HYDROLOGY

PREDICTION OF THE TRANSITION PROBABILITIES OF VARIOUS ALERT STAGES DURING RISING FLOOD

by

I. V. NAGY

Institute of Water Management and Hydraulic Engineering, Technical University, Budapest

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Introduction

Investigation into the possibilities of preparation to the anticipated critical flood conditions is of extraordinary significance for the water management and the national economy, because the mobilization of machinery and labour establishes high demands.

The methods of mathematical statistics permit to predict the probability of occurrence of the various stages and thus, to make himself prepared *one or several years in advance for the tasks of flood prevention.*

In the following the basic principles of a *new prediction procedure* based on the theory of the so-called Markov processes (kind of stochastic processes) likely to offer a solution, will be described, making use of the data sequence (of 80 to 90 years) of maximum stages of the river Tisza at Szeged, Csongrád, Szolnok, Taksony, Tokaj, Dombrád and Záhony. The schematic plan of the river Tisza and of the above gauges is shown in Fig. 1.

First, the theoretical bases of the procedure will be summarized, and then, the results of the calculations will be presented.

Theoretical consideration

A stochastic process which can be characterized either by discrete parameters (X_n, $n = 0, 1, 2, \ldots$) or a continuous variable (X_t, $t = 0, 1, \ldots$) is called Markov process if for all real values of n the condition

$$t_1 < t_2 < t_3 < \ldots < t_n$$

is satisfied and the conditional distribution of X_{t_n}, in the case of given values of

$$X_{t_1}, X_{t_2}, X_{t_3}, \ldots, X_{t_{n-1}}$$

depends only on $X_{t_{n-1}}$, that is, for every set of $x_1, \ldots, x_n$ the equality

$$P[X_{t_n} \leqslant X_n / X_{t_1} = x_1, \ldots, X_{t_{n-1}} = x_{n-1}] = P[X_{t_n} \leqslant x_n / X_{t_{n-1}} = x_{n-1}]$$

Fig. 1. General sketch

is valid. The probabilities of different possible transitions so defined are called transition probabilities.

In the case in question the stationary Markov chains, independent of n, homogeneous in time, will be used for which the one-step transition probabilities are defined by the expression

$$P[X_n = j / X_{n-1} = i] = P_{ij}$$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{1m} \\ P_{21} & P_{22} & P_{2m} \\ \vdots & \vdots & \vdots \\ P_{m1} & P_{m2} & P_{mm} \end{bmatrix}$$

called a transition matrix. Here, P_{ij} means the probability of the system being in state j at time $t + \Delta t$ if at time t it was in state i (Δt being the time interval between subsequent states). Since P_{ij} denotes one probable distribution for every fixed i value, therefore

$$0 \leq P_i \leq 1; \qquad \sum_j P_{ij} = 1; \qquad i = 1, 2, \ldots$$

Application of the theory

Let us apply the above principles for solving the following (simplified) problems of decision concerning the critical stages of the river Tisza. Let us examine the alternative cases whether *in the next year* the stage reaches a given elevation or not (or in other terms, whether a year of floods or without floods is to be expected).

In this case, the one-step transition matrix is of the form:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Denoting by State 1 the occurrence of a high water level (meaning at certain stages the danger of inundation and in other cases the necessity of putting on alert of the 1st, 2nd, 3rd degree into being) and by State 2 the occurrence of a water level lower than that on the basis of a sufficiently long time sequence of stages, the transition matrix may be given numerically.

For the sake of completeness, the meaning of the matrix elements: P_{11} is the probability that a year of flood is followed by another one; P_{12}, that a year of flood is followed by a year without flood, and so on.

The matrices of transition probabilities were computed separately for each of the seven characteristic river gauges (taking intervals of 100 cm). Computation of six matrices for e.g. Szeged gave the results shown in Fig. 2. These being one-step ($n =$ one year) transitions, the probability that the stage of 800 cm of this year will be followed in the next year by a stage of 800 cm (curve P_{11}), is $P_{11} = 14$ per cent. On the contrary, the probability that the stage of 800 cm of this year will not be repeated next year, is $P_{12} = 86$ per cent.

To determine the occurrence probability of the stages to be expected after $n = 2, 3, 4, \ldots$ years, let us express *the probabilities of possible transition* for each given value of stages after n transformations. Thus, the value of the probability of transition from State 1 to State 2 after $n = 2, 3, 4, \ldots$ years is sought for.

The system of Markov chains is solely defined by the initial distribution peculiar to the system and by the appropriate transition matrix. Transition

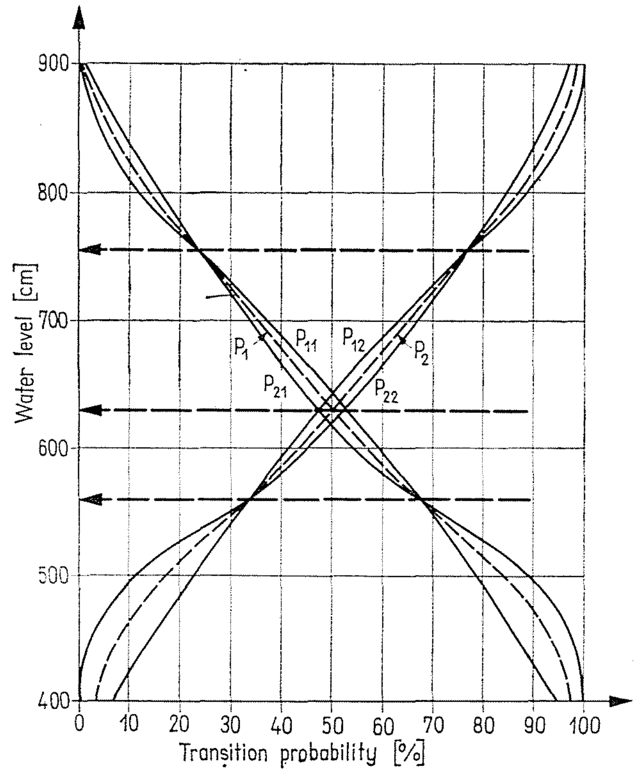

Fig. 2. Transition probabilities of Tisza water levels (Szeged, 1892 to 1970). P_{11} — flood after flood year. P_{21} — flood after non-flood year. P_{22} — non-flood after non-flood year. P_{12} — non-flood after flood year. P_2 — non-flood year. P_1 — flood year

probability for the n-th step:

$$P[X_{n+m} = j/X_m = i] = P_{ij}(n)\ ;\ n = 1, 2, 3, \ldots$$

The probabilities of an n-step transition may be calculated on the basis of the Chapman—Kolmogorov equalities (FELLER, 1968). The expression

$$P_{ij}(n) = \sum_k P^h_{ik}\, P^{n-h}_{kj}$$

valid for every i, j, n, if $0 \leqslant h \leqslant n$ yields the value of the probability that if n steps are needed for a transition from state i to state j, another h steps will lead to state k.

In this calculation, the initial matrices have been raised to powers $n = 2, 3, 4$. The *limit probabilities* (P_1 and P_2) have been attained already at the third power (further involutions did not alter the matrix values). The values P_1 and P_2 obtained by the involution are represented with dash lines in Fig. 2.

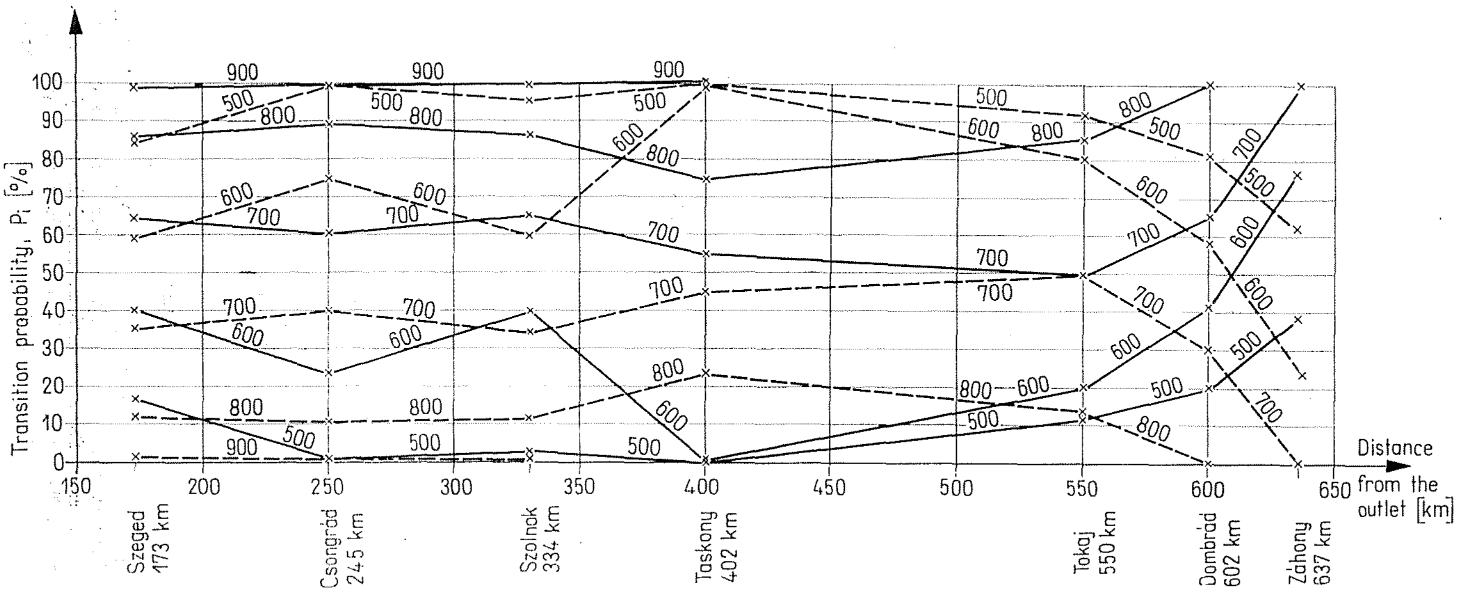

Fig. 3. Variation of transition probabilities.
Legend: — — — transition probability of occurrence,
········· transition probability of not occurrence
500 to 900 levels in cm

By definition, P_1 means e.g. the probability for the corresponding stage occurring (irrespective of whether it occurred previously or not).

The final results of the calculations are summarized in the *longitudinal section of transition probabilities* shown in Fig. 3. On the ordinate, the limit probabilities (the probability of occurrence is P_1, that of the non-occurrence P_2), whereas on the abscissa the distances from the mouth are plotted. The stage values (third variable) are assumed according to the elevations above zero on the gauge (thus, the results have not been reduced to a common reference datum).

The diagram offers a possibility to predict by $n = 1, 2, 3$ years in advance the different degrees of flood alert (associated with given stages) as a function of the maximum stage observed in the given year. Thus the resources needed by flood prevention may be mobilized in time, and significant damages may be prevented.

Remind, however, that this is only a *statistical average* of occurrences deduced from the previous time sequence. Accordingly, in each year, significant deviations from the statistical average (in dependence on the probability values) may occur.

Finally, let us mention the possibility to draw a further conclusion from the calculated limit probabilities. For example, the values correlated to the 800 cm stage at Szolnok are

$$P_1 = 0.139 \quad \text{and} \quad P_2 = 0.861.$$

Assume that in the Szolnok reach of the river Tisza, absence of the flood (i.e., of the 800-cm stage) represents a sum of Ft 10 millions (by savings in the costs of preventing damages), whereas the flood would cost Ft 3 millions in terms of the protective activities.

Thus, in this case, the net income N for a year will be:

$$N = 3P_1 + 10P_2 = 3 \cdot 0.139 + 10 \cdot 0.861 = 9 \text{ million Ft/year} .$$

The above calculation was only intended to emphasize the importance of establishing numerical values based on local data in order to see the savings (or losses) expectable from the actual development of levels for the national economy and the improvement possible through a higher degree of safety.

Summary

Transition probability and limit probability values have been calculated on the basis of time sequences of the highest stages (during 80 to 90 years) of seven gauge stations over the whole Hungarian reach of the river Tisza, by making use of the theory of the stationary time-homogeneous Markov chains. Results permitted to construct the longitudinal section of the transition probabilities which may be effectively utilized for predicting the anticipated alert stages.

References

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Prof. Dr. I. V. NAGY, 1111 Budapest, Műegyetem rkp. 3, Hungary