SOME PROBLEMS OF MODELLING REINFORCED CONCRETE STRUCTURES

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General on modelling

If certain properties of a phenomenon V (thing, event, set of relations, etc.) decisive for its analysis correspond in turn to each of the appropriately selected properties of a phenomenon M, and if the same quantitative correlation can be established between the corresponding properties of the phenomena M and V, then the phenomenon M is the *model* of V. The properties correlated are referred to as *analogous properties.*

If the quantitative relations between the decisive properties of phenomenon V may be correlated to quantitative relations between mathematical concepts, then these latter constitute with their relations the "immaterial model" of the structure. As a matter of fact, all kinds of design calculations may be considered as an analysis of the mathematical model developed on the basis of assumed properties of the designed structure.

A mathematical model is in every respect advantageous because it permits to make use of the simplifications offered by its immateriality. Thus, for example, stresses in simple bar-systems can, of course, ever be determined on "immaterial models". However, most of the advantages of the mathematical model become illusory if the relations between the analogous properties are quantitatively unreliable, hypothetical or of restricted validity. Neither can the model immateriality be made use of — although analogous relations subsist — if the mathematical model is too complex to determine the relations needed for the design.

Though the event of computers largely extended the possibilities of immaterial mathematical models, this trend of development results by no means in reducing the uses of "realistic models" constructed of some kind of material. On the contrary, the progress in electronics, the new results of automation, data processing and electrical metrology multiplied the efficiency and applicability of model tests at least as much as those of mathematical methods.

This paper is concerned with problems related to the "realistic model" analysis of common reinforced concrete structures in order to find solutions — of course, without aiming at completeness — helping the research engineer to decide over the material and the scale reduction for the model of reinforced concrete structures.

Engineering design — mathematical model of structures

The stages of development of mathematical models

Design of engineering structures consists of the following operations:

— On the basis of technological, service, etc. requirements for the structure, the design values of the static and dynamic loads acting on the structure, as well as the unfavourable but probable variations of these effects are determined.

— On the basis of favourable or unfavourable observations on structures of the same function, the basic arrangement of the structure (possibly a few alternatives considered to be equivalent) and the approximate values of the significant dimensions of the structure are assumed.

— In possession of the characteristics of the structural materials the "static model" of the structure is constructed. The static model is a model of idealized material subject to idealized loads in which the analogous properties i.e. the loads (effects) considered to be substantial may be correlated with those of the actual structure, at a sufficiently close agreement from the aspect of technical requirements.

— If the loads and stresses of the structural model are related by essentially mathematical methods of structural engineering, these relations expressed in dependence of the decisive data of the structural model yield the mathematical model of the actual structure.

— In possession of the mathematical model the alterations in the assumed structural dimensions and system needed to assure the required load capacity are determined (making use of the structural model).

Idealization and neglections in developing the mathematical model

Let us consider now the neglections introduced to the mathematical model of the structure needed by ease of handling or even by feasibility aspects.

— The loads acting on the structure are random variables forming a stochastic system of values from either magnitude, pattern or acting time aspects. Their design values may, even in case of a detailed analysis, be given as the mean values biased by the effect of the variations.

— The same considerations apply to the structural dimensions and even more to physical properties of the structural materials, especially to the physical characteristics of the concrete in reinforced concrete structures.

This effect should be considered in each case in selecting the structural material and the basic structural system.

In constructing the structural model further neglections have to be introduced.

— No exact physical properties of the structural materials can be reckoned with, even in the sense that, without making allowance for the variations, one considers the mean values to be exact. Namely, the structural material behaviour laws — first of all those of the concrete — are not yet known exactly, and the relationships describing the laws in accordance with our knowledge of materials and the actual accuracy of metrology would lead to a very intricate, mathematically untreatable structural model even for the simplest structural system. Thus, to obtain a utilizable mathematical model, only the idealized material behaviour laws, obtained by simplifying the most substantial material behaviour laws, may be applied.

— Except for the simplest cases, it is hardly possible to construct a structural model corresponding in every detail to the actual structural system, irrespective of the idealization. The structural model involving simplifications based on an "engineering mind" trained on practice will be though more comprehensible, and will contain the analogous properties important for the design at a due accuracy. (For example, three-dimensional frameworks mostly are modelled simply as plane frameworks easier to use, neglections causing but slight errors as compared to three-dimensional frameworks.)

— Mathematical models are hardly built up of exact relationships between the analogous properties of the structures; easy-to-treat mathematical relationships are applied. Sometimes, even the mathematical model composed as a system of simplified relations is to be treated by an approximate mathematical method to yield the required new relationships.

Restrictions of the mathematical model

In reference to the preceding chapter, the correlation between the structure made of a real material and its model may be realized at a closer or rougher approximation depending on the rate of neglections. In some cases the deviations are as important as to lead to qualitatively erroneous conclusions on the actual structure. Namely:

— The ideal material behaviour law of the structural model cannot be correlated to the real material behaviour laws. This is a frequent case in problems of deformation, stability and ultimate condition of reinforced concrete structures.

— The system of a structural model cannot be correlated to that of the actual structure. This is true, first of all, for such cases where the effects considered as negligible from engineering aspects are of the same order of magnitude as those considered to be primordial for the particular structural system or the rather unusual structural dimensions and proportions.

— Loads and effects ignored in idealizing the loads acting on the structure induce effects of the same order as the idealized loads. Most of the similar sources of error are due to the repetitive character of the loads and to the wrongly neglected dynamic effects.

— The results obtained by the approximate analysis of the mathematical model otherwise correctly developed involve significant deviations from the mathematically exact solution. The reason for this commonly is that sufficiently exact calculations require different approximations in analyzing the different effects (a more or less dense network of differences, allowance for a different number of terms of the infinite series of functions, application of a different number of valued numerals in cases of small differences between great numbers, etc.).

Models from real materials

Uses of model structures made with real materials

Each of the sources of error described above may strongly restrict the applicability of the mathematical model. Undoubtedly, the errors may be reduced by applying more complicated structural or mathematical models, even below the permissible value, but in the numerical analysis of the too complicated mathematical model, the deviations caused by the inevitable neglections limit the possible accuracy of the mathematical model. This is why the tests on other than immaterial models are preferred in spite of the rapid development of the analyses by mathematical models.

In general, complicated mathematical models pertain to plane and space structures: slabs, discs and shell structures. Practically, these analyses always involve the solution of partial differential equations of high order. In most cases, the solution of these differential equations of simple structure encounters difficulties if great many unknowns are to be considered in the numerical solution to obtain a sufficiently exact result. Commonly, only a few of the unknowns determined in the problem are taken as design values, therefore, in such cases it is often more comfortable and economical to resort to model tests confined to the determination of the design stresses.

New structural engineering problems required the introduction of a number of new structural designs. Significant deviations from the common dimensions and systems of structures gave prominence to effects hitherto ignored or omitted (for example, stability problems, plastic behaviour of structural materials, problems of rheology, etc.). Also to elucidate these phenomena, it is advisable to develop material behaviour laws and mathematical models based on model test results.

Extension of our knowledge in material behaviour, and progress in structural design and in computation technique demand the development of ever new mathematical models. Model tests are also useful to check the validity and limits of new methods of computation, and to establish the applicability of structures designed by the new procedures.

Model test problems

The listed manifold uses require, of course, model types adapted to the specific problem. In general, the structural models belong to three large groups.

1. Models simulating the material behaviour.
2. Mechanical models without material similitude.
3. Other models without material similitude.

— On models in the first group, those problems are analysed where material behaviour laws should exactly be considered. Thus, tests exploring phenomena peculiar to reinforced concrete structures, such as formation of cracks, ultimate load capacity, creep etc. apply models simulating material behaviour. Nevertheless, material similitude does not necessarily mean identity between materials of model and structure: on the contrary, for reinforced concrete structures, the identity between materials of model and structure will be seen to result in general, in different material behaviour.

— On models in the second group, problems are analysed where only structural correspondence between the original structure and its model is required. In such cases it is implicitly assumed that the material behaviour laws of both the prototype and the model structure may be replaced by identical idealized material behaviour laws. These models often lend themselves to avoid mathematical models requiring extensive calculations, or to determine the optimum proportions of the structure.

— Models of the third group may be used in cases where the mathematical models of the original structure and the model phenomenon are the same. These models always contain the effects of the neglections made in constructing the mathematical model, their application is justified by metrology advantages and, in some cases, by the possibility to simply and continuously vary the parameters. Such model types without material similitude consist, in general, of electric and electronic units, wherein the analogous properties are electric quantities ready to measure: voltage difference, current intensity, ohmic resistance, impedance, etc. Simple model tests based e.g. on the soap-film analogy, sandhill analogy, etc., belong also to this group.

In the following, only models belonging to the first two groups will be dealt with. The problems of application of models in the third group — after the development of the mathematical model — are rather electric and metrology problems.

Material similitude models of reinforced concrete structures

Material similitude characteristics of reinforced concrete

— The idealized design material properties should more or less approximate the properties of the actual materials. For the concrete in reinforced concrete structures, the following material characteristics are assumed to be known:

E_{b0}	Young's modulus of elasticity (initial value);
ν	Poisson's ratio;
σ_p	compressive strength of concrete (prism strength);
ε_u	ultimate compression of concrete;
σ_t	ultimate tensile stress;
$\varphi(t)$	coefficient of creep;
$\varepsilon_{sh}(t)$	specific shrinkage;

Reinforcement characteristics:

E_a	modulus of elasticity;
ν	Poisson's ratio;
σ_A	limit of proportionality;
σ_y	yield point;
σ_B	tensile strength;
ε_B	ultimate tensile strain;
ψ	coefficient of contraction.

The effect of creep in the reinforcement is mostly neglected, except for prestressed structures.

Let us see now the consequences due to differences in the above concrete characteristics for two beams of the same structure and load.

— If only the moduli of elasticity differ for perfectly crackless structures under identical loads, then the deformations differ proportionally to the initial values of the moduli of elasticity, this proportion, however, may already be altered for rather small loads. Namely:

a) the stress-strain diagrams of concretes with different initial moduli of elasticity E_{b0} deviate in different ways from the linearity according to Hooke's law;

b) due to second-order effects, the deformations do not depend exactly on the first power of E;

c) the stiffening effect of the reinforcement becomes manifest.

In the case of small loads neither of these effects are of interest, and may be ignored in practice, i.e. idealized quantities may be introduced.

— Differences in only the Poisson's ratios leave bar systems inaffected, except for the interaction between concrete and reinforcement, to be discussed later. Plane and space structures are, however, much affected by ν since:

a) it strongly influences the magnitude of deformations (and also their proportion if second order effects occur),

b) it significantly influences the development of stresses.

Since the formation of cracks in, and the getting into plastic state of concrete is considered to be bound to certain characteristic stress values (in accordance with the knowledge in material behaviour), identity between Poisson's ratios of models similar in material and of the original plane or space structure is a must.

— If only the σ_p value is different, a perfect material similitude can only be realized if the limit of plasticity is reached nowhere in the structure. This means, in general, that application as a model similar in material should be restricted to the investigation of structural cracks, not concomitant to plastic deformations.

Later on, possibility of models of material similitude made of materials of different limits of plasticity will be demonstrated.

— If only the ultimate compression ε_u differs, the material similitude may practically be assured up to the last stage of beam loading. From practical aspects, model tests are also valid in the last stage, but where the deformations at failure or secondary effects (e.g. arch-action in beams and plates) are investigated, the failure patterns of model and original structure may significantly differ, because the simulation of material behaviour is imperfect at failure.

— If only the σ_t values differ, then the cracking loads will be different. Accordingly, the cracked structures reach the plastic state for different crack patterns. From the development of cracks to that of plastic deformation, however, the behaviour of the two structures will be similar. Bearing in mind that owing to the plastic deformations, the crack widths are largest in regions in the plastic range, the internal stresses of the structure are becoming more uniform in the pre-ultimate load stage, and in the ultimate stage they can be considered to be uniform. In a structure developing significant second-order effects in the ultimate load stage, this identity will only be approximated because the initial cracks may strongly influence the final failure pattern of the structure subject to arch-action.

— For permanent static loads, the differing coefficients of creep represent a divergence from a model true to material. Though for instantaneous loads this divergence causes a negligible difference, its effect on the ultimate condition must not be left out of mind. For an important creep, much of the second-order load capacity excess due to concrete compression will be absorbed by the deformations. This phenomenon may only be analyzed by means of a true-to-material model with the same coefficient of creep $\varphi(t)$ as that of the actual structure.

— The difference between specific values and histories of shrinkage deformation represents a deviation from the material similitude for models of structures under permanent load. Use of a model of the same shrinkage $\varepsilon_{sh}(t)$ as that of the actual structure may be of special importance for analyses of the ultimate conditions of cracking.

The factors affecting the material similitude between reinforcements are as follows:

— the modulus of elasticity varies in relatively narrow limits, for mild steel wires it may be considered constant;

— the very same is true for the σ_A, σ_y and σ_B values of the reinforcement, although rolling and cold working may considerably influence first of all the limit of proportionality and the yield point;

— knowledge of the exact values of σ_B and ψ may be important for the cease of interaction between concrete and reinforcement at the ultimate condition; to our knowledge, however, no experiments yielding unambiguous results on this effect have been made yet. A forced ignoration of this effect, also influencing the material similitude, is, at all events, a source of errors in experiments on reinforced concrete models;

— a number of experiments have been performed for the determination of the effect of the Poisson's ratio, first of all for prestressed structures. In view of the fact that the Poisson's ratio of reinforcing steel is nearly constant, it is omitted from among the factors affecting the material similitude.

Criteria of material similitude

The deviation from material similitude was seen to depend on different material properties in different loading stages.

Conditions of a perfect material similitude for reinforced concrete structures are:

a) identity between Poisson's ratios of the materials of structure and model;

b) identity between specific failure deformations at failure of materials of structure and model;

c) constant ratios of material stress characteristics and moduli of elasticity of structure to model;

d) similar functions and equal final values of creep coefficients and of shrinkage deformations.

From the criteria it is evident that the perfect material similitude is conditioned by strict requirements, difficult to be met. If, for example, the reinforcement of the model consists of steel wires, all the other material properties should correspond to the concrete characteristics. The material suitable for modelling and having the same characteristics as the concrete, satisfying more or less the requirements of model construction, is the *micro-concrete*, with certain substantial physical properties corresponding to those of the concrete, provided the mixing, compaction and curing instructions have been strictly observed. Plastic materials may lend themselves for concrete modelling as materials with organic binders. To our actual knowledge, however, the only modelling material suitable for reliable simulation of material behaviour of reinforced concrete structures, is reinforced concrete itself.

Effect of scale reduction on material behaviour

Many factors affect the strength properties of concrete, accordingly, no functional relation can be established between effects and strength characteristics. A uniform effect of factors governing the strength of concrete may be reached by the application of uniform concreting technology, exact dosage and careful curing. Tests done under such conditions show strength characteristics of concrete to be rather sensitive to form and scale. The smaller the scale of the model, the more the material strength characteristics of the actual structure and model deviate. Size of the scale effect may though vary, with scale reduction the strength values definitely increase, e.g. as much as 20 per cent or so for a scale 1 : 5.

In the case of concretes of the same grading, the maximum grain size imposes a natural limit to scale reduction. A model smaller than one fifth of the actual size may only be constructed from a concrete of special grading of from micro-concrete. The properties of the micro-concrete may largely differ from those of the actual structure; with this kind of concrete the simulation of material behaviour is only partial. Further reduction, e.g. 1:20 to 1 : 25, may involve difficulties even for micro-concrete. The closer the maximum grain size to the least dimension of the structural element, the more the strength values of the structure scatter and the more the failure pattern is decided by local concrete imperfections and discontinuities. Therefore no reinforced concrete models below a scale 1 : 20 are used in practice.

Scale groups of models

In accordance with the above statements the models may be divided into groups as follows:

- large-scale models (1 : 1 to 1 : 5);
- middle-scale models (1 : 5 to 1 : 20);
- small-scale models (below 1 : 20).

Large-scale models permit a practically perfect simulation of material behaviour. The behaviour of the actual structure under load may better be determined by such model tests than by calculation procedures. Analysis of the special effects developing in the actual structure, as well as determination of the ultimate load capacity is only possible on large scale models at a sufficient accuracy. The only, though decisive disadvantage of large-scale model tests is their costliness.

Middle-scale models lend themselves to model tests of partial material similitude. Strength properties of micro-concrete being the same as those of concrete, middle-scale models may be advantageous for theoretical investigations into special strength and structural problems of reinforced concrete structures, because the calculation procedures (mathematical models) developed for the model may also be applied for the actual structure made of the real material replacing, of course, the material characteristics of the micro-concrete by those of the concrete to be applied.

— With *small-scale models*, practically no model tests of material behaviour simulation are possible, at most up to the limit of the elastic range or at quite rough qualitative correlations. In many cases, however, where the details of the structural behaviour are absolutely unknown, such model tests yield useful information for further studies. The wide scope of application of small-scale models will be discussed below.

Models without material similitude

Selection of the analogous physical properties

Models not simulating the material behaviour are mostly used for the analysis of structures in the elastic range. Accordingly, the correlated properties are loads acting on the actual structure and on the model, the elastic material constants and elastic deformations. Since the correlation is only limited by the linearly elastic behaviour of the materials, the scale of the model may be selected at will, and also the scale of loading may vary in wide limits, independently of the scale of geometric simulation.

Viewpoints in selecting the material of the model are:

a) behaviour according to Hooke's law of linear elasticity over a loading range as wide as possible;

b) low modulus of elasticity;

c) absence of anisotropy and internal stresses; and

d) workability and joinability.

Electric strain gauges are to be used on materials of good thermal conductivity.

The scale of geometry and load of the structural model are to be selected in accordance with the following considerations:

a) Inaccuracies in the construction of the model should not affect significantly the stress distribution.

b) Possibly simple instruments should suit to determine deformations of the model within their range of reliability.

c) Simple equipment should suit either continuous or gradual loading.

Substantial requirements are reproducibility at any time, especially in the case of a large number of measurements; rapidity (and automaticity); evaluability; and prevention of other effects than the analogous properties from affecting the results.

Applied materials

Materials more or less meeting these requirements are the metals, with the disadvantages, however, of relatively high moduli of elasticity and, in some cases, a poor workability. Provided careful work and design subsist, models constructed of metal — commonly of steel or aluminium — afford the most exact results at a sufficient elasticity. Models constructed of plastics, first of all of plexiglass and celluloid have several advantages, the most important ones being an easy workability and a low modulus of elasticity. As against metals, they have the disadvantages of important creep, low limit of proportionality and poor thermal conductivity.

Asbestos cement sheets are highly convenient for models of plate structures. Asbestos cement unites certain advantageous properties of metals and plastics at a low price, and sheets suitable for the construction of models are always available.

Notice that for investigations in the scope of models without material similitude, in general, models of concrete or micro-concrete cannot be used. Namely:

a) Formation of hair cracks during setting and hardening disturbs the material isotropy.

b) Beyond the relatively low ultimate tensile stress, the structure does not behave in accordance with the assumed elasticity.

c) Relatively long bases depending on the grading are needed to determine average specific deformation at a sufficient accuracy.

Problem of model selection

Complex model test programs

In design work one often has to decide over solution alternatives likely to yield the accuracy needed from technical aspects at the minimum cost. After surveying the applicability of model tests certain facts should be mentioned, often ignored in selecting the appropriate method.

Mathematical models requiring voluminous computation may mostly be replaced by structural models of appropriate scale and degree of similitude. The more so since the model is simulating automatically most of the second-order effects.

In order to simplify the calculations, design for each effect is often done separately. Although model tests lend themselves to analyse simultaneously all effects one can proceed — again for the sake of simplicity — in modelling only certain parts or properties of the actual system.

In such cases, the not simulated properties may be predicted, in general, by means of simpler, yet sufficiently accurate calculations or separate model tests.

The most effective combined model test programs are:

— Analysis of the stress distribution in the structure on small or middle scale elastic models, checking the load capacity of the structural members critical for the structure as a whole on large or middle-scale models.

— Analysis of the stress distribution in the structure according to the theory of elasticity, checking the load capacity of the structural members subject to stress maxima by model tests.

— Determination of stresses in the structure by using an elastic model and simple calculations based on the test results. The load capacity of the structural members critical for failure of the structure is to be checked by calculation or model test simulating material behaviour.

— Automatic determination of stresses in, and load capacity of the structure on small or middle-scale elastic models, directly computer processing the test data.

In the following, three model test programs will be reported of, done at the Department of Reinforced Concrete Structures, Budapest Technical University. The tests aimed at helping design offices in decisions over particular design problems related to constructions, some of which have been erected since.

Analysis of the arch-action in flat slabs by tests on a model of material similitude

Scope and problems of the model test

The load capacity of reinforced concrete slabs, on certain boundary conditions, is known to exceed the value based on the theory of elasticity or the theory of plasticity of first order for thin slabs.

The excess in load capacity can be explained by the so-called arch-action.

The problem was to investigate the excess in load capacity due to the arch-action of reinforced concrete slabs supported at certain points; this conception being more and more applied in practice.

No simple mathematical model could be used because the simulation of the particular boundary conditions and of the slab structure in the plastic range would require mathematical formulation of several, up to now unknown factors, demanding in turn first of all to determine the quality and order of magnitude of the effects of these factors, it being essentially the aim of the test series.

The actual model tests had to decide:

a) whether an arch-action significantly increasing the load capacity of a slab supported at points can develop or not;

b) what a horizontal flexural stiffness is required for a totally loaded structure adjacent to a single panel slab supported at four points to withstand the horizontal deflection in each part of the slab;

c) to what a degree the arch-action affects the mechanism of deformation of the structure after cracking;

d) what is the function of the flexural reinforcement in the development of the arch-action;

e) whether a stage where the load capacity of the slab ends by snapping through could be developed or not;

f) how the arch-action affects the punching of the slab?

Selection of the model.

For answering the questions *a)* and *b)* alone, the development of a model without material similitude would have been sufficient, while analysis of problems *c)* to *f)* required a model of material similitude.

It should be noted herein that for determining the appropriate proportions of the model slab and loading surface, a model without material similitude was applied. Here the supporting conditions were such that the field between the four supporting points behaved as a totally loaded slab of an infinite number of panels.

A scale of 1 : 15 was selected for the micro-concrete model of material similitude.

According to the presented classification the scale 1 : 15 belongs to the group of middle-scale models with their limits of application. In this instance, this scale factor was partly appropriate for the investigation of the mainly qualitative questions and the material similitude of the model was sufficient for analyzing the phenomenon, and partly, these models were inexpensive and easy to treat.

Testing procedure.

A total of 21 slabs have been constructed, the largest dimension of which was slightly over 1.0 m. The slabs were supported at four intermediate points at the corners of a square of 54.5 cm sides, over supporting surfaces of 6 cm sides.

Loads were applied mechanically on 7 slabs and hydraulically on 14 slabs.

In some slabs, the specific flexural reinforcement percentage, and in the others, made with uniform reinforcement, the horizontal flexural rigidity of the lateral support was varied.

Besides of electronic and mechanical instruments for load indication, the displacements of the characteristic points of the slab were measured by inductive transmitters, dial gauges and by levelling.

Fig. 1 shows a slab instrumented for testing to failure. On the left-hand side of the photo, the co-ordinate recorder for plotting the indications of the inductive instruments is shown.

Fig. 2 is the load-deflection diagram of the characteristic points of a slab. The curve "i" is the load-deflection curve of the slab centre recorded

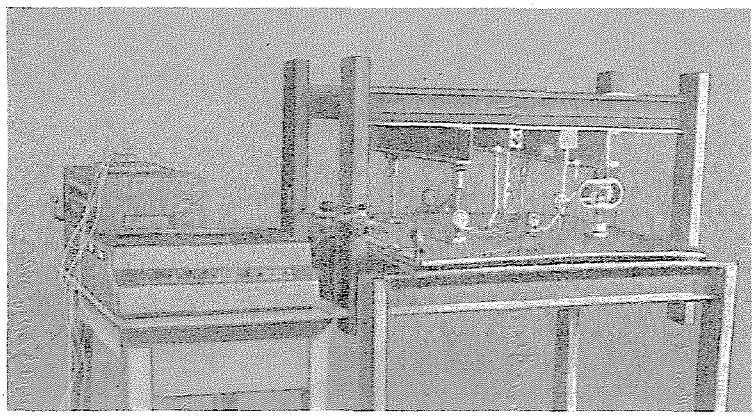

Fig. 1

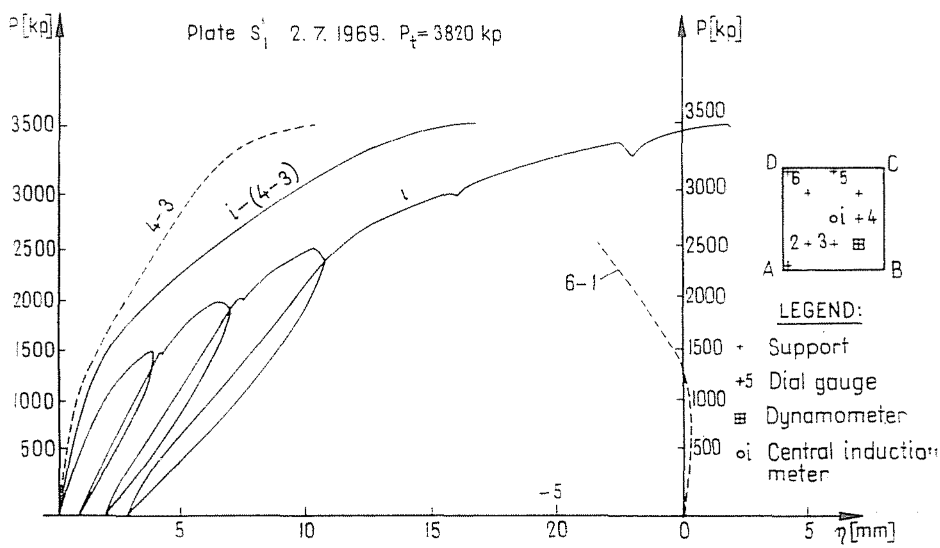

Fig. 2

by the inductive transmitter, points 3 and 4 are support mid-spacings and points 1 and 6 are at the slab corners. The curve 4—3 indicates the mean of the deflections of the two mid-spacing points; the curve 6—1 shows the mean deflections of the two slab corners; finally, the curve i(4—3) indicates the differences between the slab centre deflections and the mid-spacing mean deflections.

On all of the curves, ranges of the slab behaviour may be distinguished. After initial cracking, the slope of the load-deflection curve does not become

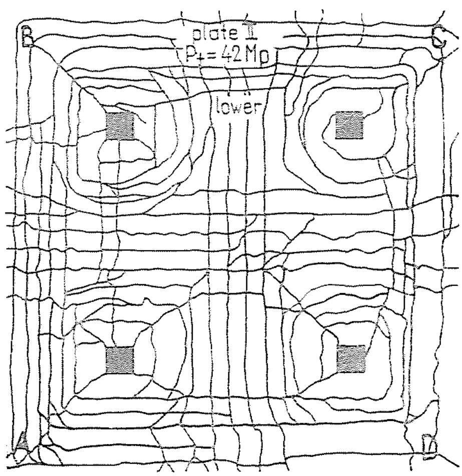

Fig. 3. Example of a model slab tested to failure (lower side)

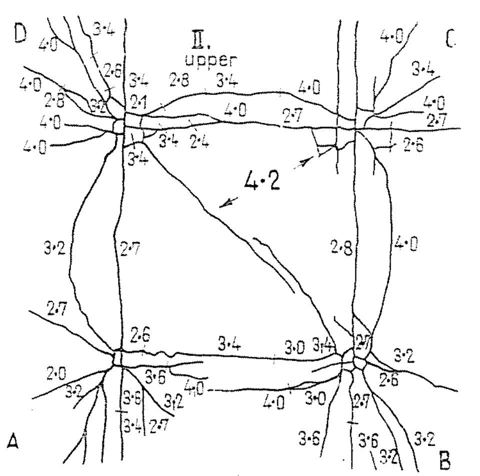

Fig. 4. Upper side of the model slab in Fig. 3. Numbers indicate cracking loads in tons

zero, but after a definite break-point the curve gradually flattens. The slope is only zeroed just before failure (provided there is not enough reinforcement suitably arranged to induce the tensile membrane effect).

Figs 3 and 4 show top and bottom surface of a slab, respectively, after failure. The study of the cracking pattern yields useful information on the load behaviour of the slab.

Interpretation of the model test results

Conclusions drawn from the model tests are, in the sequence of the questions:

a) in slabs of convenient geometry on point supports an arch-action develops, of a similar rate as in slabs of continuous edge support;

b) in a detached slab panel, arch-action develops even in the case of a relatively low lateral stiffness of the adjoining structure;

c) after formation of the cracks, the deflections of the slab are smaller than without lateral support;

d) further tests are needed to see the effect of the flexural reinforcement percentage;

e) slabs without flexural reinforcement exhibited failure by snapping through;

f) the load capacity of most slabs with flexural reinforcement depended on the ultimate punching force, this value, however, exceeded the value calculated omitting the arch-action, and the failure pattern was that of a field moment combined with a moment-shear punching.

The series of model tests cleared the problems up to the expected degree. Bearing in mind, however, that the model was of middle-scale, the partial material similitude prevented the behaviour of the model from being considered analogous to that of the structure. Nevertheless, the information supplied by the tests was sufficient to confirm the correlations of the mathematical model based on the test results applied to study the arch-action.

Analysis of a slab of particular shape by means of a model without material similitude

The problem

On commission of an industrial enterprise, the design of a reinforced concrete slab had to be checked by a method independent of the original procedure. The slab, of elliptical shape, with a major axis of 37.0 m and a minor axis of 22.80 m, was simply supported along the edge and elastically at six symmetrical, internal points. The structure was that of a pedestrian subway deck roof subject to heavy vehicle loads distributed over a relatively small surface. The original design applied the finite element method involving a system of equations for over 150 nodes solved by computer.

No solution of elliptic slabs subject to point loads equivalent in accuracy to the above mentioned method using infinite function series is reported of in the technical literature, except for slabs with restrained edges. Numerical solution of slabs of the given boundary condition is nearest to impossible by manual method. Therefore, model tests were applied to determine the stress distribution in the structure.

Selection of the model

The first step consisted in selecting type and scale of the model.

The idea of constructing a model of material similitude has been rejected because knowledge of the elastic behaviour of the structure sufficed; no analysis of secondary effects depending on the construction and material properties of the deck roof was necessary.

The choice of a model without material similitude was motivated also by the requirements of economy and urgency.

Supposing an ideally elastic behaviour of the materials of model and structure, simulation of the structural system of the roof slab was only required.

Stress values corresponding to a mathematically exact calculation could also be obtained by stress or curvature determinations on a model of appropriate scale, but at the cost of prolonged time demand.

The method of checking tests involving the test program was established so as to avoid difficulties due to both the tedious calculation work and the lack of checking by inspection, as well as to the extensometry requiring much preparation and evaluation work.

On the basis of these considerations, the known structural design method based on simplifying assumptions has been combined with tests on the model without material similitude to determine the design stresses of the special hyperstatic slab structure. The simplified mathematical models, certain parameters of which were obtained in model tests, were as follows: for the field moments, the slab structure was divided into column strips and field strips. Considering each strip as a continuous beam on elastic supports, the design moments can be determined at fair approximation from the knowledge of loads and reactions.

For the determination of the stresses about the supports, the part of the slab confined by the zero circle of the radial moments is analysed as a circular plate simply supported at the perimeter, loaded in its centre by the reacting force of the column.

Determination of the design moments is seen to require the column reaction forces of the structure of very intricate stress system in the case of different load patterns.

Testing procedure

The model test program has been established so as not to require anything but the simplest and at the same time most exact determination, namely that of the deflections by means of dial gauges.

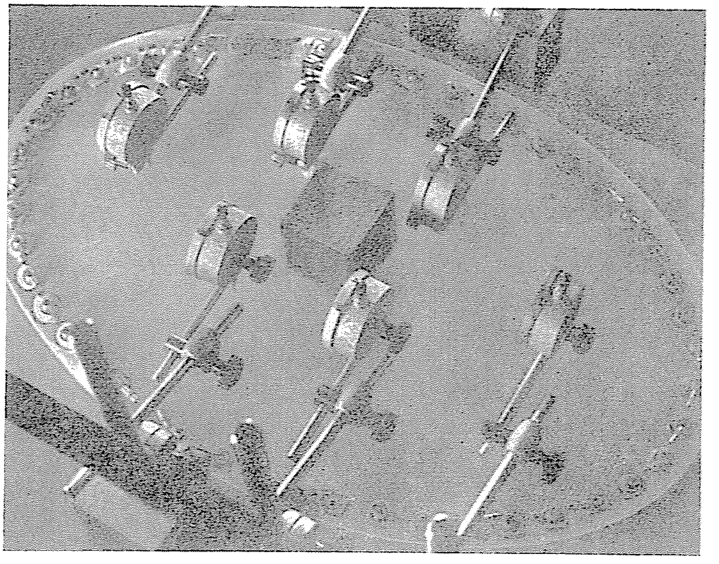

Fig. 5

The hyperstatic principal beam of the structure elastically supported at six inside points is the elliptic slab without inside supports. The further sixfold redundancy may be resolved by writing down six equations of elastic connections for the displacements and reaction forces of the supports.

The unit factors of the set of equations have been defined from the deflections due to unit loads applied on the unsupported slabs at the locations of the columns, measured at the locations of the other columns. For this purpose a loading device has been constructed which permitted to measure the deflection also at the load point.

Also the load factors i.e. the concentrated load effects were obtained from the deflection values at the fictitious supports of the unsupported slab. Such an arrangement is presented in Fig. 5.

The effect of the elastic bedding was taken into consideration by increasing the elements in the main diagonal of the unit factor matrix by the value of the coefficient of subgrade reaction.

Owing to the manifold design load pattern, it was advisable to determine the inverted matrix of the unit factor matrix, thereby the supporting forces of the flat slab could readily be determined as a product of vectors, composed of the deflection values at the fictitious support locations for the different load patterns, by the inverted matrix, and by a scalar number derivable from the model similitude law.

To determine the zero circle of the radial bending moment around the columns, the deflections have been measured by levelling at the points of the

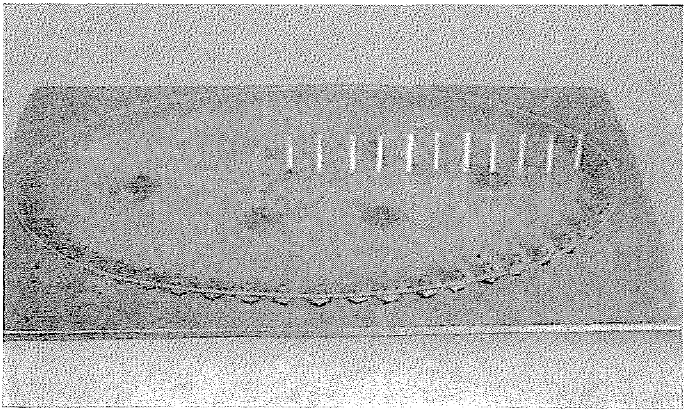

Fig. 6

radial cross-sections intersecting the supports. The model with the arrangement of the levelling riders is shown in Fig. 6. The results indicated that the position of the radial moment zero lines assumed at the one fifth of the column spacing practically hardly depended on the load, therefore, this assumption is always satisfactory in calculating the maximum negative bending moments.

For the size of this model to scale 3 : 200 made of nitrate-celluloid sheet 3 mm thick the stresses due to a concentrated load of 1—2 kips did not exceed the limit of proportionality of the material; the deflections of 2 to 3 mm could be determined at a sufficient accuracy by dial gauges of 1/100 mm sensitivity.

The Navier boundary condition could be realized by placing the slab upon a row of balls arranged along the edge of the ellipse; lifting was prevented by weights suspended on the slab edge.

In interpreting the test results, the multiple symmetry has been fully utilized.

Some results of checking tests and calculations, as well as those of the original computation are compared in Table 1.

Table 1

Column reaction forces

Load	Computer output P [Mp]	Checking calculation P [Mp]
Dead load	$P_{12} = 166.3$ $P_{77} = 153.9$	$P_{12} = 189.5$ $P_{77} = 120.0$
Concentrated load (Load pattern 5)	$P_{12} = -3.57$ $P_{77} = 67.37$	$P_{12} = 2.3$ $P_{77} = 69.8$

Load	Location	Computer output m [Mpm/m]	Checking calculation m [Mpm/m]
Support moments:			
Dead load	12	−26.2	−28.7
	77	−24.78	−18.15 (−23.2)
Concentrated load	12	− 9.04	−10.1
	77	−12.19	−7.5 (−10.6)
Field moments:			
Dead load	1	$m_y = 5.57$	
	5	$m_y = 7.64$	$m_y = 5.92$
	1	$m_x = 3.18$	
	73	$m_x = 5.92$	$m_x = 5.92$
Concentrated load	1	$m_y = 13.18$	
	5	$m_y = 10.01$	$m_y = 15.5$
	73	$m_x = 11.72$	$m_x = 19.5$

(The values in brackets are those calculated on the basis of other load patterns approximating the design values.)

These examples of moment values clearly show that simultaneous application of calculation and modelling — especially in the case of simple calculations and rapid model program — is very effective and allows an accurate analysis of complex problems which, at first glance, seem to be inaccessible to simple means.

Analysis of flat slab punching in a combined model test program

The problem

The engineering structures adjoining the underground railway in construction have been designed with flat slabs.

The supports have been designed with expensive, cast steel discs.

On commission of the design office we had to investigate, to what a degree the diameter of the steel casting could be reduced without affecting the load capacity of the structure.

Selection of the model

Tests with flat slabs conducted by the authors, as well as by foreign researchers showed most of such structures to fail by insufficience of shear or moment-shear load capacity of the slab about the supports.

The ultimate punching load of the slab, the mode of failure depends on the ratio of slab thickness to diameter of the supporting surface, on the grade of concrete and reinforcement, and on the position of this latter.

The problem was seen to be that of the failure of the structural material, therefore a model of material similitude had to be applied.

Fabrication with variable parameters and loading to failure of large or middle-scale models of material similitude of a whole floor structure would have been uneconomical and unreasonable. According to the deflection and moment diagrams, a flat slab under uniform load exhibits circularly symmetric behaviour about regularly arranged supports. Accordingly, as was mentioned above in connection with the preceding model test, the environment of the column may appropriately be investigated on the part of the slab bordered by the moment zero circle, considered independent of the rest of the slab.

The constraint represented by the material continuity can be substituted by freely rotating support at the perimeter of the zero circle.

Our problem also might be modelled by similar method. For this case, however, no references on form and size of the moment zero line are found in the literature, and are rather difficult to calculate. Namely:

— the points of support are arranged at the vertices of regular triangles;

— intensive asymmetric loads may significantly affect the form of the zero line to distort into an asymmetric configuration.

Thus, the first step was to determine the shape of the zero line of the radial moments of the elastic slab under concentrated load around the column, if supports are arranged in a network of regular triangles.

For this purpose a small-scale model without material similitude was used. It was sufficient to simulate the structural system, as well as to correlate the moduli of elasticity and the transverse contraction coefficients, since the moments had to be evaluated from the strain determinations.

The model was made of asbestos cement, to scale 1 : 25.

The column capitals together with the neoprene shoes have been scaled down according to the law of similitude.

Instead of simulating the whole floor structure, only a part of it has been analyzed, the columns being arranged in the centre and at corners of a regular hexagon (Fig. 7).

Considering that the designer intended to realize several floors of the same plan layout and since no moment influence surfaces were available for such floors, it seemed advisable to simultaneously determine the moment influence surfaces.

The developed moments have been evaluated by calculation from the specific strains indicated by resistance strain gauges placed in the axes of symmetry both sides of the slab.

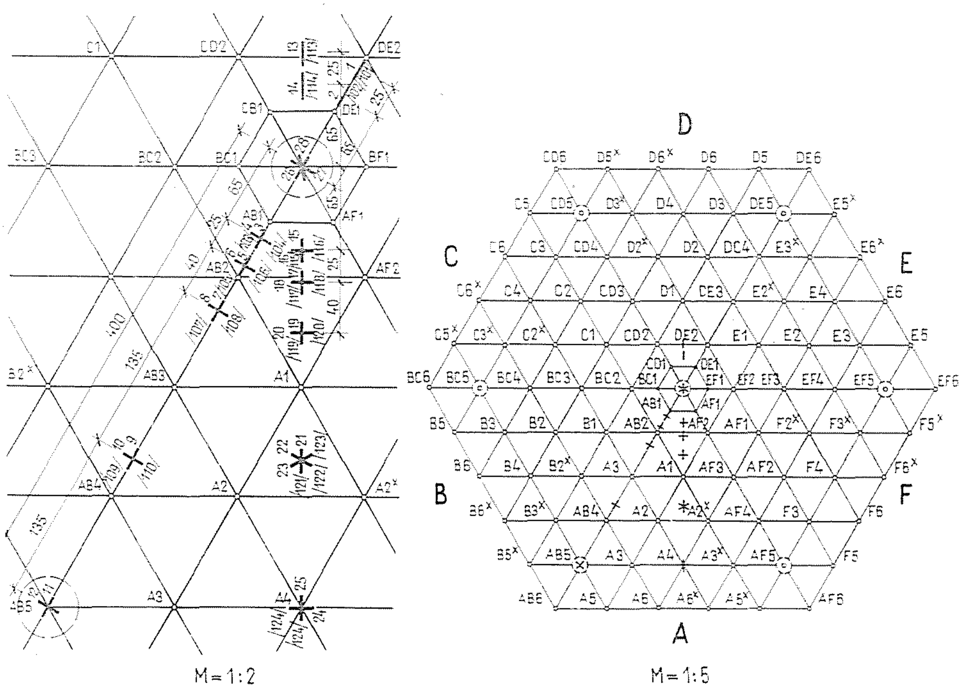

Fig. 7. Model tests of the subway plate, Baross square. Arrangement of strain gauges, loading points. Numbers in brackets refer to gauges on the lower side

Description of the model tests

Fig. 8 shows the over-all arrangement of the model test.

The loads were applied at regular network nodes.

The reliability of the moment influence charts has been checked: they led to a value of 18.6 Mpm/m for the moment at the centre of a panel under the design load, whilst the computer output was 20.12 Mpm/m. The difference of about 10 per cent has to be ascribed to the neglections inherent with the computer method of finite difference. The radial moment diagrams about the column capital under heavy loads distributed over small areas, in positions, have been determined.

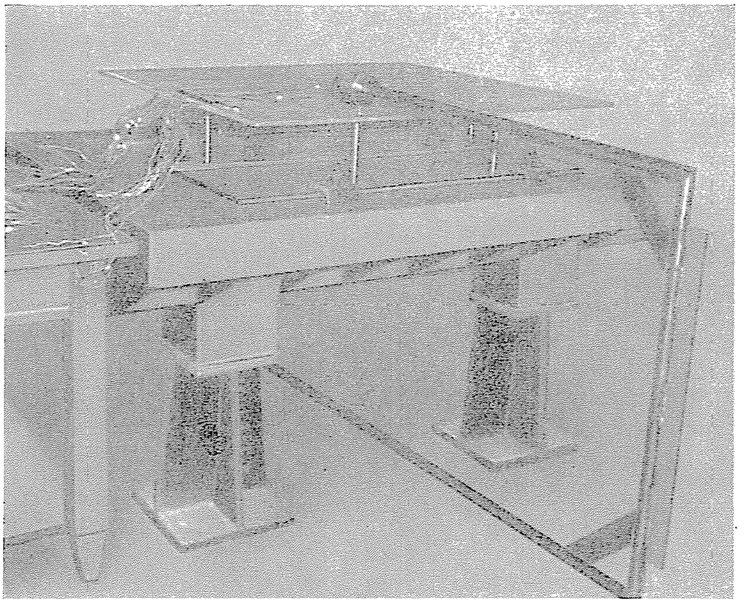

Fig. 8

In every case the radial moment zero-line was a circle at a fair approximation, with a specific radius $c = 0.179$ to 0.216, a comparable value to $c = 0.2$ to 0.22 quoted in the literature.

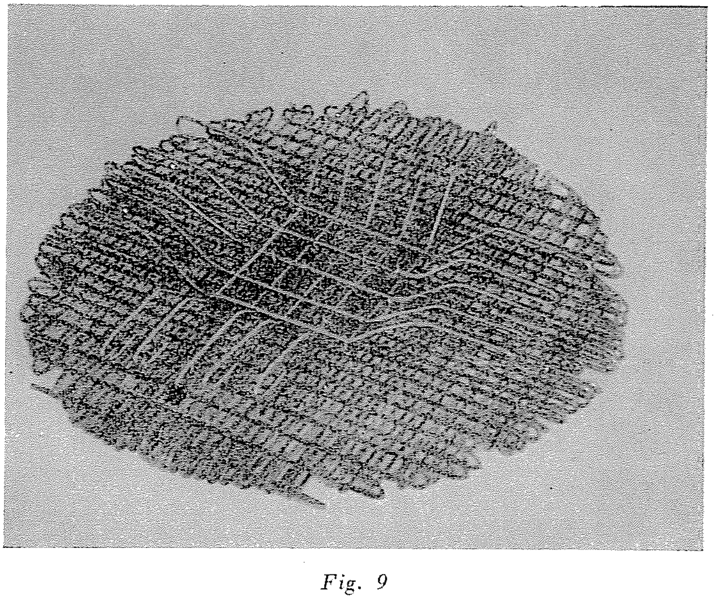

Fig. 9

Then, in accordance with the program, the models of material similitude representing the slab structure about the column, have been constructed to a scale 1 : 10.

Eight circular reinforced concrete plates of 42 cm dia., 5.5 cm thick have been cast, reinforced similarly as the original structure (Fig. 9).

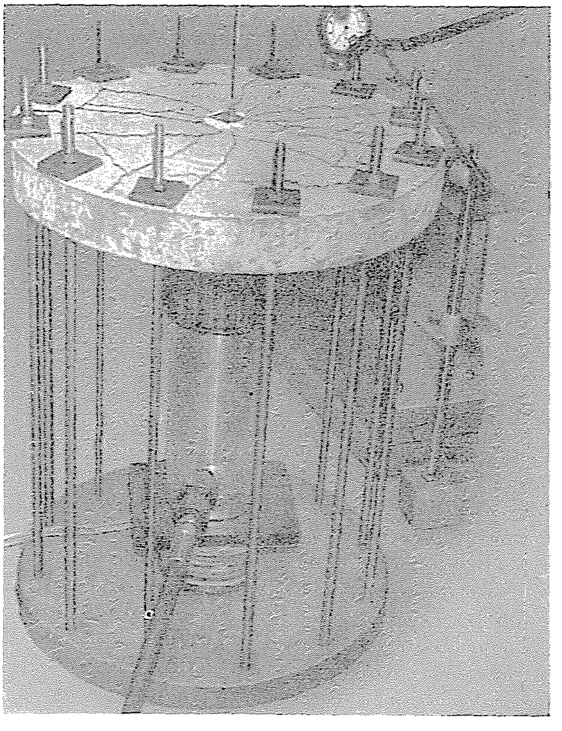

Fig. 10

In accordance with the column capitals of three different diameters, the plates were supported at their centres — with the intermediary of circular steel plates 8 mm thick of 10.7 and 4 cm diameters, respectively, — on 4 cm diameter circular discs proportional to the actual column diameter, and the arrangement was placed in the hydraulic testing machine.

The circular plates were fixed at 12 points along the moment zero circle of 38 cm diameter of the model.

The test setup is shown in Fig. 10.

During loading, the displacements of the plate centre vs. load were measured.

In Fig. 11, load-deflection diagrams are plotted on the basis of tests on three supporting discs of different diameters.

It is obvious that the 10 cm dia. disc was punched after a significant plastic deformation (moment failure), punching of the 4 cm dia. discs was of brittle nature (shear failure), while the disc of 7 cm dia. failed by moment-shear. The disc diameter also influenced the ultimate load value.

In accordance with the model test results, a proposal to improve design economy has been developed.

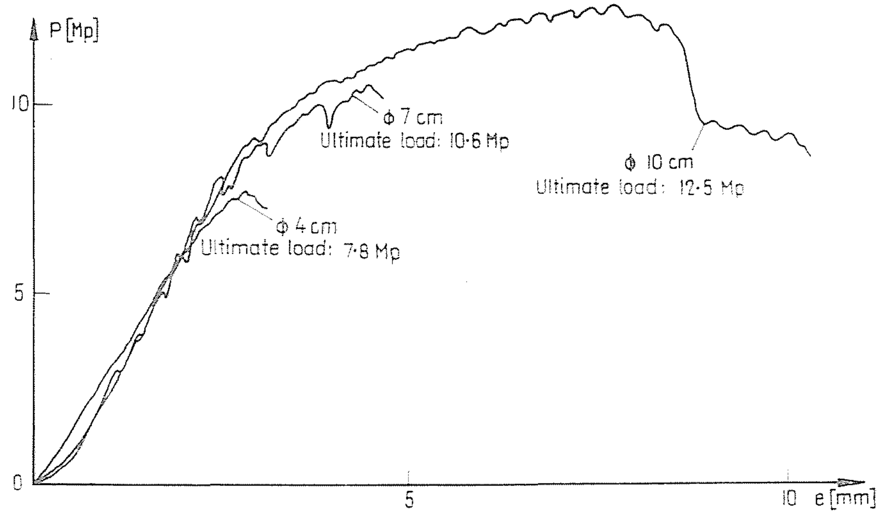

Fig. 11

Summary

Models of appropriate type, material and scale significantly help the work of the designer, as authors experienced it themselves.

Models true to material are advantageous by exhibiting phenomena of material character of the structure, permitting thereby to check by inspection .

As against complex, computerized mathematical models, the advantage of the real model is to make needless the determination of other than direct design values, provided the real model is appropriately constructed and the testing program is duly established.

In reporting the three model tests conducted at the Department of Reinforced Concrete Structures, three distinct application possibilities of the model tests have been presented. In each of the three cases, the selected method is superior in economy and efficiency to the corresponding, purely mathematical analysis.

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