HYDROLOGICAL DESIGN OF A TRANSDANUBIAN WATERCOURSE NAMED "CSÁSZÁRVÍZ", BASED ON 26 YEARS OF HYDROLOGICAL DATA*

by

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Hydrological data and theoretical (Pearson III), negative binomial as well as empirical distributions of the examined watercourse are seen in Table 1 and Fig. 1, respectively.

Table 1

Discharges of "Császárvíz" (1934 to 1959)

	Discharge during the summer half-year		Discharge during the winter half-year		Annual discharge	
Year	m³/sec	Q (10⁶ m³)	m³/sec	Q (10⁶ m³)	m³/sec	Q (10⁶ m³)
1934	0,047	0,741	0,850	13,403	0,897	14,144
35	0,079	1,246	0,747	11,779	0,826	13,025
36	0,196	3,091	0,963	15,185	1,159	18,276
37	0,174	2,744	1,445	22,785	1,619	25,529
38	0,150	2,365	1,598	25,197	1,748	27,563
39	0,164	2,586	0,609	9,603	0,773	12,189
40	0,392	6,181	1,420	22,390	1,812	28,517
41	0,111	1,750	1,955	30,826	2,066	32,576
42	0,184	2,901	1,814	28,603	1,998	31,504
43	0,106	1,671	0,494	7,789	0,600	9,460
44	0,174	2,744	0,789	12,583	0,972	15,327
45	0,096	1,514	1,508	23,778	1,604	25,292
46	0,112	1,766	0,710	11,195	0,822	12,961
47	0,006	0,095	1,862	29,360	1,868	29,455
48	0,135	2,129	0,482	7,600	0,617	9,729
49	0,040	0,631	0,418	6,591	0,458	7,222
50	0,024	0,378	0,976	15,390	1,000	15,768
51	0,240	3,784	1,102	17,376	1,342	21,160
52	0,136	2,144	0,677	10,675	0,813	12,819
53	0,169	2,665	1,330	20,971	1,499	23,636
54	0,179	2,822	0,443	6,985	0,622	9,807
55	0,224	4,532	0,874	13,781	1,098	17,313
56	0,184	2,901	1,585	24,992	1,769	27,893
57	0,151	2,381	1,065	16,793	1,216	19,174
58	0,169	2,665	0,723	11,400	0,892	14,065
59	0,073	1,151	0,662	10,438	0,735	11,589

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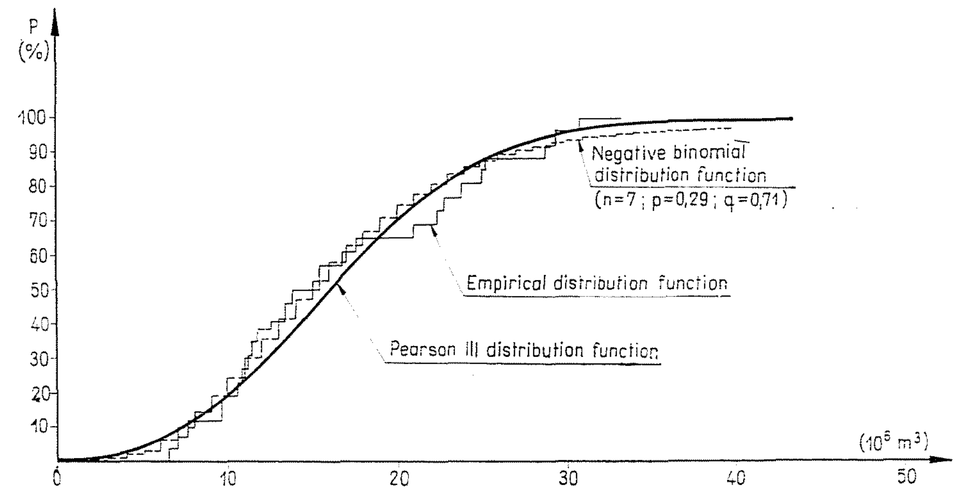

Fig. 1

Mathematical construction of the model

Let the random variable set $\{\xi_k\}_{k=0}^{n}$ denote the rainfall in the wet seasons of consecutive hydrological years, i.e. by ξ_k the rainfall in the wet season of the k-th hydrological year. The set of random variables is assumed

1. to be independent and uniformly distributed;

2. there being no water withdrawal from but only water inflow into the reservoir during the wet season. If the water level is beyond a maximum level, constant during the storage process, the entering water is discharged through the spillway;

3. to be no inflow during the dry period but only withdrawal of a water volume M. It should be noted that the water volume M can be withdrawn at any rate of flow, hence, at any instant of the dry season.

Denote by η_k the water content in the reservoir at the beginning of the k-th hydrological year ($k = 0, 1, \ldots, n$) i.e. the water volume in the reservoir before filled up volume ξ_k becomes fed in (Fig. 2).

Let K denote the reservoir capacity.

In conformity with the above, the following relationships hold:

$$\eta_{k+1} = \begin{cases} \eta_k + \xi_k - M, & \text{for} \quad M \leqslant \eta_k + \xi_k \leqslant K \\ 0 \quad , & \text{for} \quad \eta_k + \xi_k \leqslant M \\ K - M \, , & \text{for} \quad \eta_k + \xi_k > K \end{cases}$$

Values assumed for the random variables $\{\eta_k\}_{k=0}^{K-M}$ indicate the degree of fullness of the reservoir. According to assumption 1, the set of random variables $\{\eta_k\}$ constitutes a homogeneous Markov chain, that is, the probability of

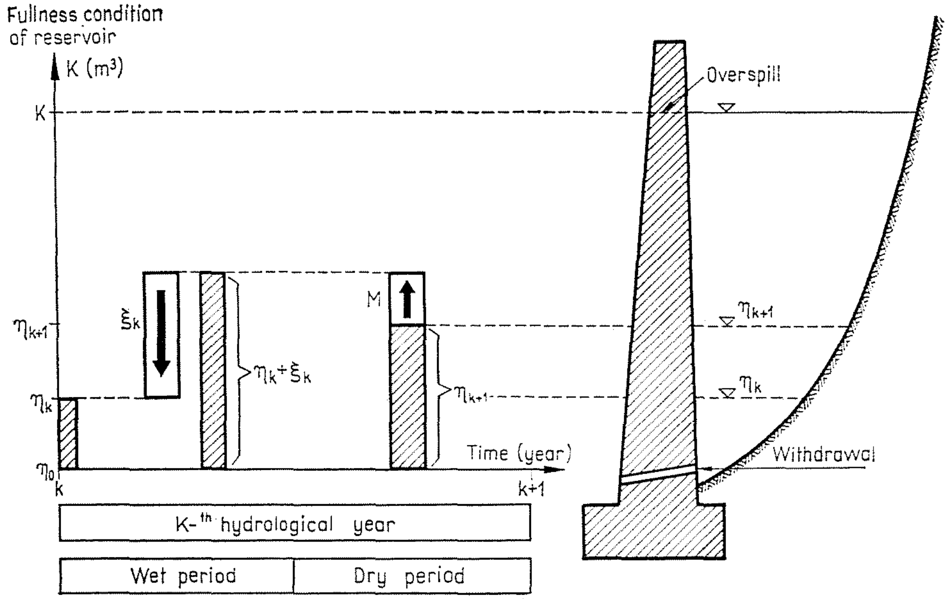

Fig. 2

transition

$$P_{ij} = P \quad (\eta_{n+1} + j / \eta_n = i)$$

is independent of n.

Denote the probability matrix of the Markov chain transition by π

$$\pi = \begin{bmatrix} P_{0,0} & P_{0,1} & \dots & P_{0,K-M} \\ P_{1,0} & P_{1,1} & \dots & P_{1,K-M} \\ \vdots & & & \\ P_{K-M,0} & P_{K-M,1} & \dots & P_{K-M,K-M} \end{bmatrix}$$

Here an element e.g., P_{ij} denotes the probability of the reservoir condition changing from i to j in one step (Fig. 2).

Determination and processing of limit probabilities offered by the model

The analysis is intended to determine limit probabilities $\{P_k\}_{k=0}^{K-M}$ meeting the equation system

$$P_j = \sum_{k=0}^{K-M} P_k \, P_{kj}; \qquad j = 0, 1, \dots, K-M$$

and the equation $\sum_{j=0}^{K-M} P_j = 1.$

The value of limit probabilities $\{P_k\}_{k=0}^{K-M}$ indicates the probability for the reservoir to assume conditions $0, 1, \ldots, K - M$, after a long series of condition changes.

Rather than by solving the established equation system, the limit probabilities have been determined by raising the matrix π to its power, a much simpler and faster procedure.

Elements of matrix π are taken from the empirical distribution function determined from the hydrological data series for the wet season (Fig. 3).

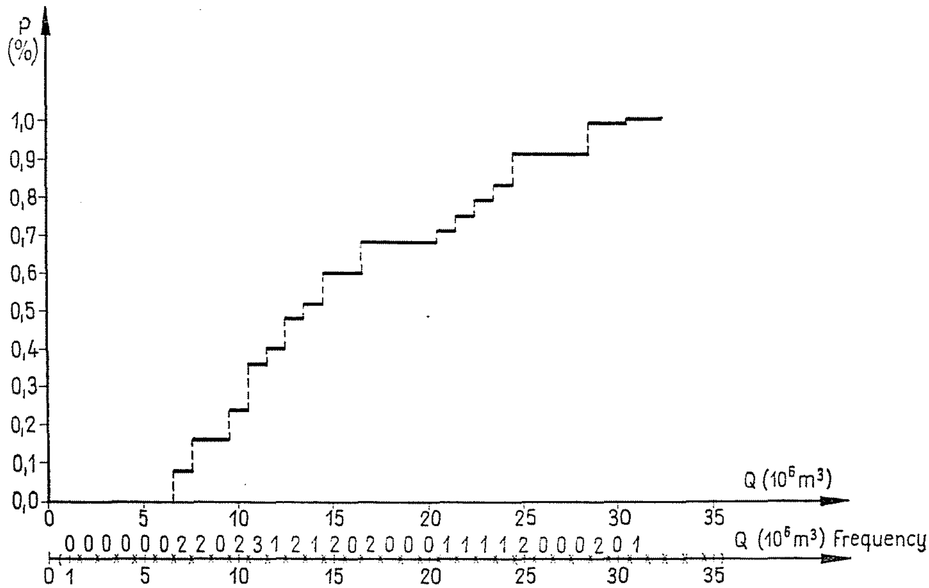

Fig. 3

Column elements of the 16th power of the matrix agreed at an accuracy of 3 decimals. The calculations involved the following K and M values:

$$K = 10 \cdot 10^6\, m^3 \begin{cases} M = 8 \cdot 10^6\, m^3 \\ M = 9 \cdot 10^6\, m^3 \end{cases}$$

$$K = 15 \cdot 10^6\, m^3 \begin{cases} M = 9 \cdot 10^6\, m^3 \\ M = 10 \cdot 10^6\, m^3 \\ M = 11 \cdot 10^6\, m^3 \end{cases}$$

$$K = 20 \cdot 10^6\, m^3 \begin{cases} M = 11 \cdot 10^6\, m^3 \\ M = 12 \cdot 10^6\, m^3 \\ M = 13 \cdot 10^6\, m^3 \end{cases}$$

$$K = 25 \cdot 10^6\, m^3 \begin{cases} M = 12 \cdot 10^6\, m^3 \\ M = 13 \cdot 10^6\, m^3 \\ M = 14 \cdot 10^6\, m^3. \end{cases}$$

Diagrams of limit probabilities for a reservoir of $K = 25 \cdot 10^6$ cu.m capacity and a discharge $M = 14 \cdot 10^6$ cu.m/half year are shown in Fig. 4.

Discharge M for probabilities

$$p = 0.01$$
$$p = 0.05$$
$$p = 0.10$$

has been determined by linear interpolation from limit probabilities. Reservoir capacity curves are indicated by circles in Fig. 5.

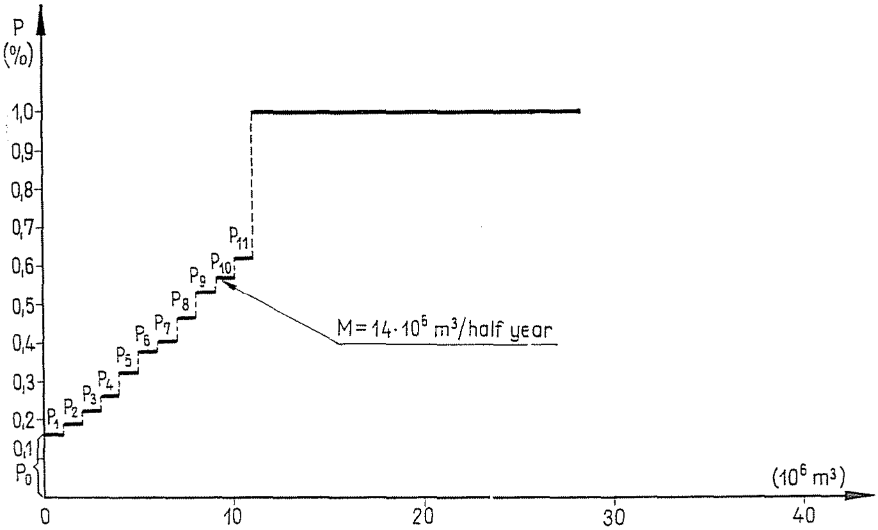

Fig. 4

Plotting the capacity diagram permits the hydrological design of the reservoir. According to Fig. 5, if e.g., a water volume $M = 10 \cdot 10^6$ cu.m/half a year is to be provided at a probability $p = 0.90$, a reservoir of capacity $K = 13.8 \cdot 10^6$ cu.m is to be built.

Checking the model

For checking the model, the function

$$\eta_{k+1} = \eta_k + \xi_k - M$$

has been processed in a digital computer for correlated K and M values.

For $\eta_{k+1} \leqslant 0$ the output was zero, for $\eta_k + \xi_k > K$ the M value was deduced from the fixed K value.

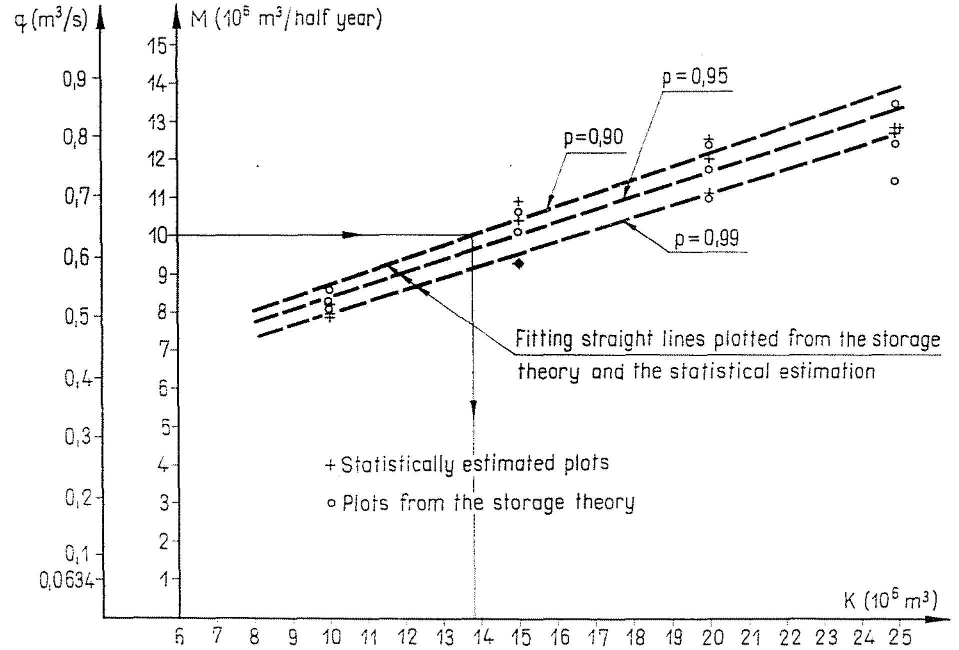

Fig. 5

The number of zeros indicated the frequency of emptying the reservoir. M values for relative frequencies

$$p = 0.01\,,$$
$$p = 0.05\,, \quad \text{and}$$
$$p = 0.10\,,$$

have been obtained by linear interpolation. Correlated M and K values are shown in Fig. 5 by (+) evidencing that the real reservoir conditions during the period of 26 years fairly approximate the conditions determined according to the reservoir theory.

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Summary

The mathematical model for Moran's storage theory has been presented and applied to a design based on the concrete hydrological data set in Table 1. The theory has been checked on a simple, realistic model and the results plotted in a graph intended for the use of design engineers. The graph simplifies hydrological design.

References

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