizontal forces of intensities varying with time, become decisive components of the design load.

Theoretically, wind load may affect the towers from any direction, therefore it is advisable to design them with a circular or a regular polygon cross-section. For TV and lookout towers, objections are often made to the "concrete tube" appearance, though optimum for stability and strength, and a more articulated design is preferred. Undoubtedly, the latter are more pleasant but at the expense of their wind loads being much higher — even two or three times — than the optimum values.

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Recently, it happened that certain design considerations did not permit the use of circular or annular foundations for relatively tall, tower-like structures, the foundation slab being thus rectangular or square in form. Since such problems may often emerge, let us present here our studies on rectangular foundation slabs.

The analysis is based on the premise that the horizontal dimensions of a foundation slab are determined by the safety to tilting and the bearing capacity of soil. If, however, architectural, aesthetical, telecommunication or allowable settlement considerations require increased dimensions, then of course these latter are the significant ones. In this case adequacy of the foundation for safety and load bearing capacity requirements has to be demonstrated, rather than to determine these dimensions.

Direction of design wind load

If — for some reason — a tower-shaped structure is to have a rectangular cross-section according to Fig. 1, then wind forces W_A and W_B in the two principal directions will have different values. (Numerical determination of the wind pressure will not be considered here.) The considerations below will concern the most general case where dimensions A and B of the founda-

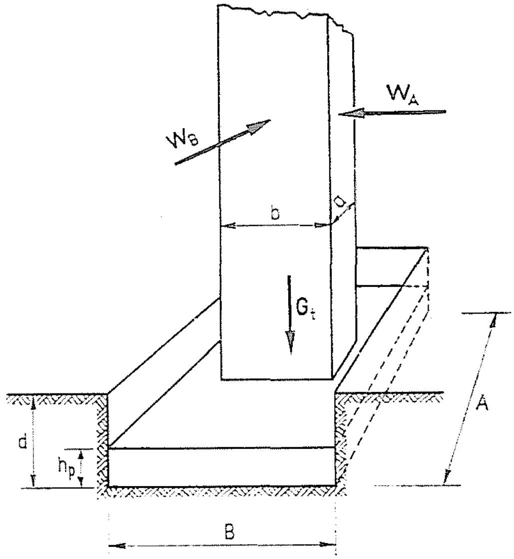


Fig. 1

tion slab are solely determined by the quoted requirements of stability and bearing capacity of soil. Finally, it is assumed that the *maximum* wind effect has no prevailing direction that could follow from local conditions and be pre-estimated.

In general, it is sufficient to assume that the wind force vectors of different directions plotted in ground plan would have end points approximately along an ellipse. Consequently, significant moments developing from them — and from incidental effects — also define an ellipse of similar position. With notations in Fig. 2, let M_A and M_B be maximum moments due to wind effects normal to sides A and B of the foundation, resp., and to the corresponding incidental effects. For practical aspects of the calculation procedure, let M_B by definition denote the higher value: $M_B < M_A$. The "moment ellipse" equation can be written as:

$$\left(\frac{M_x}{M_A}\right)^2 + \left(\frac{M_y}{M_B}\right)^2 = 1 \tag{1a}$$

and

$$M_y = M_B \cdot \left[1 - \left(\frac{M_x}{M_A}\right)^2\right]^{1/2}. \tag{1b}$$

Contact pressure maximum at corner point 1 due to affin moments M_x and M_y, to the tower part G_B, to the slab dead load $A \cdot B \cdot h_p \cdot \gamma_b$ and

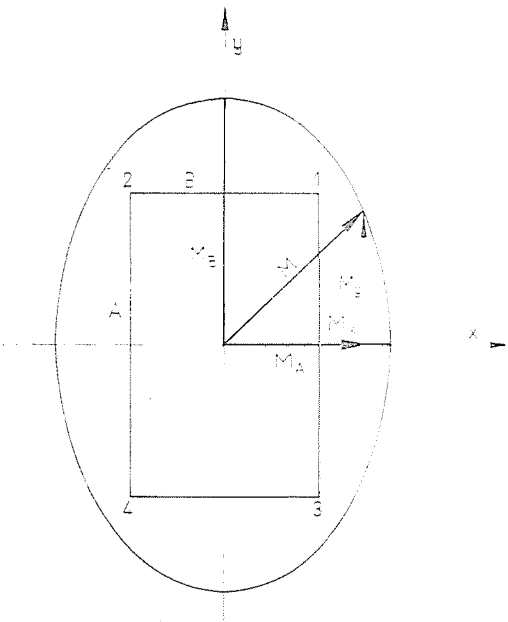


Fig. 2

to the load of the backfill $(d - h_p) \cdot \gamma_e \cdot (A \cdot B - a \cdot b)$ is (assuming a linear contact pressure distribution throughout):

$$p_1 = \frac{6\ M_x}{A.\ B^2} + \frac{6\ M_y}{B.\ A^2} + \frac{G_T + A.b.\ h_p.\ \gamma_b + (d\text{-}h_p) \cdot \gamma_e \cdot (A.\ B\text{-}a.\ b)}{A \cdot B} \tag{2}$$

γ_b and γ_e being design densities of concrete and earth, respectively.

Magnitude and ratio of the design moment couple $M_x - M_y$ are defined by the requirement to produce maximum p_1. Substituting the M_y value according to (1b) into (2), deducing the derivative dp_1/dM_x and zeroing it leads to the equation (omitting deductions):

$$M_x = M_A \cdot \sqrt{\frac{M_A^2}{M_A^2 + \left(\frac{B}{A}\right)^2 \cdot M_B^2}} \tag{3}$$

The moment component in the other direction is obtained by substituting (3) into (1b):

$$M_y = M_B \cdot \sqrt{\frac{\left(\frac{B}{A}\right)^2 \cdot M_B^2}{M_A^2 + \left(\frac{B}{A}\right)^2 \cdot M_B^2}} \tag{4}$$

The design moment being:

$$M = \sqrt{M_x^2 + M_y^2} \tag{5}$$

Direction and magnitude of the design wind force is seen to depend on both the ratio between sides of the foundation slab and that between the forces affecting the superstructure.

In knowledge of the previous results, the following requirements are to be met:

I. Contact stress on the foundation slab is compressive throughout with the resultant located at the boundary of the core, hence, at corner 4 opposite to point 1 the contact pressure is zero.

II. The peak contact pressure at point 1 does not exceed the ultimate stress of the soil.

Assume the thickness h_p of the foundation slab to be approximately known and introduce notations:

$$G = G_T - a \cdot b \cdot (d - h_p) \cdot \gamma_e \tag{6}$$

and

$$g = h_p \cdot \gamma_b + (d - h_p) \cdot \gamma_e \, . \tag{7}$$

(Effect of eventual buoyancy due to high water level may be accounted for in these equations.) Furthermore, let e_A be the eccentricity of the resultant force due to M_x. Requirement I leads to the equation:

$$\frac{e_A}{A} = \frac{1}{6} - \frac{e_B}{B} \tag{8}$$

permitting to substitute the following quantities based on previous results:

$$\frac{e_A}{A} = \frac{M_y}{A \cdot (G + g \cdot A \cdot B)} = \frac{M_B}{A} \sqrt{\frac{\frac{B^2 \cdot M_B^2}{A^2 \cdot M_A^2 + B \cdot M_B^2}}{G + g \cdot A \cdot B}} \tag{9}$$

$$\frac{e_B}{B} = \frac{M_x}{B(G + g \cdot A \cdot B)} = \frac{M_A}{B} \sqrt{\frac{\frac{A^2 \cdot M_A^2}{A^2 \cdot M_A^2 + B^2 \cdot M_B^2}}{G + g \cdot A \cdot B}} \tag{10}$$

If these conditions are met, then, according to requirement II:

$$2 \frac{G + g \cdot A \cdot B}{A \cdot B} = \sigma_H \tag{11}$$

σ_H being the ultimate soil stress. (Its calculation method is assumed to be known.) The four equations so obtained contain four unknowns: A, B, e_A and e_B. Thus, theoretically, the problem can be solved.

Without particulars of the tedious deductions, the following equation is obtained:

$$(M_B \cdot \sigma)^2 \cdot B^4 + (G \cdot M_A)^2 = \left(\frac{G^2 \cdot \sigma_H}{12 \sigma}\right)^2 \cdot B^2 \tag{12}$$

using the simplification

$$\sigma = \frac{\sigma_H - 2g}{2} \tag{13}$$

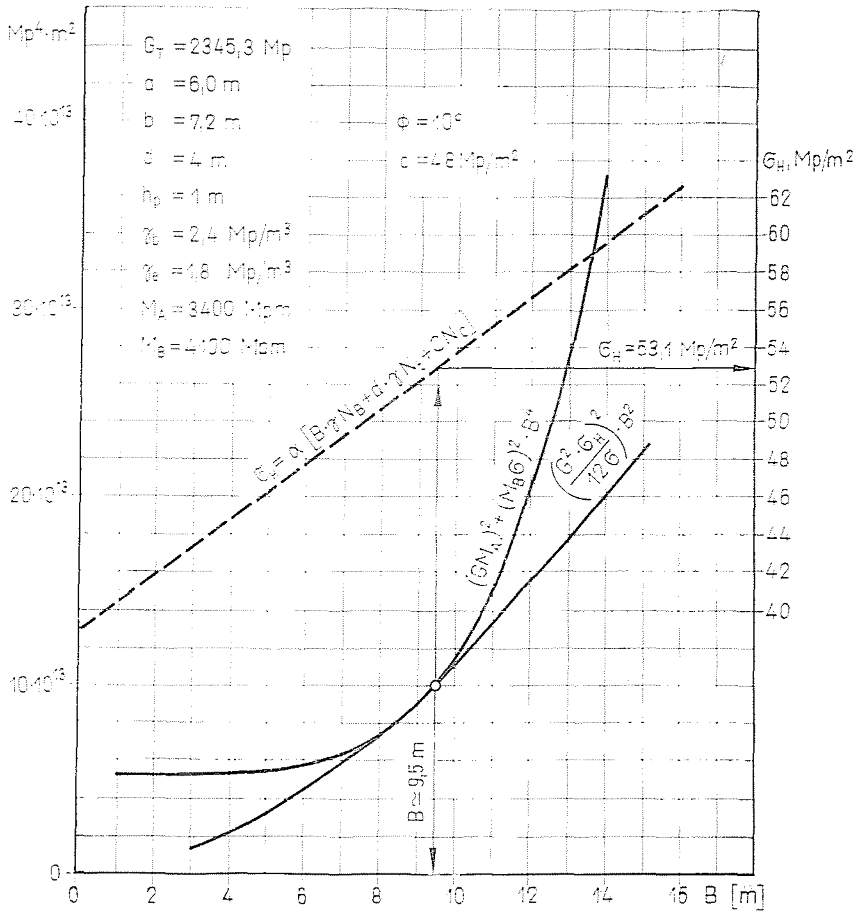
Since in general σ_H itself depends on B (except for the case when the angle of internal friction of the soil $\Phi = 0$), calculation of the root B in Eq. (12) is less simple than it seems at first glance.

According to the established practice, it is advisable to substitute various B values into both sides of Eq. (12) and to plot the variation of both sides as a function of B. The desired root B is at the intersection of the curves, as seen by the numerical example in Fig. 3.

Basic data of the numerical example are seen in the figure. The two curves have two intersections, among them the higher, $B = 9.5$ m, is significant. For the sake of comprehension, the variation of σ_H as a function of B is also plotted in the same figure. It is easy to read off that solution of the problem involves $\sigma_H = 53.1$ Mp/m^2. Then, the other side length is given by

$$A = \frac{G}{B \cdot \sigma}$$

σ being a magnitude according to (13). For the example, $A = 11.86$ m.



3 Fig.

The figure permits another conclusion: Eq. (12) has not necessarily a real solution. This would be the case in the numerical example if e.g. M_A were some higher. Thus, there may be circumstances where requirements I and II cannot be met simultaneously. Then either the soil bearing capacity is not fully used up — hence, Eq. (11) is frustrated, — or the significant resultant is inside, rather than at the boundary, of the core, frustrating condition (8).

Assume e.g. that the soil bearing capacity is not yet used up, when the resultant of significant position reaches the boundary of the inner core. Then the two equations may be written:

$$\frac{6\,M_x}{A \cdot B^2} + \frac{6\,M_y}{B \cdot A^2} = \frac{G + g \cdot A \cdot B}{A \cdot B} \tag{14a}$$

and

$$\frac{M_y}{A \cdot G + g \cdot A \cdot B} + \frac{M_x}{B \cdot G + g \cdot A \cdot B} = \frac{1}{6} \tag{14b}$$

where M_x and M_y are given by Eqs (3) and (4), respectively. Hence, Eqs (14) contain two unknowns: A and B, they can thus be solved. Expansion of particulars of this solution — in knowledge of the previous examples — is felt to be superfluous.

In the other case, where the resultant remains within the core and the corner pressure equals the allowable soil stress σ_H, the following procedure may be applied. Introduce equality $A = K \cdot B$ where $K \geq 1$ is a proportionality factor *unknown for the time being*. Using previously given expressions it can be written:

$$\frac{6\, M_x}{K \cdot B^3} + \frac{6\, M_y}{K^2 \cdot B^3} + \frac{G + g \cdot K \cdot B^2}{K \cdot B^2} = \sigma_H = f(B, K) \tag{15}$$

This equation has a single unknown, i.e. size B. Calculating it by assuming various K values, that one furnishing the minimum foundation surface $F = K \cdot B^2$ may be found. Thus, Eq. (15), theoretically with two unknowns, will be associated by a condition equation

$$\frac{dF}{dK} = 0 \tag{16}$$

This problem is rather simple to solve graphically.

Summary

Tower-shaped structures are typically exposed to wind loads. Wind may blow from any direction but value and distribution of contact pressures are only independent of its direction for structures with circular cross-section. Here the case of a rectangular building cross-section and foundation is considered. The allowable contact pressure is decisive for the foundation slab size, this latter determining both value and direction of the design wind load, a fact up to now ignored in wind force calculations. A rather easy treatment is suggested for the analysis of the required foundation dimensions.

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