INVESTIGATION OF REFRACTION IN THE LOW ATMOSPHERE

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1. Significance of the refraction in surveying

Light crosses vacuum and media of constant (homogeneous) physical state in a straight line, but in the free atmosphere the propagation of light is perceptibly influenced by temperature, humidity, pressure and carbon dioxide content of the air.

Exploration of the atmosphere became necessary for the theoretical and practical geodesy to achieve a higher precision. In the course of centuries, instruments and methods of surveying made a remarkable advance. Theoretical and technical conditions of a high precision seemed to be assured, but the assumption made on the ambiency of measurements i. e., on the free atmosphere, differed from reality.

The atmosphere envelops the earth in strata of different densities, in first approximation in the form of concentric spherical shells. Optical properties of the strata of air of different densities, and thereby refractive indexes are also different. According to the known optical law, the light beam passing from a medium of a smaller refractive index to a medium of a greater one is deflected towards the normal to the interface, and inversely.

The free atmosphere is, however, not composed of spherical shells of different densities but the physical factors affecting the refractive index are changing continuously and thus, the light beam follows a curvilinear path. In geodesy, the path of propagation of the light is referred to as refraction curve, whilst the angle between the rectilinear propagation and the real path of light is called the angle of refraction; the ratio of the radius of earth assumed to be spherical to that of the refraction curve assumed to be a circular arc, is called the coefficient of refraction.

Already in 1671, Picard treated of the effect produced by the atmospheric refraction in trigonometric levelling in his work "Mesure de la terre". The notion "coefficient of refraction" was used first by Maupertuis in 1736 and 1737 in his surveyings in Lapland trying to determine the form of the earth. The astronomical refraction was taken into account by Tobias Mayer in 1751. A significant advance was made by Gauss in his 1823 geographical degree measurements at Hanno—verGöttingen, by determining the mean value of the coefficient of refraction — 0.1306 —, in actual use.

As a consequence of the rapid development in meteorology, in the first half of our century, atmospheric refraction was not regarded any more a purely geometry problem.

The detailed analysis of the physical components of the atmosphere introduced the study of the physical problem of refraction.

The free atmosphere is characterized by different states of physical components at different levels above earth surface. In surveying, the collimation line passes through air strata of differents levels, therefore the refraction diversely affects the results. Accordingly, the examined refraction can be divided, according to air-strata affected by the measurements, into:

a) levelling refraction;

b) terrestric refraction;

c) astronomic refraction.

In this paper the refraction in the vertical plane will be treated, with particular consideration to the refraction in the atmosphere near the ground.

2. Meteorological factors producing refraction in the atmosphere near the ground

Study of the levelling refraction requires first the knowledge of microclimatic conditions in the atmosphere near the ground. Both theory and practice show a decisive importance of air temperature, air humidity, carbon dioxide content and air pressure among atmospheric factors affecting refraction. Refraction is much less affected by the oxygen, nitrogen and rare gas contents of air.

21. *Temperature variation.* The intensity of the solar radiation changes the whole day. Considering that the non-transparent, solid soil surface has a much greater heat-absorption capacity than has the atmosphere, during the period of solar radiation the ground surface grows warm. The atmosphere obtains a great part of its temperature by heat transfer from earth surface, consequently, the temperature of the earth surface has a crucial effect on the thermal state of the atmosphere. Shortly after sunset, the ground surface gets cooler and during the period of heat emission, cooling also affects the atmosphere above the ground.

During the period of insolation, the most efficient way of heat transfer is that by air current and mixing. During mixing, air particles getting warmer near the ground surface are moving upwards, heat exchange starts in the form of turbulent air movement, the refractive index changes rapidly and irregularly. The first derivative of the temperature function with respect to the height, i.e. the vertical gradient of temperature becomes negative, the atmospheric equilibrium unstable, and the lower air layers will be lighter than the upper ones. The temperature gradient is at its minimum around noon, then its absolute value diminishes and by one or two hours before sunset it will transiently be zero. When the earth surface cools down, again the lower air body will be the cooler, the state of equilibrium becomes stable. The temperature gradient soon approaches its positive maximum and keeps it almost invariably all along the night. Its maximum develops immediately before sunrise, then its value decreases and one or two hours after sunrise it will be zero again, and so on.

22. *Change in air humidity.* The daily variation of the vertical air humidity gradient is similar to that of the temperature gradient. (By humidity content of the air the relative humidity is understood.) During sunrise, the vertical gradient of humidity is zero, then its value becomes negative, i.e., the humidity content of the air diminishes with sun height. It reaches its minimum at noon; at sunset it becomes again zero, remaining positive until sunrise.

23. *Change of the carbon dioxide content of the atmosphere.* The normal atmosphere contains 0.03 per cent by volume of carbon dioxide. The atmosphere is considered as normal if its vertical temperature gradient is -0.65 °C/100 m and its pressure reduced to sea level is 760 Hg mm at a temperature of 15 °C. According to LUNDEGÅRD [1], the carbon dioxide content may strongly vary. In wood, with dense undergrowth, it may be as high as 0.07 per cent. At sights near the soil level the carbon dioxide content of air may commonly be considered as normal.

24. *Change in air pressure.* For levelling and terrestric refractions, in case of horizontal or nearly horizontal lines of collimation, the vertical gradient of air pressure does not affect the refraction. Though the horizontal gradient of air pressure caused by wind ought to be taken into account if the gusts exceeded 6 degrees on the Beaufort scale, but in fact, since under such conditions no surveying work is done, this can be left unconsidered.

According to MEGGERS and PETERS [2], between refractive index, temperature, pressure, humidity and carbon dioxide content of the air the following interdependence exists:

$$n-1=\frac{0{,}000\,2923}{1+0{,}00368\,t}\cdot\frac{B}{760}-0.000\ 041\,\frac{e}{760}+0.000\ 0016\,k\,\frac{B}{760} \qquad (1)$$

where n = refractive index of the air;
t = air temperature in °C;
B = air pressure in Hg mm;
e = humidity content of the air in Hg mm;
k = percentage by volume of the atmospheric carbon dioxide.

According to Lorenz, the refractive index of vapour is 1.000 2500. On this basis he suggests the correction term $-$ 0.000 041 $e/760$. In possession of the refractive index of carbon dioxide, the correction term may be found as $+$ 0.000 0016 k $B/760$.

Eq. (1) is strictly valid to the wavelength of 556 nm which corresponds to the maximum luminous intensity of white light.

In order to determine the effect of the differential change in the temperature (t), air pressure (B), humidity (e), and carbon dioxide (k) of the air on the refractive index (n), Eq. (1) has to be derived with respect to each variable.

We assumed the atmospheric variations in a height of 3 m above ground level and at a distance of 100 m (this latter being the distance of two neighbouring change points) to range to $\mathrm{d}t = \pm 2$ °C; $\mathrm{d}e = \pm 2$ Hg mm; $\mathrm{d}k = \pm 0.02$ per cent and $\mathrm{d}B = \pm 0.05$ Hg mm. Under average weather conditions suitable for surveying, the extreme values of variations practically keep within the 50 to 60 per cent of these ranges.

The above values replaced into the differential equations yield the differential variation of the refractive index due to:

a) temperature variation: $\mathrm{d}n = 1.92 \cdot 10^{-6}$;
b) atmospheric humidity variation: $\mathrm{d}n = 0.108 \cdot 10^{-6}$;
c) carbon dioxide variation: $\mathrm{d}n = 0.032 \cdot 10^{-6}$;
d) air pressure variation: $\mathrm{d}n = 0.018 \cdot 10^{-6}$.

It is obvious that the effect of temperature variation is decisive over the other atmospheric factors on refraction, to the following proportions: Tempera ture to humidity to carbon dioxide content to air pressure = 100 : 6 : 2 : 1

Thus, it can be stated that from among the atmospheric factors occurring in the microclimate, it is sufficient to take into account the effect of the temperature variation; that of the other factors may be omitted.

3. The vertical temperature gradient

The vertical temperature gradient i.e. first derivative of the temperature function with respect to height, $\mathrm{d}t/\mathrm{d}z$, is approximately equal to the change of temperature per unit of the difference of height: $\Delta t/\Delta z$. Its value γ is reduced in the international use to 100 m, its dimension being accordingly °C/100 m.

Observation data during several decades show an either positive or negative temperature gradient always to exist in the atmosphere, with values rather strongly changing in short intervals.

From the viewpoint of surveying, the temperature gradient has some characteristic values. Among these, the adiabatic lapse rate is the most significant one. Its value may be determined from the equation expressing the static equilibrium in the atmosphere:

$$\frac{c_p}{A} \Delta t = - g \Delta z \tag{2}$$

wherein:

c_p = 0.239 cal g^{-1} °C^{-1}, specific heat of the air at constant pressure;
A = 2,389 · 10^{-4} cal cm^{-2} g^{-1}, sec^2, the thermal equivalent of work;
Δt = the difference of temperature °C;
Δz = the difference of height;
g = 9.81 m sec^{-2}, the gravity acceleration in Budapest.

The quotient from the differences of temperature by height from Eq. (2) yields the adiabatic gradient:

$$\frac{\Delta t}{\Delta z} = - \frac{A \cdot g}{c_p} = \frac{2.389 \cdot 10^{-4} \cdot 981 \cdot 10^{-2}}{0.239} = - 0.974 \text{ C}°/100 \text{ m} \tag{3}$$

Namely, the unsaturated air cools down in rising by 0.974 °C for every 100 m.

In practice, the adiabatic value of the temperature gradient is assumed to be − 1 °C/100 m.

If in an air column 100 m high, the change in temperature by 1 °C is just proportional to the height, then the atmosphere is in an indifferent state of equilibrium. If the decrease in temperature along the increase of height is less than 1 °C/100 m, then the atmosphere will be in a state of stable equilibrium. On cloudless, warm days, however, the decrease in temperature along height increase exceeds 1 °C/100 m, the updraft movement of the air will be accelerated. This is an accompanying phenomenon of the unstable state of equilibrium.

Besides, the adiabatic value of the temperature gradient $\gamma = -3.42$ °C/100m is the most characteristic value in the atmosphere near the ground where the line of collimation is rectilinear. When this value is exceeded in negative sense, the air density increases with the height, an unstable lower layer develops, and as a result, the coefficient of refraction becomes negative, the refraction curve will be concave seen from above. On the contrary, for a positive value, the coefficient of refraction becomes positive, and the refraction curve will be convex regarded from above.

From the viewpoint of surveying, the critical value of the gradient of temperature is −1.7 °C/100 m, namely, in this case, the curvature of the line of collimation can be described by a constant radius of curvature, thus it is a circular arc.

For a temperature gradient $\gamma = -1.14$ °C/100 m, the curvature of the line of collimation may be expressed by a linear function of the height.

4. Effect of the atmosphere near the ground on precise levelling

The refraction in the atmosphere near the ground causes random or systematic changes along the line of collimation, proving to be random or systematic sources of error in precise levelling.

Random sources of error are the phenomena of atmospheric vibration and shimmer, well known in levelling.

The refraction in the atmosphere near the ground has, however, a more dangerous form of appearance than random sources of error, deviating the light beam in a regular way in comparison with the rectilinear line of collimation. This systematic source of error, with magnitude and sign depending on the temperature gradient, is referred to as levelling refraction. Theory and tests show the value of the levelling refraction to be proportional to the vertical temperature gradient, to the square of the sight distance, and approximately proportional to the difference of height. It is at its maximum about one hour after sunrise and about an hour before sunset, when it is temporarily extincted. At night it comes up to about the half of the day peak but of course, with reversed sign. During the day exhibiting definitely negative temperature gradients, the refraction curve is concave seen from above, and during the night, period of inversion with positive gradient, it is convex seen from above. Because of levelling refraction a negative coefficient of refraction causes the measured elevation difference to seem less than its real value and inversely.

Notice that in periods of non-systematic refraction errors no precise levelling is made, in conformity with the instructions on surveying, whilst in the period of systematic errors — though difficult to reckon with accurately, and thus, commonly neglected in practice, — precise levelling is carried out. The most advantageous period for surveying begins about 15 to 20 minutes after sunrise — when air vibration stops — and lasts in the forenoon until the beginning of air vibration, and in the afternoon, by dying down of the air vibration, it continues till dusk and ends at 15 to 20 minutes before sunset.

The increased accuracy requirements for precise levelling motivate to change the practice followed so far. Neglection of the levelling refraction would require the period of surveying to be limited, prejudicial, however, to the efficiency of surveying. Namely, the coefficient of refraction is positive during about the half of the forenoon and afternoon periods, and negative in the other half, likely to change systematically the measured elevation difference by a variable value of identical sign. It seems to be more practicable to establish easy-to-treat relationships for the consideration of the refraction, unlike to prolongate the field work by simultaneous meteorological observations, but likely to enhance the possible precision.

In the 1930's, the advancement in the instruments technique resulted in the development of up-to-date, more precise levelling instruments; the surveying precision, however, could not be increased because of the uncleared problem of levelling refraction. LALLEMAND [3] did pioneering work in the relevant theoretical investigations; he expressed the change of the temperature in dependence on the height as:

$$t = a + \log (z + c) \tag{4}$$

where: t = temperature,
z = height above ground level, and
a, b and c constant values.

HUGERSHOFF [4] expressed the dependence by using a second degree polynomial:

$$t = a + bz^2 \tag{5}$$

This relationship has been transformed by KOHLMÜLLER [5] and was preferred by REISSMANN [6] in his investigations:

$$t = a + bz + cz^2 \tag{6}$$

After the establishment of the temperature equations (4), (5) and (6) the climatic investigation of the atmosphere near the ground has much developed, and precision in systematic thermometry has also increased.

KUKKAMÄKI [7] made use of the results observed by Best in South England, as well as of his own experimental measurements in Finland. His equation for expressing the change in temperature is:

$$t = a + bz^c \tag{7}$$

wherein a, b and c are constant values similarly as in the previous equations.

From the interdependence between height and temperature, the horizon of the instrument as well as the change of the coefficient of refraction between forward and backward readings on the rod and also the refraction-borne error of height difference measured, can be determined. Correcting the measured value by the systematic error due to refraction theoretically permits a refraction error-free determination of the elevation difference.

From the mathematical point of view, the theoretical derivation of the correction for refraction may be considered to be exact, i.e., the neglect corresponds to the precision required, but it assumes the temperature function to be generally valid. This assumption may only be justified on the basis of mathematical statistics, and though the continuous variation of microclimatic conditions prevents it from reflecting the truth, on the basis of the law of averages it gives an exact idea of the trend of the temperature change as a function of height. In correcting for refraction, the main difficulty is that thermometry in two or three given levels depending on the form of the function simultaneous with the levelling work at an accuracy of at least $\pm$ 0.1 °C is a precondition.

This additional observation of the temperature significantly protracts the field work, and thus, it may indirectly be harmful to the levelling precision [8]. That is why, above all, levelling refraction remains unobserved in practical surveying.

5. Observation of the levelling refraction by systematic thermometry

For a closed geographic area, like that of Hungary, the expected value of the coefficient of refraction can be determined on statistical basis in dependence on the season and hours of the day. This determination requires a systematic microclimatic thermometry.

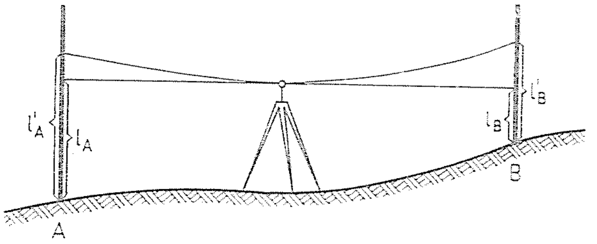

Fig. 1. Effect of levelling refraction on the height difference, in case of a negative coefficien of refraction . $\Delta m = l_A - l_B$; $\Delta m' = l'_A - l'_B$; $\Delta m' < \Delta m$

At the *Microclimatic Obsevatory* in *Erdőhát* of the *Department of Climatology of the Eötvös Loránd University of Natural Sciences*, air temperature is systematically observed in heights of 10, 50, 100 and 200 cm above ground level, eight times a day.

Selecting from among the observed data those relating to the months June and September of five years from 1963 to 1967, the mean values of temperature gradients in height intervals of 10 to 50, 50 to 100 and 100 to 200 cm have been determined. The gradient has been investigated directly in lieu of the temperature, because the former is less affected by local and timely temperature differences.

Separate data processing affected the mean values of gradients of temperature for clear and cloudy days, and for all of the days.

Brock's formula [9] was applied to determine the coefficients of refraction from the mean gradient values:

$$k = 5.03 \frac{B}{T^2} \cdot (3.42 - \gamma) \tag{8}$$

where B = air pressure in Hg mm,
T = temperature in °K

The very same result is obtained from the equation of PELLINEN [10]:

$$k = 668.7 \frac{B}{T^2} \cdot (0.0342 - \gamma) \tag{9}$$

where B = air pressure in mbars,
γ is understood in °C/m.

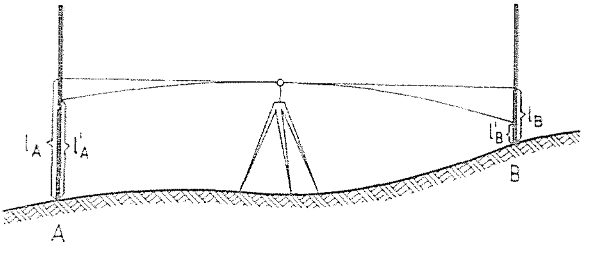

Fig. 2. Effect of levelling refraction on the height difference in case of a positive coefficient of refraction. $\Delta m = l_A - l_B$; $\Delta m' = l'_A - l'_B$; $\Delta m' > \Delta m$

From the September 7h a.m. values it is interesting to see that close to the ground level, the lower unstable stratum and the negative coefficient of refraction already begin to develop, whilst the higher strata are dominated by the inversion conditions, characteristic to the positive coefficient of refraction.

The factors of major significance, likely to modify the expected mean value of the coefficient of refraction are: permeability of the atmosphere, solar heat stored in the soil as a function of its thermal properties and of the overgrowth density.

Considering, however, that lines of precise levelling cross the verge of roads, earth roads, dams etc., where the influence of the soil and vegetation varies for each instrument position and levelling section, it is difficult to observe factors affecting temperature gradients, and use of mean values seems to be more practicable.

The tabulated values of the coefficients of refraction are, at any rate, a meteorologically justified basis for determining the levelling refraction values.

Correction for refraction of a height difference by precise levelling may be determined at an accuracy of calculation of 0.01 mm:

$$\Delta m_{\text{corr}} = \Delta m + \frac{d^2}{2r} (k_2 - k_1) \tag{10}$$

Table 1

Expected values of the coefficients of refraction

June Time h	Height range cm	Mean values of all the days	Mean values of clear days	Mean values of cloudy days
7	10— 50	—3.41	—4.75	—1.76
	50—100	—0.65	—1.87	—0.54
	100—200	—0.28	—0.88	—0.16
11	10— 50	—8.24	—10.92	—4.61
	50—100	—1.99	—3.00	—1.40
	100—200	—1.13	—1.51	—0.74
14	10— 50	—7.07	—9.76	—5.19
	50—100	—2.51	—3.20	—2.09
	100—200	—1.19	—1.62	—1.01
19	10— 50	—0.79	—1.48	—0.42
	50—100	—0.49	—0.76	—0.25
	100—200	—0.14	—0.55	—0.04
21		Isothermy		

Table 2

September Time h	Height range	Mean values of all the days	Mean values of clear days	Mean values of cloudy days
7	10— 50	—1.14	—1.50	—0.73
	50—100	—0.17	—0.63	+0.83
	100—200	+0.47	+0,33	+0.64
11	10— 50	—3.94	—5.50	—2.69
	50—100	—1.59	—1.74	—1.00
	100—200	—0.67	—0.90	—0.51
14	10— 50	—4.07	—5.60	—2.49
	50—100	—1.50	—1.60	—0.60
	100—200	—0.58	—0.68	—0.34
19		Isothermy		
21	10— 50	+0.71	+1.32	+0.45
	50—100	+0.56	+0.86	+0.35
	100—200	+0.37	+0.56	+0.30

k_2 and k_1 being the mean values of the critical coefficient of refraction at the instrument level in the backward and forward rod readings, respectively.

Thus, besides the easy treatment of the corrective equation, the disadvantage involved in the additional field temperature observation may be omitted and a higher precision obtained.

Summary

From among the meteorological factors causing levelling refraction, the change in temperature is of basic significance. The change in temperature, depending on the height, may be described by the vertical temperature gradient. The static equilibrium of the atmosphere and the curvature of the path of light, i. e., the shape of the curve of refraction depend on the value of the temperature gradient.

The refraction, the most dangerous source of errors in precise levelling, causes random and systematic errors. The effect of the systematic levelling refraction may be taken into account by the function of the temperature versus height, if, simultaneously with levelling, the air temperature is determined in two or three given levels. This additional thermometry, however, considerably protracts the levelling and calculation work, that is why it is commonly omitted in practice.

From systematic temperature data delivered by a microclimatic observatory, five years' mean values of temperature gradients in three different height ranges have been determined and the expected value of the coefficient of refraction calculated. An easy formula is suggested for taking into account the correction for refraction.

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