INVESTIGATIONS ON THE CHANGE OF PLUMB-LINE DEVIATION CAUSED BY CHANGE OF MASS AROUND THE POINT

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In many a problem of physical geodesy, taking into account the mass between the actual Earth surface and the chosen Earth model, or, in general, the effect of some excess or deficiency of mass causes difficulties. Theoretical correction formulae are applied to this purpose.

The result of an actual analysis will be presented below which has not required theoretical solution, on the contrary, data of an actual change of mass could be used as criteria for the theoretical solutions. Namely the change of plumb-line deviation in consequence of the change of mass around the point was investigated at the geodetic triangulation point named Ság-hill.

The Ság-hill is at the south border of the Kis-Alföld, about 2 km south-west of Celldömölk. Originally it was a nearly regular truncated cone basalt mountain, 150 m high above its environment, ending in an almost circular plane platform. The foot line of the hill is nearly circular, with a diameter of about 1.5 km. In the last few decades, up to the beginning of the fifties, a quarry was operated on the hill to exploit the good quality basalt and about 5 million cu.m of material had been removed. During this period the shape of the hill changed substantially, because the top basalt columns have been quarried. Average height of the hill in its present shape is about 40 m less than originally and even its highest point is 12 m lower than before.

There is a possibility to compute the change of plumb-line deviation given by several surveys made on the Ság-hill in the last 90 years, survey series comprising the operating period of the quarry. Survey data used for the calculations are compiled in Table 1. In addition, two topographic maps of Ság-hill and its environment have been available, one of them prepared in the second half of the last century by the K.u.K. Militär-Geographisches Institut to the scale (R.F.) 1 : 25.000, provided also with "key-lines" and contour lines, the other being a topographic map prepared in 1952, completed in 1966 by tacheometer survey results.

Of the available data, the plumb-line deviation change could be computed in different ways:

a) from geographical co-ordinates;
b) from difference of masses;
c) from differences of gravity anomalies.

Table 1

Date of survey	Result	Executed by
May 1884	Geographical latitude azimuth	K.u.K. Militär-Geographisches Institut
1891	Torsion-balance measurement at 6 points	Tangl, Kövesligethy, Bodola
1930—1950	Quarry operation	
1955	Geographical latitude, longitude, 2 azimuths	Enterprise for Surveying and Mapping, Budapest
1966	Relative gravity by gravimeter	Department of Geodesy (Varga) Budapest Technical University

1. Computation of the plumb-line deviation change from geographical co-ordinates

Our analyses could be based on the results of geodetic astronomy surveys for two different points at two different times. Survey in 1884 had been centred on the triangulation point TP_{1884} of Ság-hill. Thus, here the latitude of equipotential surface and the azimuth for TP of Magos-hill is known, as seen in Table 1, with a reliability of this latter given as $\pm$ 0.33″. The mean error of the latitude does not figure in the old record books. It has to be noted, however, that in 1884 the equipotential surface co-ordinates were not corrected with polar movement correction, unfortunately impossible to be posteriorly computed now [11]. The correction may be as high as about 0.1″ in latitude and 0.3″ in azimuth, either positive or negative.

Measurements in 1955 were centred to Ság-hill TP_{1951}, then, after recompleting the plane co-ordinates of TP_{1951} to the arisen TP_{1966}. TP_{1966} and TP_{1884} are by far not identical, the two points differ in height by about 12 m and horizontally by about 136 m.

The plumb-line deviation computed from the 1955 measurements can only be compared to the value calculated for 1884, by converting the two plumb-line deviation values for one and the same point. In our computations the measurement results of 1955 centred on TP_{1966} were converted to TP_{1884}. To calculate latitude and longitude values the table of ALBRECHT, and for transfer of the azimuths, relationships known from the literature were used.

A further correction was made because of the plumb-line curvature by means of the Pizetti formulae [11].

If the ellipsoid co-ordinates of TP_{1884} are known, components of the plumb-line deviation in direction north-south ξ and in direction east-west η can be computed from the survey results in 1884 and 1955, centred in both cases on TP_{1884}. Subtracting the corresponding components from each other, the required plumb-line deviation change is obtained:

$$\Delta\xi = \xi_{1884} - \xi_{1955} = -0.82''$$
$$\Delta\eta = \eta_{1884} - \eta_{1955} = -\,1.08''$$

2. Computation of plumb-line deviation change from the difference of masses

The method to compute plumb-line deviation is known from the literature [4], according to which mass inequalities, elevations and depressions on the Earth surface, referred to sea level, i.e., visible mass excesses and mass deficiencies are taken into account. In the present case, the method may be applied in a way that topography maps are constructed for both the condition before mass change and the actual state, computations are made for each separately; the difference of the two results is giving the change of the plumb-line deviation. The original method takes into account mass inequalities of the whole Earth, in present case, however, the computation has to affect only the immediate environment of the point, because the change of mass is there in a circle of 600 m radius.

The computation itself consists in splitting the mass into approximately prismatic bodies, by fictitious concentric cylinders, around the vertical of the tested point and with vertical planes intersecting that vertical, calculating the effect of each prism and summarizing them. With adequately close partitioning, the height of each prism is approximated by the average height above sea level of that plot. The lines of intersection of the plane of the map with the concentrical cylinders and the vertical planes can be plotted as circles and a row of radii passing through the point; this is a so-called raster. Dimensions of the raster are given in Table 2, according to the partition by Heiskanen and Vening-Meinesz, where each annulus is divided into equal parts, corresponding to the given number of compartments.

Computations have been carried out in horizontal sense for TP_{1884} using both maps, and the change of plumb-line deviation was obtained as the difference between values calculated from data of the two maps.

Hence:

$$\Delta\xi_t = -\,0.004''$$
$$\Delta\eta_t = +\,0.017''$$

Table 2

Circle symbol	Circle radius [m]	Number of compartments
A	2	1
B	68	4
C	230	4
D	590	6
E	1280	8
.	.	.
.	.	.
.	.	.

The outcome is seen to rather differ from that computed from the geographical co-ordinates. Looking for the cause of the difference it might be questionable, whether the estimated average height of the relatively large compartments was true enough for the rather indented relief. Control was carried out by determining the change of mass by planimetry and simple cubing from the two topographic maps and the same was done by using the area of the compartments and the respective heights above sea level. A difference of about 30% between the two mass differences resulted. This circumstance indicated a need of refinement for the computation method. It seemed to be expedient to densify the raster, in this way the area of each compartment decreased and the mean height above sea level of each compartment could be estimated more exactly. Densifying was achieved by increasing the number of compartments within each annulus. Contrary to the partition into equal parts of the annuli of the previous raster, this partition has been carried out according to the conditions:

$$(\sin \alpha_2 - \sin \alpha_1) = C_1 \text{ constant}$$

and

$$(\cos \alpha_2 - \cos \alpha_1) = C_2 \text{ constant, respectively.}$$

Choosing the constant values as $C_1 = C_2 = \frac{1}{8}$ a raster according to Fig. 1 was obtained. The raster to be used for computing component η is similar and only it has to be rotated by 90°. With the raster plotted in this way, the mean heights above sea level for each compartment had been evaluated again and then the change of pump-line deviation was computed, resulting in:

$$\Delta\xi_t = -0.046''$$
$$\Delta\eta_t = -0.012''$$

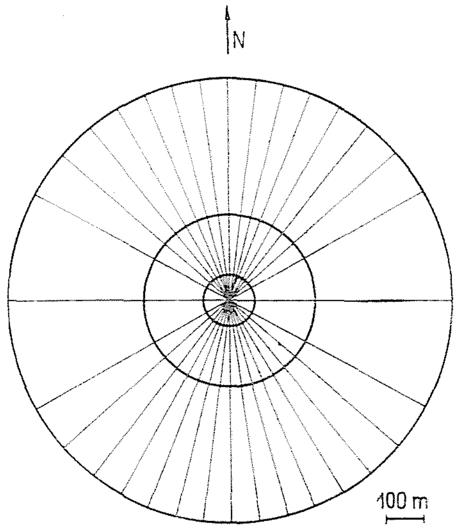

Fig. 1

A control computation has been carried out again, leading to merely half of the previous difference of 30%.

3. Computation of the plumb-line deviation change from gravity anomalies

In the computation based on gravity anomalies, the method elaborated by EREMEEV for solution of the Vening-Meinesz functions known from the literature was applied. This method lends itself to determine the components of plumb-line deviation pertaining to the given reference ellipsoid in any point, if the gravity anomaly and the position of the reference ellipsoid relative to the mass centre of the Earth are known. The co-ordinates between the reference ellipsoid and the mass centre of the Earth are unkown, so they are considered to be zero and with this supposition, the ellipsoid will be positioned earthly and the obtained plumb-line deviations will be absolute. In our case, however, the relative plumb-line deviation change is needed, because the results of the other two methods are the same. But considering that the absolute or relative deviation depends merely on whether the reference ellipsoid is absolute or relative, it is indifferent what reference surface is used for the computations, the change has to be the same in every case. Therefore, using the Eremeev method to compute the absolute plumb-line deviation change by means of the Faye-anomaly maps, computed and constructed from the gravity determined on the original and the actual Ság-hill, the results can be compared with the previous relative plumb-line deviation changes. Our scope is therefore to construct the two anomaly maps and to carry out the computations.

The Faye-anomaly map of the actual Ság-hill was plotted from the gravity measurements made by the *M.Á. E.L. Institute of Geophysics*, and from further densified gravimetric measurements. The resulting Faye-anomaly map covered an about 3 sq.km area of the Ság-hill, to a scale of 1 : 2000, with the average point density of 0.15 sq.km/point and distance of isoanomaly lines 1 mgal. Between points of the map of smallest and greatest anomaly, the anomaly difference is about 15 mgal.

For plotting the anomaly map corresponding to the original state of Ság-hill, results of the 1891 torsion balance measurements directed by Eötvös were used. The plotted map covers an area of about 3 sq.km of Ság-hill, to a scale of 1 : 2000, with the average point density of 0.17 sq.km/point, distance of isoanomaly lines 1 mgal. The anomaly difference between the point of smallest and greatest anomaly is about 18 mgal.

The Eremeev method substitutes the surface integration indicated in the Vening-Meinesz functions by summation, so that the area to be integrated is divided into compartments by means of concentric circles around the investigated point and a row of radii crossing the investigated point; the value behind the integral sign is computed for each compartment and values are summed up. The considered territory was also in this case at a distance of 600 m from the tested point, the mass changes being here located. As the radii of concentric circles of the Eremeev raster are of the km order, a raster with one tenth part of the original dimensions was constructed to the scale of the anomaly maps.

Before carrying out the computations, it was examined how the coefficients of the basic computation formulae were affected by the changed raster dimensions. It was found that the correct relationships corresponding to the raster adopted were identical with the originally deduced formulae, as the coefficients of the equations and the parts to be summarized did not include dimensions of radii of the circles, the computed values depending only on

Table 3

Annulus symbol	Inner radius m	Outer radius m	Number of sectors
I.	50	73	16
II.	73	107	16
III.	107	156	16
IV.	156	227	16
V.	227	332	16
VI.	332	484	16
VII.	484	707	16

Table 4

Computation data	$\Delta\xi$	$\Delta\eta$	$\mu_{\Delta\xi}$	$\mu_{\Delta\eta}$
Results of geodetic astronomy	$-0.82''$	$-1.08''$	$\pm 0.5''$	$\pm 1''$
Mass difference I	$-0.004''$	$+0.017''$		
Mass difference II	$-0.046''$	$-0.012''$		
Gravity anomalies	$-0.380''$	$+0.635''$	$\pm 0.1''$	$\pm 0.1''$

the mean values of anomaly read for the compartments, and on the magnitude of the azimuths pertaining to the radial median of the compartments [11]. Hence, computations could apply the original Eremeev functions, dimensions of the raster used are given in Table 3.

The change of plumb-line deviation was obtained as the difference of plumb-line deviation values computed, using the old and actual anomaly maps:

$$\Delta\xi = -\ 0.380''$$

$$\Delta\eta = +\ 0.635''$$

For the sake of comparison, the values of plumb-line deviation change obtained by different methods and their computed or estimated mean errors are compiled in Table 4. (To compute or estimate the mean error for values computed from mass differences was omitted, it being rather difficult.) Examining results obtained in different ways and taking into account estimated reliability values as well as circumstances referred to [11], it can be concluded that the quarry operation on the top of the basalt cone of Ság-hill might change the plumb-line deviation by a few tenth seconds. This statement is especially supported by the very low value of a few hundredth seconds of the plumb-line deviation change computed in two different ways from the quarried mass. It has to be noted that neither the difference between the values computed in two different ways from the mass differences exceeded a few hundredth seconds.

Comparing the magnitude of obtained results with the determination reliabilities, it may be stated that though there is a change in the numerical value of plumb-line deviation on the Ság-hill, but not more than just detectable.

Summary

An actual numerical investigation was carried out to establish to what extent the plumb-line deviation value was affected by mass change in the immediate environment of the point. Change of plumb-line deviation was computed by several methods:

a) from geographical co-ordinates;

b) from difference of mass;

c) from the difference in gravity anomalies.

Analyses led to the statement that in the given case the mass change of about 5 million cu.m produced in the plumb-line deviation a change of the order of the probable reliability of the computation.

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