INVESTIGATION INTO THE PERIODICITY OF HYDROLOGICAL TIME SERIES

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(Received February 8, 1972)

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In planning hydrological engineering projects and using hydrological models of water management, much stress is laid on the inherent properties of hydrological time series. Inherent periodicity of time series may be physically explained in each case by physics of the complicated motion phenomena of the Sun and celestial bodies. A new asset in the investigation of periodicity of time series was the extensive use of computers, easing the often lengthy calculations.

1. Test methods

To analyze the inherent properties of time series, the following methods have been used:

a) Autocorrelation $[r(k)]$

b) Variance function $[q^2/(k)]$

c) Spectrum function $[S'(T), S(T)]$

d) Fourier analysis $[k(T), P_s(T), P_w(T)]$

a) The autocorrelation function is the application of the widely used linear correlation to the time series. Let $X_1, X_2, \ldots, X_i, \ldots, X_n$ be a series of observations of some hydrological element (e.g. the medium water discharge per annum). Let $x_1, x_2, \ldots, x_i, \ldots, x_n$ stand for the deviation of the above values from the average ($x_i = X_i - \overline{X}$).

The correlation between data at k intervals (years) is

$$r(k) = \frac{E\{X_i X_{i+k}\}}{[E\{X_i^2\} E\{X_{i+k}^2\}]^{1/2}} \tag{1}$$

term $E\{\cdot\}$ indicating the expected value.

For the reliability of the autocorrelation Anderson suggests the following formula:

$$CL(r_k)_p = -\frac{1}{n-k} \pm \frac{\sqrt{n-k-1}}{n-k} U_p \tag{2}$$

where U_p is the value of the standard normal distribution function at $p\%$. Calculating the autocorrelation variance according to REZNIKOVSKY:

$$CL'(r_k)_p = \pm \frac{1-r_k^2}{\sqrt{n-k}} U_p. \tag{3}$$

The above expressions written in a convenient form can be used to determine the probability level of autocorrelation.

b) The variance of consecutive data is defined by the following expression:

$$q^2(k) = \frac{0,5}{n-k} \sum_{i=1}^{n-k} (X_{i+k}-X_i)^2. \tag{4}$$

According to HALD, for $k = 1$ the expected value of q^2 is equal to the standard variance expressed as:

$$E\{q^2(1)\} = \frac{3n-4}{(n-1)^2} \cdot \sigma^4. \tag{5}$$

Introducing the term

$$r' = \frac{q^2}{\sigma^2} \tag{6}$$

$r' = 1$ may be expected and the variation:

$$E\{r'\} = \frac{n-2}{(n-1)(n+1)}. \tag{7}$$

For a hydrological time series the characteristic function is

$$r'(k) = \frac{E\{(X_{i+k}-X_i)^2\}}{2E\{X_i^2\}}. \tag{8}$$

The probability level can be assessed as:

$$CL(r'_k)_p = 1 - \frac{U_p}{\sqrt{n-k+2}}. \tag{9}$$

c) To better point out the periodicity of the autocorrelation function, it will be approximated by cosine functions of different period lengths and taking the sum of the products of ordinates of both functions, a spectrum function vs. period length is obtained. The spectrum function has a peak at those period lengths, where the cosine functions fit best the autocorrelation function. The spectrum function depends also on the number m of the maximum intervals of the autocorrelation function. The formula of the spectrum function is:

$$S(T, m) = \frac{1}{2\pi}\left[1+2\sum_{k=1}^{m} r(k)\cos\frac{2\pi k}{T}\right] \tag{10}$$

where T = period length.

Suggested by REZNIKOVSKY, the spectrum function is also in a rewritten form in use, taking the uncertainty of autocorrelation functions of larger intervals into account:

$$S'(T, m) = \frac{1}{2\pi}\left[1+2\sum_{k=1}^{m}\left(1-\frac{k}{m+1}\right)\cdot r(k)\cdot\cos\frac{2\pi k}{T}\right]. \tag{11}$$

For the evaluation of the spectrum functions no probability criteria exist.

d) The Fourier analysis considers the time series as a superposition of trigonometric series. Let the sum of products of the time series with different period lengths and cosine functions be $A(T)$ and $B(T)$, respectively:

$$A(T) = \frac{2}{n}\sum_{i=1}^{n} X_i \cos\frac{2\pi i}{T} \tag{12}$$

$$B(T) = \frac{2}{n}\sum_{i=1}^{n} X_i \sin\frac{2\pi_i}{T}. \tag{13}$$

The periodicity can be characterized by the sum of the square of the above two functions:

$$k(T) = \frac{A^2(T)+B^2(T)}{\frac{4}{n}E\{X_i^2\}}. \tag{14}$$

To characterize the probability of periodicity, SCHUSTER suggests the reliability level

$$P_s(T) = e^{-k(T)} \tag{15}$$

whereas WALKER suggests the reliability level

$$P_w(T) = 1-(1-e^{-k(T)})^{n/2}. \tag{16}$$

Of the two probability levels, the latter sets stricter conditions.

2. Periodicity analysis results

As the presented computation methods have gained ground but recently, the first calculations served to establish their usefulness. In Figs 1, 2 and 3 results from hydrological studies of the Balaton, in Fig. 4 the time series of the ground water level at Kondoros have been plotted.

Fig. 1 shows autocorrelation functions of the time series for the total rainfall and average temperature per annum for the Balaton, showing the

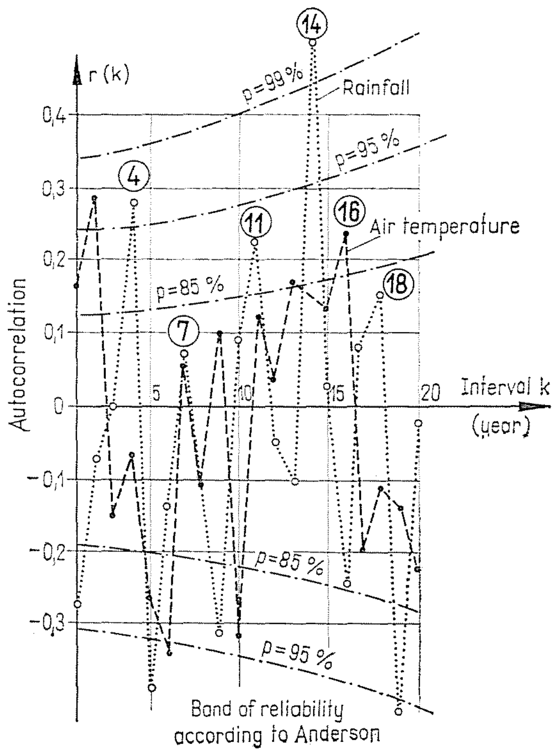

Fig. 1

reliability range according to ANDERSON. For the rainfall a shorter period of 3 to 4 years and a larger period of 14 years seem to be characteristic. The average yearly air temperature shows a large period of 15 to 16 years.

In Fig. 2 autocorrelation functions of the time series for the natural water reserve change of the Balaton without drawoff on the Sió are shown, together with the reliability levels suggested by ANDERSON and REZNIKOVSKY, respectively. It is most characteristic that the 25-year large period can be established at a probability higher than 99% according to the fairly long observation time series of 104 years. Because of the large period, the periodicity of rainfall can be seen faintly but recognizably, "copied" on the time series of the water

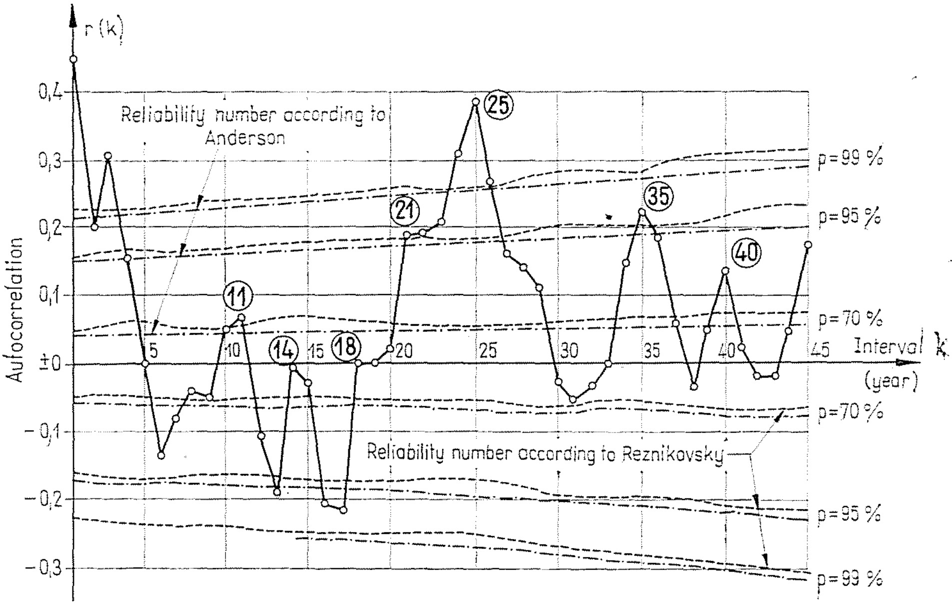
r(k)
Reliability number according to Anderson
Autocorrelation
Interval k (year)
p=99 %
p=95 %
p=70 %
p=70 %
p=95 %
p=99 %
Reliability number according to Reznikovsky
0,4
0,3
0,2
0,1
±0
-0,1
-0,2
-0,3
5
10
15
20
25
30
35
40
45
11
14
18
21
25
35
40

Fig. 2

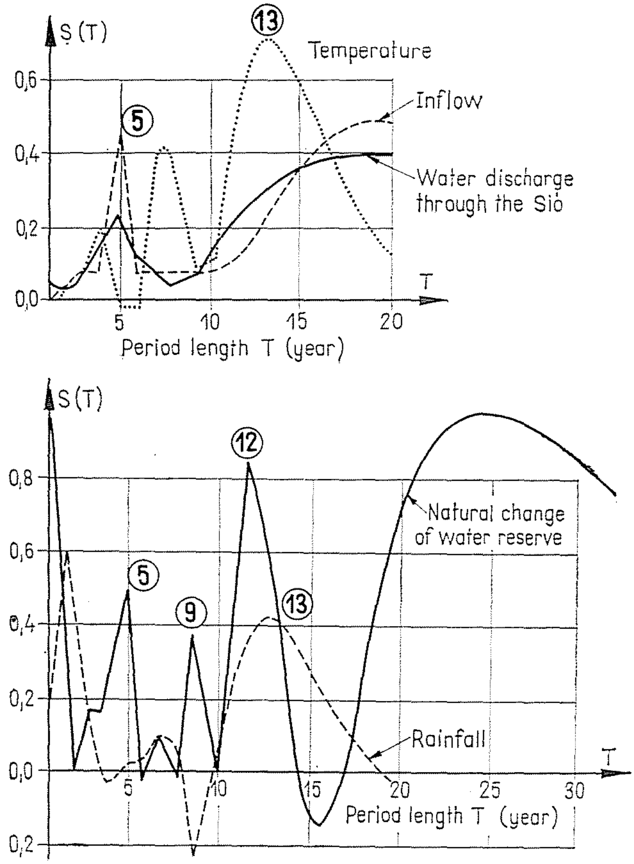
S(T)
Temperature
Inflow
Water discharge through the Sió
Period length T (year)
T
0,6
0,4
0,2
0,0
5
10
15
20
13
5
S(T)
Natural change of water reserve
Rainfall
Period length T (year)
T
0,8
0,6
0,4
0,2
0,0
0,2
5
10
15
20
25
30
12
5
9
13

Fig. 3

reserve change. To investigate the inherent periodicity of the autocorrelation functions, the approximation by Eq. (10) was used. The results are plotted in Fig. 3. 5-year periodicities of the afflux, the change of water reserve and drawoff by the Sió are most characteristic. The spectrum function of the air temperature has maxima at 4, 7.5 and 13 years, the rainfall implies a 13-year large period. Also the spectrum function of the natural water reserve change indicates a 25-year large period and a 12-year half period; it must be stressed, however, that the function $S(T)$ depends also on m — the maximum considered

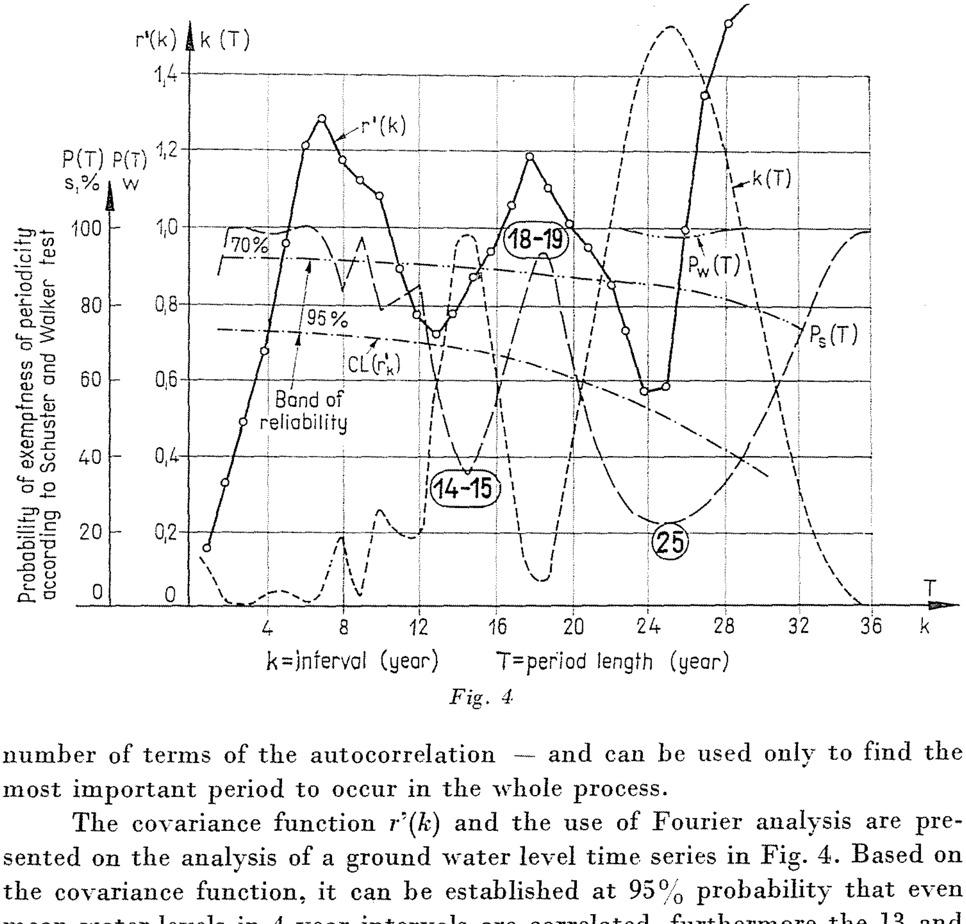

Fig. 4

number of terms of the autocorrelation — and can be used only to find the most important period to occur in the whole process.

The covariance function $r'(k)$ and the use of Fourier analysis are presented on the analysis of a ground water level time series in Fig. 4. Based on the covariance function, it can be established at 95% probability that even mean water levels in 4-year intervals are correlated, furthermore the 13 and the 24 or 25 year periods do appear.

Based on the parameter $k(T)$ of the Fourier analysis (Eq. 4) and the probability decision $P_s(T)$ proposed by SCHUSTER, periods of 14.5 and 25 years appear. The much stricter Walker test shows an appreciable periodicity only at about 25 years, evidently since the length of the data series was merely 36 years.

Summary

The presented period analysis methods and some hydrological applications let conclude on a periodicity common for the inherent structure of hydrological phenomena, likely to be of use in forecasts and in water management. The affinity between periodicities of the water balance elements and rainfall or temperature points to the determining role of the evaporation or the rainfall. The tests carried out so far already support the existence of a 25-year large period.

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