DIMENSIONING OF BRANCHING IRRIGATION NETWORKS BY MEANS OF LINEAR PROGRAMMING

by

I. IJJAS

Institute of Water Management and Hydraulic Engineering, Technical University, Budapest

(Received February 8, 1972)

Presented by Prof. I. V. NAGY

In recent years, the method of linear programming has been used in several countries for dimensioning branching pipe networks. Mathematical models to this aim have been described by ZDRAZIL and SPITZ [13], LABYE [11, 12], KARMELI, GADISH and MEYERS [7], KORSUN and SPITZ [9], KIKACHEYSHVILI [8], CHUDZIK and OPALINSKI [1].

In Hungary linear programming has been used since 1966 for dimensioning branching networks. In the following, the mathematical model applied by the above authors and by the writer of this paper [2] with more or less changes will be presented, referred to as the first model, followed by the description of the new model developed by the author, referred to as the second model [5], permitting wide use of the method of linear programming.

1. Notations

H_k = lift of pumping station in the k-th service condition;
H_l = pumping level (on sucking side);
H_{bk} = head loss of pumping station (from pump shaft to junction at main conduit in the k-th service condition;
p = mark of critical point (critical hydrant);
H_p = elevation of critical point;
k = mark of service condition critical to point p;
H_{hp} = service pressure above ground level to be maintained at the critical point;
h_p = head loss permissible up to critical point;
v_{min} ..., v_{max} ... = permissible minimum and maximum velocity, respectively, in pipes;
j = number of pipe section considered;

by *sections* longitudinal elements of a pipe system are understood;
- exempt of branching;
- along which water discharge is constant in every service condition (no drawoffs o branches are found within the length);
- wherein no hydrants to be checked for service pressure are inserted (e.g. from the viewpoint of geodetic elevation).

Section symbols

l_j = length of section;
d_i = i-th pipe diameter in the section;
r_j = number of different pipe sizes in the section;
l_{ij} = length of portion with the i-th diameter;

Q_{kj} = discharge in the i-th service condition;
$E_i[Q_{kj}]$ = specific head loss over the portion of i-th diameter in the k-th service condition;
h_{kj} = head loss along the section in the k-th service condition;
n = number of sections in the pipe network;
b_{ij} = specific construction cost of pipe with the i-th diameter applied in the j-th section;
B = construction cost, understood as optimum cost of construction;
B_{max} = construction cost involving maximum pipe diameters.

2. The first mathematical model

The problem of dimensioning the pipe network of the irrigation system is as follows:

Determine the optimum pipe sizes in a branching network (thus, in the sections) if the head in the main pipe at the pumping station; the service pressure to be maintained at the hydrants; the critical service conditions and the corresponding overall discharge; as well as the critical points and their elevations are known.

2.1. Mathematical model in the case of a known pump lift

The problem is to select the pipe sizes so that the required discharge and head is available even at the hydrants in the worst position, at a minimum cost of construction for the pipe network.

The permissible head loss from the pumping station to the p-th critical point is known:

$$h_p = H_1 + H_2 - H_{bk} - H_{hp} - H_p . \qquad (1)$$

The available pipe sizes (which may be used for the construction of the pipe network) and the minimum and maximum velocity permissible in the pipes are given.

Consider an arbitrary conduit section and assume it to be constructed telescope-like of pipes of all the diameters available so that the velocity of water cannot exceed the maximum permissible velocity v_{max} even for the maximum discharge in the section, but it is higher than the permissible minimum velocity v_{min}.

In the *section* of length l_j, the portions built of different pipe sizes may be of arbitrary length l_{ij}, provided two conditions are met:

$$l_{ij} > 0 \qquad (2)$$

and

$$\sum_{=1}^{r_j} l_{ij} = l_j . \qquad (3)$$

Head loss in the conduit section may be written as follows:

$$h_{kj} = \sum_{i=1}^{r_j} E_i[Q_{kj}] \cdot l_{ij} \tag{4}$$

where $E_i\,[Q_{kj}]$ = specific head loss along the portion of i-th size in the k-th service condition.

The head loss between the pumping station and the p-th critical point (hydrant) must not surpass the value obtained from (1):

$$\sum_{p} \sum_{i=1}^{r_j} E_i[Q_{kj}] \cdot l_{ij} = h_p \tag{5}$$

where p indicates that along the critical line between the pumping station and point p the head losses of all the sections are to be summed.

The sections of the pipe network should be built of pipe sizes such as to minimize the *construction cost*, hence:

$$B = \sum_{j=1}^{n} \sum_{i=1}^{r_j} b_{ij} \cdot l_{ij} = \text{minimum}. \tag{6}$$

Accordingly, the mathematical model of the problem consists of the *hydraulic conditions* type (5), *geometric conditions* type (3) and of the *target function* type (6). As many hydraulic conditions should be written down as there are critical points. (Exceptionally, several hydraulic condition equations may be related to the same critical point.) The geometric conditions equal in number the sections. To meet conditions type (2) is self-evident.

If needed, also upper limiting hydraulic conditions of head may be incorporated in the model.

Solution of the model starts from a secondary target function, the first step being the search of a possible solution, that is, determination of the lengths of portions l_{ij} meeting geometric condition equation type (3) and corresponding to conditions type (4). Hereafter, the computation follows the original function according to the so-called modified simplex method.

2.2. *Mathematical model for the determination of the optimum lift of the pumping station*

In the previous mathematical model the lengths l_{ij} made with pipes i are unknown. Now, a new unknown value is emerging, i.e., the lift H of the pumping station. The purpose of the analysis is to minimize the water lifting costs during T years. Thus, the target function (6) takes the form:

$$K = B + T\ddot{U} = \sum_{j=1}^{n} \sum_{i=1}^{r_j} b_{ij} \cdot l_{ij} + T \cdot H \cdot C, \tag{7}$$

where B = cost of construction of pipe network;
$\ddot{U}$ = annual cost of pumping;
T = local refund standard limit (per year);
C = yearly cost of lifting the total amount of water to unit height.

Also the *hydraulic conditions* are transformed:

$$H - \sum_{p} \sum_{i=1}^{s_j} E_i[Q_{kj}] \cdot l_{ij} \geqq H_h + H_p - H_1 + H_b, \tag{8}$$

The *geometric condition equations* are unchanged.

3. The second mathematical model

Development of the model set out from that in every section of the network the largest pipe size complying with the limit velocity $v_{\min}$ has been applied. In this case, the *construction cost* of the pipe network is

$$B_{\max} = \sum_{j=1}^{n} b_{lj} \cdot l_j . \tag{9}$$

Head loss to critical point p:

$$h'_p = \sum_{p} E_1[Q_{kj}] \cdot l_j \tag{10}$$

where $E_1\ [Q_{kj}]$ = specific head loss along the j-th conduit section for the largest pipe size (meaning of p is here the same as in (5)).

If the head loss concomitant to the largest pipe size is higher than the permissible one, i.e.

$$h_p < h'_p , \tag{11}$$

then the problem cannot be solved by the utilization of the given series of pipes.

Application of the largest possible diameters in the pipe network would be uneconomic. Therefore, as far as possible, in certain sections smaller diameters should be applied. But neither the total length of the new, narrower portions can be longer than the entire length of the section. Thus, the following condition is to be met:

$$\sum_{i=2}^{r_j} l_{ij} \leq l_j, \tag{12}$$

wherein l_{ij} = length to change over to the i-th diameter (the largest pipe size $i = 1$).

Changing over from the largest to a smaller diameter results in an increased head loss. The amount of this increase along the length to the critical points cannot exceed the difference between the head available and the head loss due to the application of the largest pipe size.

$$\sum_p \sum_{i=2}^{r_j} \left(E_i[Q_{kj}] - E_1[Q_{kj}]\right) \cdot l_{ij} \leq h_p - \sum_p E_1[Q_{kj}] \cdot l_j. \tag{13}$$

Besides, pipe sizes should be changed in a way to achieve a total of construction cost saving; for each section as big as possible, in order to provide the smallest first cost possible, as expressed by the target function:

$$B' = \sum_{j=1}^{n} \sum_{i=2}^{r_j} (b_{lj} - b_{ij}) \cdot l_{ij} = \text{maximum}. \tag{14}$$

Solving the mathematical model consisting of geometric conditions type (12), hydraulic conditions type (13) and target function type (14), the optimum cost of construction of the pipe network is:

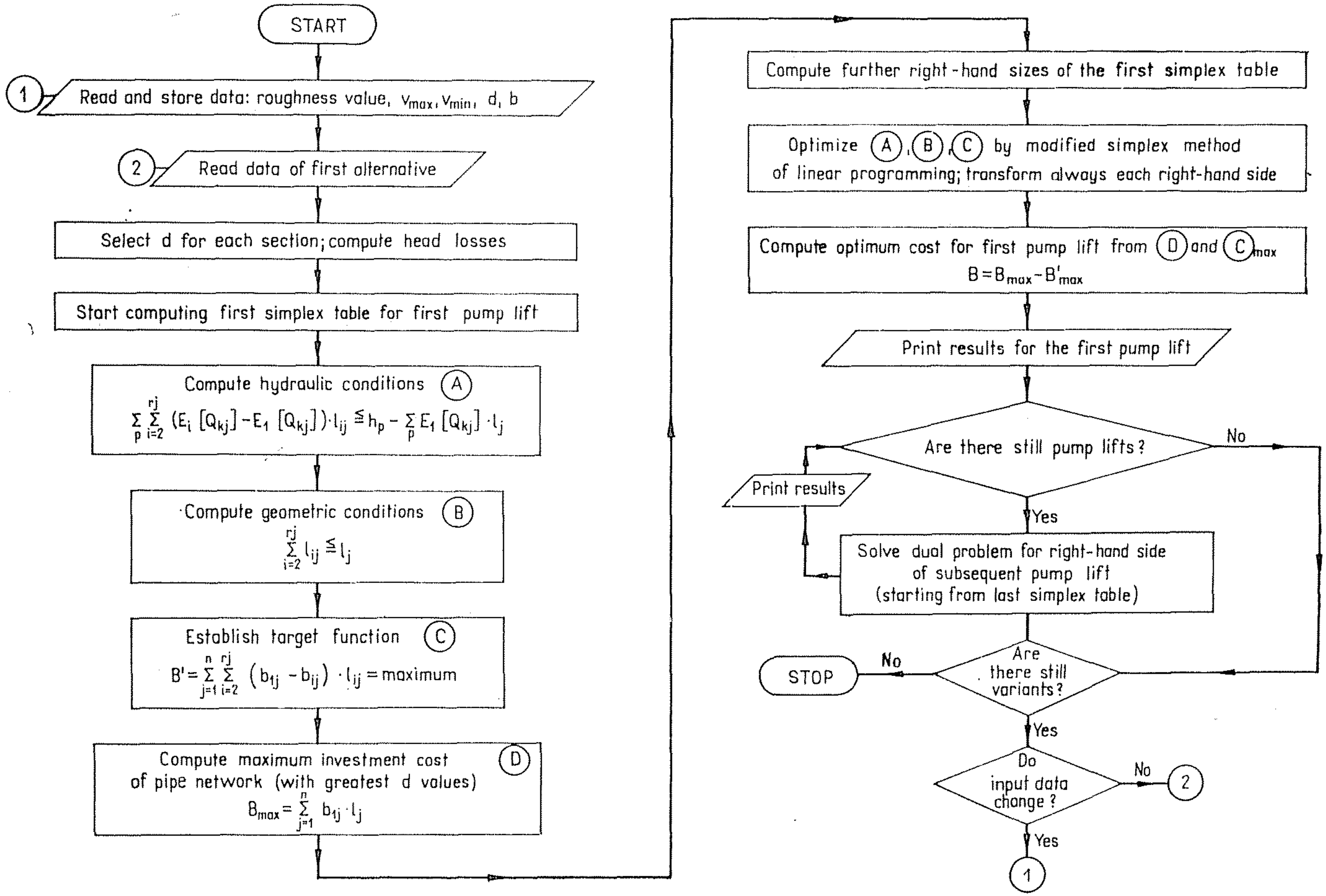
$$B = B_{\max} - B'_{\max}, \tag{15}$$

where $B_{\max}$ = maximum cost of construction if the largest diameters are applied (9); and $B'_{\max}$ = maximum of the target function (14).

The primal variables of the simplex table consisting of the matrix containing the coefficients of conditions (12) and (13), the right-hand sides of the conditions and the target function are *those portion lengths of the sections where the change-over from the largest to minor diameters is realized.*

The variables corresponding to the condition rows are, from the viewpoint of the simplex algorithm, dual variables, *those corresponding to the equality rows are, however, primals with respect to the outcome.* Among them those kept *in the optimum program mean the lengths where the largest diameters possible in the section result in the optimum cost of construction.*

If it is wanted to solve the problem for *several pump lifts*, then the right-hand side of the initial simplex table, corresponding to all pump lifts, is to be produced. Only right-hand sides of the hydraulic conditions differ, that


START
1
Read and store data: roughness value, v_{max}, v_{min}, d, b
2
Read data of first alternative
Select d for each section; compute head losses
Start computing first simplex table for first pump lift
Compute hydraulic conditions (A)
$\sum_p \sum_{i=2}^{rj} (E_i [Q_{kj}] - E_1 [Q_{kj}]) \cdot l_{ij} \leqq h_p - \sum_p E_1 [Q_{kj}] \cdot l_j$
Compute geometric conditions (B)
$\sum_{i=2}^{rj} l_{ij} \leqq l_j$
Establish target function (C)
$B' = \sum_{j=1}^{n} \sum_{i=2}^{rj} (b_{1j} - b_{ij}) \cdot l_{ij} = \text{maximum}$
Compute maximum investment cost of pipe network (with greatest d values) (D)
$B_{max} = \sum_{j=1}^{n} b_{1j} \cdot l_j$
Compute further right-hand sizes of the first simplex table
Optimize (A), (B), (C) by modified simplex method of linear programming; transform always each right-hand side
Compute optimum cost for first pump lift from (D) and $(C)_{max}$
$B = B_{max} - B'_{max}$
Print results for the first pump lift
Are there still pump lifts?
No
Yes
Print results
Solve dual problem for right-hand side of subsequent pump lift (starting from last simplex table)
Are there still variants?
No
STOP
Yes
Do input data change?
No
2
Yes
1

of the geometric conditions is invariable. From the right-hand side belonging to the first pump head these pertaining to the subsequent ones may be calculated by adding the consecutive pump head differences.

The mathematical model will be solved by means of the modified simplex method of linear programming, nevertheless in certain details of the method the peculiarities of the mathematical model of pipe network design, the special structure of the hydraulic and geometric conditions permitted the application of unusual steps of algorithm, *at a substantial reduction of the solution time, hence, of the computer time.*

The *optimization* begins according to the right-hand side corresponding to the lift of the first pump. After achieving the optimum program and printing the outcome, the right-hand sides corresponding to the subsequent pump lifts are handled as solution of the dual problem. Thereby the computer time required for the second and further heads could be reduced to a fraction of that needed for the first head.

The operations appear from the flow chart on p. 354.

4. The second model and peculiarities of the method applied

The second model is more convenient than the first one by its fewer iterations, the smaller extent of the problem and the shorter computer time.

For this problem, several programs have been developed in ALGOL—60 language which are applied by the Water Management Organization and Computing Bureau and by the Institute for Water Management and Hydraulic Engineering of the Technical University, Budapest.

The model is computer constructed eliminating the cumbersome data setup.

The program permits the determination of optimum diameter combinations corresponding to lifts of several pumps, involving *several right-hand sides.* The optimum related to the first lift having been determined, computation affects the second right-hand side, i.e., the dual problem. The algorithm *quickly* approximates the optimum for the second and further pump lifts. The number of iterations and thereby, the computer time needed for the second and further lifts make up a minor fraction of those required for the first lift.

This program helped already to design sprinkler irrigation systems for an area over 150 000 ha in Hungary. Economy in first costs amounted to 2500 forints per ha.

5. Comparison of the linear programming and other methods for optimizing branching irrigation networks

For the optimization of branching networks, programs operating according to different methods have been established (LABYE's discontinuous method, linear programming, dynamic programming) [2 to 6]. In the case of identical input data and conditions, the programs operating according to different methods gave identical results. Each method has its field of application.

The benefit of the dynamic programming [6] is to deliver at the same time the optimum trace and cost of the conduit.

The advantage of the discontinuous method and dynamic programming is a reduced computer time.

The linear programming is the most general and most versatile method. Cases where e.g. some conduit sections have different water discharges under different service conditions, can only be approximated either by dynamic programming or by the discontinuous method. This is a problem for linear programming.

Dynamic programming and the program using the discontinuous method help to find the optimum cost of construction for all heads possible at the pumping station (minimum cost polygon of the system) while the program based on linear programming offers the optimum cost of construction corresponding to given lifts.

Summary

In Hungary, linear programming has been applied since 1966 for dimensioning branching irrigation pipe networks. The presented mathematical model (first model) has been used in several countries with more or less modifications, yet, another mathematical model (second model) has been developed at the Institute of Water Management and Hydraulic Engineering of the Technical University, Budapest. For the optimization of the branching pipe networks, programs using several different methods (Labye's discontinuous method, linear programming) have been established. In the case of identical input data and conditions, the programs operating according to different methods gave identical results. The application fields of the methods are different. In many cases, the choice of the convenient method depends on the nature of the problem.

References

1. CHUDZIK, B.—OPALINSKI, CZ.: Techniczno-ekonomiczna optimalizacja projektu sieci rurociagów deszczownianych. Zeszyty Naukowe Wyższej Szkoly Rolniczej we Wroclawiu, Wroclaw, 1970. Nr. 90.
2. IJJAS, I.: Investigation of underground pipe network of sprinkler irrigation systems. (In Hungarian). (Dr. Techn. Thesis 1966).
3. IJJAS, I.: Determination of the optimum lift of pumping stations by means of dynamic programming. Periodica Polytechnica, Civ. Eng. **14** (1970) 243.
4. IJJAS, I.: Okreslenie srednic podziemnej sieci rurociagów deszczowni. Zeszyty Naukowe Wyższej Szkoly Rolniczej we Wroclawiu, Wroclaw, 1970. Nr. 90.
5. IJJAS, I.: Application of computers in agricultural water resources management. (In Hungarian). Lecture notes for special engineering students. Tankönyvkiadó 1971
6. IJJAS, I.: Determination of the optimal route of the main conduit of branching irrigation pipe networks by means of dynamic programming. ICID VIII. European Congress, 1971.

7. KARMELI, D.—GADISH, Y.—MEYERS, S.: Design of optimal water distribution networks. Proc. ASCE 1968. PL 1.
8. KIKACHEYSHVILI, G. E.: Tehniko-ekonomichesky raschet razvetvlennikh vodoprovodnikh setey metodom lineynogo programmirovaniya. Vodosnabzhenie i Sanitarnaya Teknika, 1969/6.
9. KORSUN, S.—SPITZ, P.: Pouziti lineárniko programováni pri dimenzováni závlahovych trubnich siti. Vodni hospodárstvi—A, **2** (1969).
10. MOLNÁR, L.: Some experiences on the sprinkler irrigation system in Hajdúszovát. (In Hungarian). Vízügyi Közlemények, **3** (1970).
11. LABYE, Y.: Étude des procédés de calcul ayant pour but la distribution d'eau. La Houille-Blanche, **5** (1966).
12. LABYE, Y.: L'utilisation en France des calculatrices électroniques pour les projets d'irrigation et d'assainissement. Essai de synthèse, ICID Symposium, Mexico City, 1969.
13. ZDRAZIL, K.—SPITZ, P.: Ekonomické dimenzováni trubni site metodou lineárniko programováni. Vodni Hospodárstvi **9** (1965).

Senior Ass. Dr. István IJJAS, 1111 Budapest, Műegyetem rkp. 3, Hungary