DETERMINATION OF THE OPTIMUM LIFT OF PUMPING STATIONS BY MEANS OF DYNAMIC PROGRAMMING

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Determination of the optimum lift of pumping stations feeding conduit systems is based upon economic technical dimensioning methods such as the marginal programming, the method by sections, and the linear programming. In this paper, the technique of application and the possibilities offered by a new method, that of dynamic programming [2, 3], in the practical economic

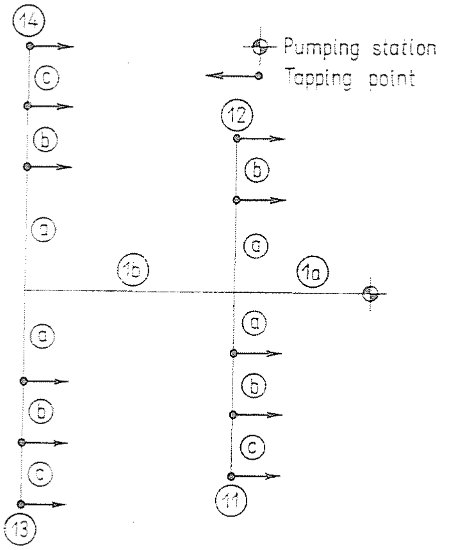

Fig. 1

dimensioning of branching pipe networks and in determining the optimum lift of pumping stations is presented.

To find the optimum lift of the pumping station for the conduit system shown in Fig. 1, the variation of the optimum capital investment for the construction of the network in dependence of the lift of the pumping station and of the pressure loss in the conduits is to be determined.

In designing the network, all the pipe diameters should be accounted for conveying the discharges of each section within certain limits of velocity (for example, in Hungary, 0.5 to 2.5 m/sec in sprinkler irrigation). This sets limits also from the point of view of the head loss in the conduit system.

At the tapping points, a service pressure depending on the function of the network should be produced. The head loss permitted in the network up to the tapping points is the difference in pressure between that existing in the delivery conduit branched off the pumping station and that to be maintained in service.

Applying the minimum diameter permissible in all of the sections of the network, the head loss in the conduits and as a result, the required lift of the pumping station will be the maximum (H_{max}), and the construction cost of the conduit system the minimum (B_{min}), and vice versa.

If the lift of the pumping station is of some intermediate value, the network may be composed of conduits of very different diameters. Nevertheless, for every case a combination of diameters, optimum for construction costs, can be found. The co-ordinate values $H-B_{min}$ so obtained are located on a polygon, termed the polygon of minimum cost, which will be verified in the following.

1. Construction of the polygon of minimum cost of a network

Let us establish the polygon of minimum cost of the conduit system in Fig. 1 by dynamic programming starting from the point corresponding to the maximum pumping lift.

Reduce the lift of the pumping station by a value Δh, and increase accordingly the conduit diameters stepwise to obtain the optimum cost of construction of the conduit system, all along the polygon of minimum cost.

As a matter of course, the conduit diameters are to be changed so that the required service pressure is realized at every critical tapping point. Higher pressures than required are admitted if no pipes of smaller but just sufficient diameter are available.

Definitions and symbols:

section: conduit between tapping point and branch point, between two tapping points or branch points;
branch: conduit without laterals;
critical route: conduit between the pumping station and tapping points;
policy: possibility to change pipe diameter so that the head loss uniformly changes along each of the critical routes (safe in lack of pipes of smaller diameter);
optimum policy: policy resulting in optimum cost;
subpolicy: policy along one conduit section or branch;
optimum subpolicy: subpolicy resulting in optimum cost;
ir = mark of the policy;
α_i = change in construction cost of the conduit system for unit change in the lift of the pumping station, i.e., absolute value of the slope of the polygon of minimum cost in the case of the i^{th} policy;

k = mark of branches or sections where the i^{th} policy implies to change the pipe diameter;
α_k = change in the investment costs per unit head loss upon replacing the n^{th} diameter by the $n+1^{st}$ one in section k or in one section of the k^{th} branch:

$$\alpha_k = \frac{\Delta b}{\Delta \varepsilon}$$

where

$$\Delta b = b_{n+1} - b_n,$$
$$\Delta \varepsilon = \varepsilon_n - \varepsilon_{n+1}.$$

From the foregoing it follows:

$$\alpha_i = \sum_k \alpha_k .$$

Evidently, starting from the alternative with the least diameters, the condition of policies and subpolicies to be optima is to minimize the increase in capital investment per unit head loss (or lift of the pumping station), i.e., the coefficients α_i and α_k.

For one *conduit section* as many subpolicies may exist, as many conduit diameters can be substituted for the given diameter. If a smaller diameter is to be replaced by a larger one, then the condition of the optimum subpolicy is:

$$\alpha_k = \text{minimum}.$$

Within a branch, for every conduit section there is an optimum subpolicy i.e. a minimum diameter. Thus, for the given branch, the optimum subpolicy consists in changing the diameter along that section where there is a minimum increase in capital investment per unit head loss reduction upon increasing the diameter.

The conduit system shown in Fig. 1 is built in a nearly plain area. It has four critical routes which consist of the following sections and branches:

Critical route 1: 1*a* + 11
Critical route 2: 1*a* + 12
Critical route 3: 1*a* + 1*b* + 13
Critical route 4: 1*a* + 1*b* + 14

The conduit diameters should be altered so as to provide a uniform change of head loss on each of the four critical routes. Accordingly, three main policies i.e. diameter changes along the following sections and branches may be selected:

Policy 1: 1*a*
Policy 2: 1*b* + 11 + 12
Policy 3: 11 + 12 + 13 + 14

These three policies consist of subpolicies, because the branches are composed of sections. Now, an essential theorem of dynamic programming is that an optimum policy must consist of optimum subpolicies (evident in our case).

For the network as a whole, the optimum policy is where

$$\alpha_i = \text{minimum}.$$

This policy may be found by considering the previous policies in the occurrence of optimum subpolicies.

The respective α_1 factors of the three policies are:

$$\alpha_1 = \alpha_{1a}$$
$$\alpha_2 = \alpha_{1b} + \alpha_{11} + \alpha_{12}$$
$$\alpha_3 = \alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14}$$

Each of the policies is made up of as many optimum subpolicies, as there are sections where the adoption of the policy involves diameter changes. (In the above relationships the α_k factor of the optimum subpolicies should be substituted.)

Consequently, the optimum policy is that with the least α_i factor. At the same time this is the absolute value of the slope of the polygon of minimum cost.

According to the principle of the lowest ascent, this slope can be followed to the next break point corresponding to the adoption of the larger diameter all along a section pertaining to a subpolicy of the optimum policy.

Now, the optimum subpolicy to be adopted, hence the optimum policy for the conduit system, should be found as shown in the foregoing, by following the corresponding slope.

By this means, the polygon of minimum cost of the conduit system can be stepwise produced to lead to the point with the co-ordinates $H_{\min}$, $B_{\max}$.

The computation may also be started from this point. Then, the principle of the steepest slope is valid, and the optimum policies are defined by the condition

$$\alpha_i = \text{maximum}, \quad \text{i.e.,} \quad \alpha_k = \text{maximum}.$$

The problem is a rather cumbersome one for manual calculation, therefore computer programs have been developed, already applied to plot the polygon of minimum cost for hundreds of conduit systems.

2. Determination of the optimum lift of a pumping station

For determining the optimum lift of a pumping station, the construction cost of the conduit system and the cost of pumping are to be confronted. This is advisably done by the common method of comparison of technically equivalent project alternatives, stating that the economically most favourable project is where the cost of water delivery

$$K = B + T\ddot{U} = \text{minimum}.$$

Herein:

B = construction cost of the conduit system;
T = local refund standard limit;
$\ddot{U}$ = annual cost of pumping, expressed as:

$$\ddot{U} = C_1 \cdot Q \cdot H$$

where

Q = quantity of water lifted by the pumping station;
H = lift of the pumping station;
C_1 = yearly average cost of lifting unit volume of water to unit height.

The delivery cost of water can also be expressed as:

$$K = f_1(H) + f_2(H)$$

where

$$f_2(H) = T \cdot C_1 \cdot Q \cdot H = C \cdot H$$
$$f_1(H) = B\,.$$

This latter function is the polygon of minimum cost of the conduit system. It is to be determined in the way described above, by means of dynamic programming, using a computer.

According to the above it is clear that the function $B = f_1(H)$ is a polygon made up of straight runs even for quite complex networks.

The function $f_2(H)$ is linear, thus the sum function also consists of straight runs:

$$K = f_1(H) + f_2(H).$$

Consequently, the optimum permissible head loss has to be equal to a co-ordinate of a break point of the polygon $B = f(H)$ of minimum cost. In special cases where the lowest run of the polygon of minimum cost is horizontal, an arbitrary H value at one of or between the two break points is the optimum.

This may occur at certain values of the C factor. Be H_n and H_{n+1} coordinates of two adjacent break points of the polygon of minimum cost and B_n, B_{n+1} the respective construction costs, then the special case mentioned above occurs if:

$$B_n + CH_n = B_{n+1} + CH_{n+1}$$

Hence, the critical value of the C factor:

$$C = \frac{B_n - B_{n+1}}{H_{n+1} - H_n}$$

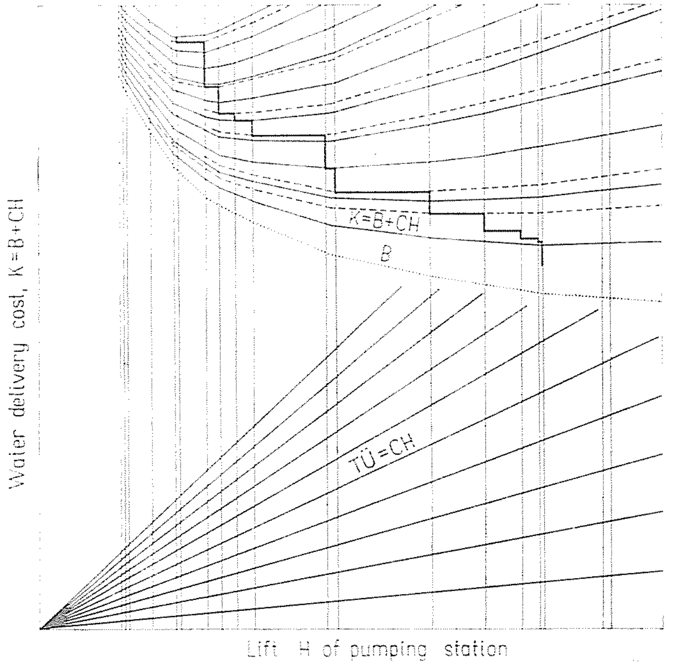

Fig. 2

The stepwise diagram in Fig. 2 shows the variation of the optimum H as a function of C, for a given conduit system.

In the case of a given project, it is rather difficult to determine C (owing to the implications involved in the estimation of the yearly average number of the service hours, of the effectiveness of the pumping stations etc.). It is more feasible to determine the range of possible C values and their effect on H optima. In our opinion, from among pumps of different lifts available, that one will be the most economical where H is as near to the upper limit of the optimum range as possible, it involving the lowest immediate construction costs.

If the optimum lift of the pumping station is to be determined for a single C value, then the polygon of minimum cost can be established by adding the C value to the optimum policy α_i, directly resulting in the curve of total cost $(B + CH)$ and the computation ends at the optimum value of H.

3. Other uses of dynamic programming for branching networks

The dynamic programming may be applied to several interesting analyses to be described below.

Is the given combination of conduit diameters the optimum?

The outlined features of the polygon of minimum cost imply the statement related to its break points:

$$\overleftarrow{\alpha_i} > \overrightarrow{\alpha_i}$$

where $\overleftarrow{\alpha_i}$ and $\overrightarrow{\alpha_i}$ are factors corresponding to the optimum policy in the direction of smaller and higher head losses, respectively.

Thus, starting from the corner, a steeper ascent may be applied along the polygon in the direction of lower, than of higher head losses.

Setting out of the combination of pipe diameters considered, the values

$$\overleftarrow{\alpha_i} = \text{minimum} \quad \text{and} \quad \overrightarrow{\alpha_i} = \text{maximum}$$

representing the optimum policy should be determined. If these values satisfy the outlined condition, then the given combination of pipe diameters is the optimum.

Finding the optimum starting from an arbitrary combination of conduit diameters

The arbitrary combination of pipe diameters is not understood in a quite general way but assuming that the head loss permitted by the lift of pumping and the required pressure along the critical routes have been considered in design.

The optimum belonging to a given pumping lift will be found either in direction of the lower or the higher head losses with an

$$\text{ascent } \overleftarrow{\alpha_i} = \text{minimum or}$$

$$\text{slope } \overrightarrow{\alpha_i} = \text{maximum},$$

respectively.

The first step will be to determine the limiting values of the α factor. If these satisfy the condition

$$\overleftarrow{\alpha}_i > \overrightarrow{\alpha}_i$$

then the solution selected is the optimum.

Otherwise the set of the feasible solutions will be followed to the first corner according to the factor $\overleftarrow{\alpha}_1 = \min$.

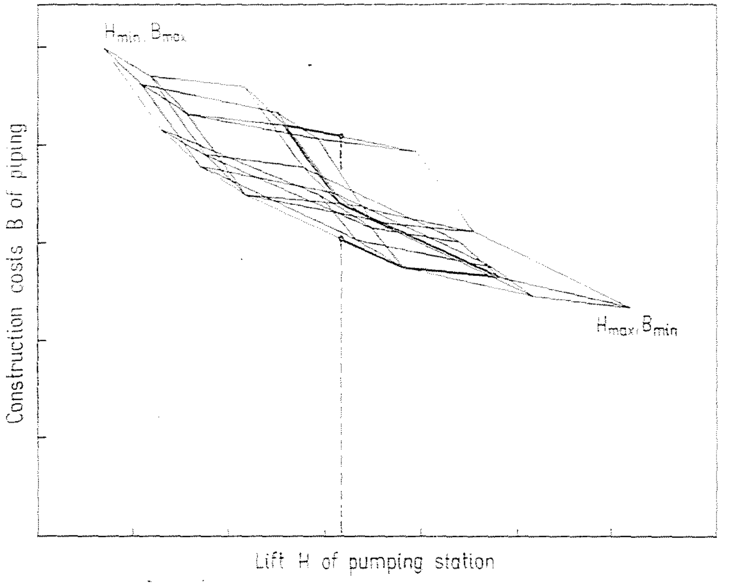

Fig. 3

Here the factor $\overrightarrow{\alpha}_i$ = maximum should be determined and this policy followed to the next corner. If the pertaining head is lower than H, then the factor $\overrightarrow{\alpha}_i$ = maximum should be recalculated and the process continued. The corner above H will be left according to the condition $\overleftarrow{\alpha}_i$ = minimum.

This alternation should be repeated until

$$\overleftarrow{\alpha}_i = \overrightarrow{\alpha}_i$$

giving the optimum solution.

The method is illustrated in Fig. 3.

The set of coupled $B - H$ values corresponding to possible combinations of diameters in the conduit system is represented by a conjugate set of diagrams bordered below by the polygon of minimum cost.

Fig. 3 is related to a simple conduit network, in more intricate cases it is difficult to produce such a diagram, owing to the great number of alternatives. Each of the straight runs between the diagram corners represents a policy.

Summary

Dynamic programming lends itself to an economical design of pipe networks and determination of the optimum lift of pumping stations, permitting

— to establish the polygon of minimum cost;

— to decide whether a given combination of pipe diameters is the optimum or not;

— to find the optimum solution starting from an arbitrary combination of pipe diameters.

The author directed the development of programs in linear, sectional and dynamic systems for various computer types to economically design conduit networks and to determine the optimum lift of pumping stations. In recent years, these have been applied to design sprinkler irrigation systems over about 20,000 hectares of area. In some cases, savings over 20 per cent of the construction costs of the conduit network have been achieved by selecting the optimum solution from a number of routing alternatives.

Investigations confirmed dynamic programming to be a highly efficient method for establishing the polygon of minimum cost, hence, for determining the optimum lift of the pumping station.

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*In Hungarian.

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