SAFETY TO BRITTLE FAILURE IN PRESTRESSED CONCRETE STRUCTURES WITH BOND

By

A. WINDISCH

Department of Reinforced Concrete Structures, Budapest Technical University

(Received April 8, 1970)

Presented by: Prof. Dr. E. BÖLCSKEI

Introduction

Two methods for the design and analysis of structures has been developed. The method of permissible stresses was the first, later a better knowledge of internal forces in structural materials and behaviour of structures served as basis to develop the method of ultimate design.

Recently, this latter method has much come to the foreground also for the design of prestressed concrete structures. Also the FIP—CEB recommendations specify analyses for two fundamental ultimate conditions: those at failure and in service.

Analyses based either on the permissible stresses or on the working load method assume the structure to behave elastically, to have an ideal cross-section and allow within given limits to analyze cracked cross-sections according to the elastic stress distribution. An accurate and realistic design can be provided for, as a rule, by a possibly true consideration of the quasi-elastic behaviour of structural materials, of eventual cross-sectional cracks and by a careful selection of strength and strain characteristics.

In the design of prestressed concrete structures, the knowledge of cross-sectional stresses (strains) in each load state is indispensable, namely these affect the prestress, the deformations, the crack formation, this latter being of special importance for prestressed structures extremely sensitive to corrosion.

Analysis of the ultimate condition at failure, knowledge of the real load capacity of the designed structure are equally of great importance. Nevertheless, knowledge of the ratio of the ultimate load of a given probability to the maximum stress due to outer loads of a given probability still does not mean that the behaviour under a random overload of the structure meets the requirements of safe service.

The ultimate design has to involve safeguarding of a given toughness of the beam or the whole structure to indicate the imminent failure, a requirement for avoiding brittle failure.

In what follows, the problem of the safety to brittle failure in bending of bonded-wire prestressed concrete structures will be considered.

Legend

Cross-section characteristics

$$\left.\begin{array}{ll} b & [\mathrm{cm}] \\ b_0 & [\mathrm{cm}] \\ b_1 & [\mathrm{cm}] \\ v & [\mathrm{cm}] \\ v_1 & [\mathrm{cm}] \\ h & [\mathrm{cm}] \\ h_1 & [\mathrm{cm}] \\ y_f & [\mathrm{cm}] \\ y_a & [\mathrm{cm}] \\ e & [\mathrm{cm}] \end{array}\right\}\ \text{Figs 1—2.}$$

A_c [cm^2] area of the concrete cross-section;
I_c [cm^4] moment of inertia of the concrete cross-section about its median;
x [cm] depth of the neutral axis;
q [cm] lever arm of internal forces;

$$\xi = \frac{x}{h}$$

$$\zeta = \frac{q}{h}$$

$$\Delta = \frac{y_a}{y_f}$$

$$\eta = \frac{e}{h_1}$$ specific eccentricity;

$$\theta = \frac{1}{1+\Delta} + \eta = \frac{h}{h_1}$$

$$\varrho = \frac{i^2}{h_1^2}$$ cross-section efficiency;

$$i^2 = \frac{I_c}{A_c}\ [\mathrm{cm}^2]$$

$$\varkappa = \frac{A_c}{bh_1}$$ shape factor.

Concrete characteristics

σ_k [kp/cm^2] cube strength at 28 days;
σ_{cu} [kp/cm^2] ultimate stress in compression;
σ_{ct} [kp/cm^2] bending-tensile strength;
σ_c [kp/cm^2] compressive stress for strain ε_c;
ε_{cc} [‰] characteristic point of the approximate $\sigma - \varepsilon$ diagram;
ε_{cu} [‰] ultimate strain of compression.

$$\alpha = \frac{\sigma_{ct}}{\sigma_K}$$

$$\lambda = \frac{\sigma_{cu}}{\sigma_K}$$

Tendon characteristics

A_p [cm^2] prestressed wire area;
μ [%] $= \frac{A_p}{bh_1} 100$ specific area of prestressed wires;
$\sigma_{0,2}$ [kp/cm^2] nominal yield point;
σ_B [kp/cm^2] tensile strength;
σ_{su} [kp/cm^2] wire stress at beam failure;
$\varepsilon_{0,2}$ [‰] total strain at nominal yield point;
ε_B [‰] ultimate strain;
ε_l [‰] wire strain excess due to outer load;
ε_{lu} [‰] ultimate strain excess;

$$\psi = \frac{\sigma_{su}}{\sigma_{0,2}}$$

$\beta = \frac{\sigma_{0,2}}{\sigma_{cu}}$

$\delta = \frac{\sigma_B}{\sigma_{0,2}}$.

Reinforcement characteristics

A_s [cm^2] tensile steel area;

$\mu_s = \frac{A_s}{bh_1}$ 100 [%] specific steel percentage;

σ_F [kp/cm^2] yield point;

$\beta_s = \frac{\sigma_F}{\sigma_{cu}}$.

Prestress characteristics

σ_P [kp/cm^2] wire prestress;

$\gamma = \frac{\sigma_P}{\sigma_{0,2}}$;

$\nu\gamma\varepsilon_{0,2}$ [‰] working strain in tendons after losses of prestress.

Internal forces

M_u [kpm/m] Mörsch ultimate moment in the prestressed cross-section;

M_r [kpm/m] cracking moment in the prestressed cross-section.

Concept of brittle failure

By brittle failure that kind of failure is meant where exhaustion of load capacity is indicated neither by marked crack width nor by some major plastic deformation at a non-catastrophal rate.

Obviously, absence of tensile stresses due to outer loads of the tensile extreme fibre of the beam cross-section under working loads — maybe a tension lower than the permissible one — does not exclude the possibility of brittle failure.

Tensile strengths much over the permissible concrete tensile stresses, specified low because of safety aspects and quality scatters, are known to be encountered in tests on real structures and on laboratory flexural specimens; first visible cracks occurred under loads higher than calculated. Let us mention, though it is not perfectly cleared yet, that the concrete tensile strength is related to the grade and percentage of the reinforcement in the tensile flange of the structure.

Often, in structures designed with total prestress, compressive stresses on the whole cross-section may arise even under service loads. Accordingly, first cracks in these structures will appear only under working loads much greater than service loads.

Obviously, analyses based on crack control or on ultimate condition do not exclude brittle type beam failures i.e. where appearance of the first crack and total failure are rather close together both as to load intensity at, and instant of failure.

To demonstrate different toughness of prestressed beams with bond, load-deflection curves for three beams are shown in Fig. 1 based on [1]. Beams

OB. 34.043; OB. 44.158; and OB. 44.032 behaved each quite differently under loading. According to notations in [1], all three beams were simply supported post-tensioned ones, with bond. The first group of numerals refers to the nominal value of prestress and to the location of outer loads, while the second one indicates the specific reinforcement percentage.

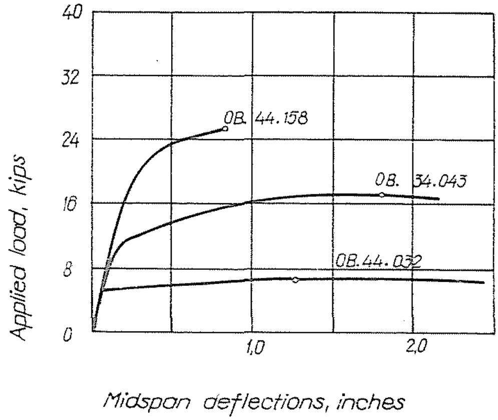

Fig. 1

The load-deflection curve of beam OB. 34.043 has three characteristic sections. Throughout the first section up to the appearance of the first crack, steel strain in the homogeneous concrete cross-section is in the elastic range of the $\sigma - \varepsilon$ diagram. Once the concrete has cracked (stress condition II), in the cracked cross-section the depth of the neutral axis rapidly decreases, still the steel behaves elastically, the beam has a rather variably ascending load-deflection curve. In the final deformation state also the steel is plastic, the load-deflection curve has an about constant slope, cracks widen, and after the initial visible concrete crushing (indicated in the curve by small circles) further deformation may occur up to total failure. Beams of this behaviour are known as *normally reinforced* ones.

Notice that wires prestressed for higher percentages of the nominal yield stress do not exhibit the second deflection state, here the steel is plastic once the beam cracked.

Beam OB. 44.158 exhibited only the first two states, beam failed by crushing before the wires yielded. This is characteristic to *overreinforced* beams.

After appearance of the first crack both beams required still significant load increase to reach ultimate load.

Wires in beam OB.44.032 yielded immediately after the first crack appeared, and the beam abruptly failed at a high rate of deformation, without further load increase. Such beams are termed *underreinforced* ones.

From among the listed three beams, the two latter underwent brittle failure according to the previous definition.

The problem of brittle failure can be examined by either the ratio of cracking to ultimate load (moment) or by the steel (prestressed wire) percentage.

Either of these means are indicated in specifications for brittle failure control.

Ratio of ultimate to cracking moment of bonded-wire prestressed beams

Let us examine according to methods in [1] and [2] how different characteristics of bonded-wire prestressed beams affect ratio of ultimate to cracking moment.

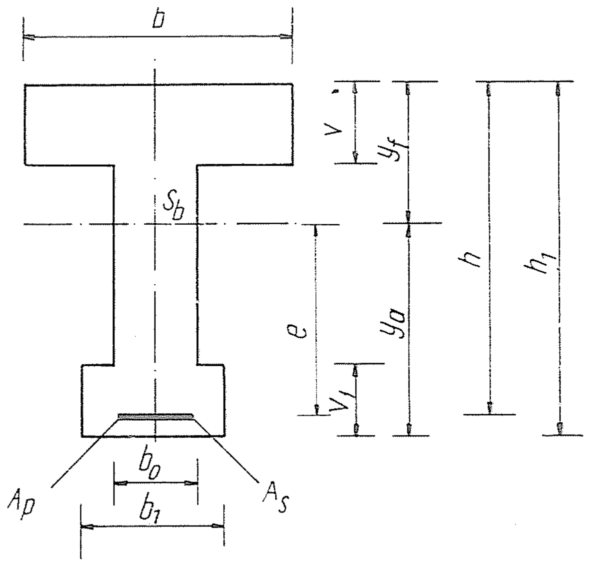

Fig. 2

Relationship will be deduced first for the general cross-section shown in Fig. 2. Initial assumptions are:

— the Bernoulli—Navier hypothesis is valid at the instant of failure;

— the $\sigma-\varepsilon$ diagram of concrete is composed of a second-degree parabola and a tangential straight line (Fig. 3). Notice that the analysis is the same as for other stress-strain diagrams;

— concrete tensile strength is omitted in the calculation of ultimate moment;

— $\sigma - \varepsilon$ diagram of wires is approximated by two straights as seen in Fig. 4;

— tendons act concentrated in their centres of gravity;

— initial and effective values of prestress strain are known;

— gravity centre of the tensile supplementary reinforcement is at the same level as those of prestressing wires;

— effect of reinforcement is neglected in the calculation of cross-section values;

— there are no repeated loads acting.

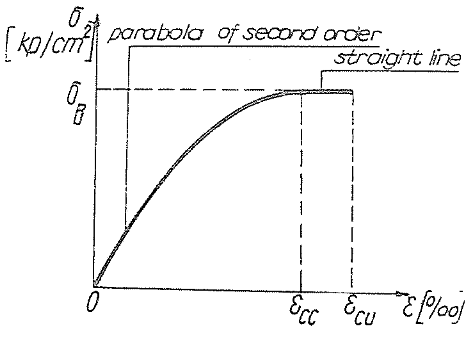

Fig. 3

Determination of the ultimate moment

Now, the Mörsch ultimate moment of a beam with a compressed flange of constant width will be determined ($x < v$).

For the general case, the ultimate moment can be expressed as:

$$M_u = A_p \sigma_{su} q = A_p \sigma_{su} \zeta h \tag{1}$$

where

A_p area of prestressed wires;

σ_{su} stress in wires at beam failure;

$q = \zeta h$ lever arm of internal forces.

Notice that by the time, no ordinary reinforcement is taken into account.

From ultimate moments calculated from the ultimate concrete compression or on the basis of different specifications for the ultimate strain of prestressing steel, the smaller one is assumed as M_u.

Let us write the value of each factor in expression (1), beginning with that based on the maximum strain increment $\varepsilon_l = \varepsilon_{lu}$ of the prestressing steel due to the outer load (Fig. 5).

Provided that the total wire strain exceeds that belonging to the nominal yield point, the ultimate stress in the prestressing wire is:

$$\sigma_{su} = \psi\sigma_{0,2} = \sigma_{0,2} + \frac{\nu\gamma\varepsilon_{0,2} + \varepsilon_{lu} - \varepsilon_{0,2}}{\varepsilon_B - \varepsilon_{0,2}} (\sigma_B - \sigma_{0,2}) =$$

$$= \sigma_{0,2}\left[1 + \frac{\nu\gamma\varepsilon_{0,2} + \varepsilon_{lu} - \varepsilon_{0,2}}{\varepsilon_B - \varepsilon_{0,2}} (\delta - 1)\right] \tag{2}$$

(Specific strain values are to be substituted in permillage.)

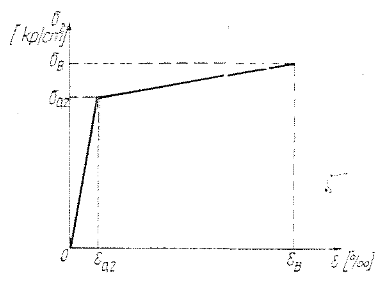

Fig. 4

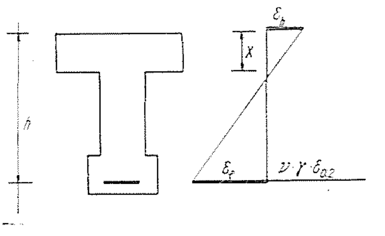

Fig. 5

This expression is valid only for $\psi \geqq 1$.

Concrete compression belonging to this ultimate condition of the cross-section (ε_c):

$$0 < \varepsilon_c \leq \varepsilon_{cu}.$$

Provided the stress-strain diagram of concrete corresponds to that in Fig. 3, the ε_c value can be calculated from the equilibrium expression about an axis parallel to the beam axis. The $\sigma - \varepsilon$ diagram of concrete being a composite one, the relationship has to be parted:

$$0 < \varepsilon_c \leq \varepsilon_{cc}.$$

and

$$\varepsilon_{cc} < \varepsilon_c \leq \varepsilon_{cu}.$$

Since

$$\xi = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_{lu}} \tag{3}$$

expressions for ε_c are:

$$0 < \varepsilon_c \leq \varepsilon_{cc}$$

$$\frac{\varepsilon_c^2}{\varepsilon_{cc}(\varepsilon_c + \varepsilon_{lu})}\left(1 - \frac{\varepsilon_c}{3\varepsilon_{cc}}\right) = \frac{\mu\beta}{\vartheta}\psi \tag{4a}$$

and for $\varepsilon_{cc} < \varepsilon_c \leq \varepsilon_{cu}$

$$\frac{\varepsilon_c}{\varepsilon_c + \varepsilon_{lu}}\left(1 - \frac{\varepsilon_{cc}}{3\varepsilon_c}\right) = \frac{\mu\beta}{\vartheta}\,\psi\,. \tag{4b}$$

Expression $(4a)$ or $(4b)$ delivers ε_c.
Lever arm of internal forces $q = \zeta\, h$. (5)
As a function of ε_c, in the range $0 < \varepsilon_c \leq \varepsilon_{cc}$

$$\zeta = 1 - \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_{lu}}\,\frac{8\varepsilon_{cc} - 3\varepsilon_c}{12\varepsilon_{cc} - 4\,\varepsilon_c}\,, \tag{6a}$$

in the range $\varepsilon_{cc} < \varepsilon_c \leqq \varepsilon_{cu}$

$$\zeta = 1 - \frac{1}{\varepsilon_c + \varepsilon_{lu}}\,\frac{6\varepsilon_c^2 - 4\varepsilon_c\varepsilon_{cc} + \varepsilon_{cc}^2}{12\varepsilon_{cc} - 4\varepsilon_c}\,. \tag{6b}$$

Assuming an ultimate compression $\varepsilon_c = \varepsilon_{cu}$ in the compressed extreme fibre of the cross-section, the factors of the ultimate moment are:

$$\sigma_{su} = \psi_1\sigma_{0,2} = \sigma_{0,2}\left[1 + \frac{\nu\gamma\varepsilon_{0,2} + \varepsilon_{cu}\left(\dfrac{1}{\xi} - 1\right) - \varepsilon_{0,2}}{\varepsilon_B - \varepsilon_{0,2}}\,(\delta - 1)\right] \tag{7}$$

$$\frac{\varepsilon_{cu}}{\varepsilon_l + \varepsilon_{cu}}\left(1 - \frac{\varepsilon_{cc}}{3\varepsilon_{cu}}\right) = \frac{\mu\beta}{\vartheta}\,\psi_1 \tag{8}$$

The ε_l value can be determined from (8) through several steps, it being implicit for ε_l . For sake of simplicity, let us substitute $\psi_1 = 1$, and since

$$\varepsilon_l \leq \varepsilon_{lu}\,,$$

the error is within 1 per cent.

$$\zeta = 1 - 0{,}39\,\xi\,, \tag{9}$$

where

$$\xi = \frac{3\varepsilon_{cu}}{3\varepsilon_{cu} - \varepsilon_{cc}}\,\frac{\mu\beta}{\vartheta}\,\psi_1\,. \tag{10}$$

Also here, the simplifying assumption $\psi_1 = 1$ causes little error.

If at failure the stress in the prestressing steel is below the nominal yield point, i.e.:

$$\psi < 1\,,$$

the nominal ultimate steel stress is:

$$\sigma_{su} = \frac{\nu\gamma\varepsilon_{0,2} + \varepsilon_{cu}\left(\frac{1}{\xi} - 1\right)}{\varepsilon_{0,2}}\,\sigma_{0,2}\,. \tag{11}$$

Steps to determine the ultimate moment of the bonded-wire prestressed beam are:

1. Since in general, the increment of ultimate strain of the tendon produces the least ultimate moment, σ_{su} is determined from (2).

2. Thereafter ε_c is determined from (4*a*) or (4*b*). Calculation is eased by the use of the graph of function

$$\varepsilon_c = f\left(\frac{\mu\beta}{\vartheta}\,\psi\right)$$

plotted by means of standard strength and deformation characteristics.

3. For

$$\varepsilon_c \leq \varepsilon_{cu}\,,$$

the ζ value is obtained from (6*a*) or (6*b*), eventually by means of the established $\zeta = g(\varepsilon_c)$ curve.

4. The ultimate moment value is obtained by substituting each factor into (1).

If in the second step

$$\varepsilon_c > \varepsilon_{cu}\,,$$

the ultimate moment will be calculated as follows:

1. Determine σ_{su} from (7):
2. ε_l from (8);
3. ζ from (9) and (10).
4. Substitute them into (1).

For $\sigma_{su} < \sigma_{0,2}$ the σ_{su} value will be obtained from (11).

Determination of the cracking moment

At time $t = \infty$, the cracking moment is known to be delivered by

$$M_r = \frac{I_c}{y_a}\left(\sigma_{ci} + \frac{\nu P}{A_c} + \frac{\nu P e y_a}{I_c}\right). \tag{12}$$

Substituting the introduced notations:

$$M_r = A_c\varrho h_1 \frac{1+\Delta}{\Delta}\left[\alpha\sigma_K + \frac{\mu\nu\gamma}{\varkappa}\,\sigma_{0,2}\left(1 + \frac{\eta}{\varrho}\,\frac{\Delta}{1+\Delta}\right)\right] \tag{13}$$

Effect of the variation of geometry and strength characteristics of bonded-wire prestressed beams on the ratio of ultimate to cracking moment

Let us examine the fraction

$$G = \frac{M_u}{M_r} \tag{14}$$

i.e. the ratio of ultimate to cracking moment in dependence on the beam characteristics. Substituting (1) and (13) into (14) leads to:

$$G = \frac{A_p \sigma_{su} \cdot \zeta h}{A_c \cdot \varrho h_1 \dfrac{1+\varDelta}{\varDelta}\left[\alpha\sigma_K + \dfrac{\mu\nu\gamma}{\varkappa}\sigma_{0,2}\left(1+\dfrac{\eta}{\varrho}\,\dfrac{\varDelta}{1+\varDelta}\right)\right]}$$

or, with further simplifications:

$$G = \frac{\mu\beta\left(\dfrac{1}{1+\varDelta}+\eta\right)\psi\zeta}{\varkappa\varrho\,\dfrac{1+\varDelta}{\varDelta}\left[\dfrac{\alpha}{\lambda}+\dfrac{\mu\beta}{\varkappa}\nu\gamma\left(1+\dfrac{\eta}{\varrho}\,\dfrac{\varDelta}{1+\varDelta}\right)\right]} \tag{15}$$

Let us examine those factors and characteristics in (15) which affect the G value:

Cross-section shape:

$\varDelta = \dfrac{y_a}{y_f}$

$\varkappa = \dfrac{A_c}{bh_1}$

$\varrho = \dfrac{i^2}{h_1^2}$

(the two latter can be termed the cross-section efficiency);

μ prestressed steel percentage;

$\beta = \dfrac{\sigma_{0,2}}{\sigma_{cu}}$ ratio of yield strength and of ultimate strength characteristics of tendon to those of concrete;

η specific eccentricity;

$\alpha = \dfrac{\sigma_{ct}}{\sigma_K}$ ratio of bending-tensile to cube strength of concrete;

$\gamma = \dfrac{\sigma_B}{\sigma_{0,2}}$ ratio of tensile strength to nominal yield point of prestressing steel, the accordance of the standard values with the real ones;

γ rate of wire prestress;
ν ratio of effective prestress at time $t = \infty$ to that at time $t = 0$;
ε_{cu} specified ultimate concrete compression, approximate character of the assumed stress-strain diagram of concrete;
ε_{lu} ultimate strain in prestressing wire due to outer loads; contribution of ordinary reinforcement (disregarded in (15]).

Let us examine the effect of each factor.

Effect of the cross-section shape

ϱ, Δ and $\varkappa$ values for usual cross-sections will be compiled in conformity with [2]:

Cross-section shape	ϱ	Δ	$\varkappa$
□	0.0833	1	1
I	0.10–0.14	1	0.15–0.7
T	0.08–0.10	1.2–1.6	0.2–0.4
⊥	0.08–0.10	0.6–0.9	3–8

From the outcome of (15) it is obvious that from shape particularity aspects the T section is the most favourable one, since it possesses sufficient moment bearing capacity after appearance of the first crack — assuming a sufficient prestressed steel percentage —, whereas the cracking moment of ⊥ sections is significantly higher for otherwise identical characteristics, while its capacity beyond cracking is lower. In what follows, only rectangular sections will be considered.

Effect of prestressed steel percentage

The most important variable of this analysis is the specific steel percentage, the effect on G of all other factors will be examined as a function of μ. It seems advisable to discuss jointly the effect of β and μ, since from (15) they appear to have identical effect.

In the following deductions $\mu\beta$ will be the principal independent variable.

Let us substitute shape factors of the rectangular cross-section into (15):

$$G = \frac{\mu\beta(0{,}5 + \eta)\psi\zeta}{0{,}166\left[\dfrac{\alpha}{\lambda} + \mu\beta\nu\gamma(1 + 6\eta)\right]}. \qquad (15a)$$

Fig. 6 shows the variation of G as a function of $\mu\ \beta$, based on [3]. Starting values of the calculation are those specified in the 1968 Tentative Recommendations for Hungarian Highway Bridges, accordingly:

$$\lambda = 0.7, \qquad \varepsilon_{cc} = 2^0/_{00}, \qquad \varepsilon_{cu} = 2.5^0/_{00},$$

$$\varepsilon_{0,2} = 7^0/_{00} \qquad \varepsilon_B = 70^0/_{00}, \qquad \varepsilon_{lu} = 5^0/_{00}.$$

Curves differ by relative excentricities $\eta = 0;\ 0.2;\ 0.4$, other factors being

$$\varkappa = 0.1, \qquad \gamma = 0.8, \qquad \nu = 0.8.$$

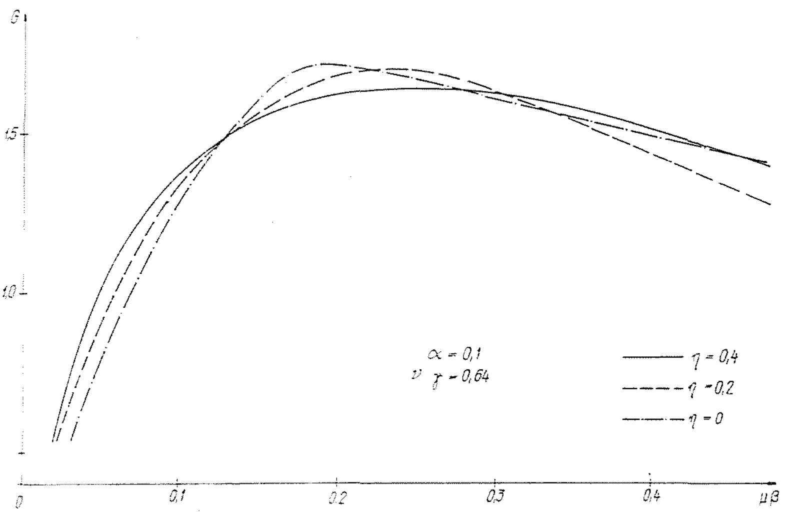

Fig. 6

Fig. 6 permits two conclusions:

a) Of course, curves for the range $G < 1$ are of a different shape, namely for $\mu\,\beta = 0$ the ultimate moment of the cross-section is identical to the cracking moment due to the concrete tensile strength, hence $G = 1.0$, but initial restrictions included omission of the concrete tensile strength. Though, curve sections in the range $G < 1.0$ evidence the risk of brittle failure.

b) The ratio G cannot be increased beyond a limit defined by other factors, even for infinite values of the prestressed steel percentage $\mu\,\beta$, and even decreases at a high rate over given values of this latter.

From among test data of 81 beams published in [1], those referring to beams with prestressed, bonded wires alone are plotted in Fig. 7. These three curves differ by the rate of prestress γ. There is a striking similarity between curves in Figs 6 and 7.

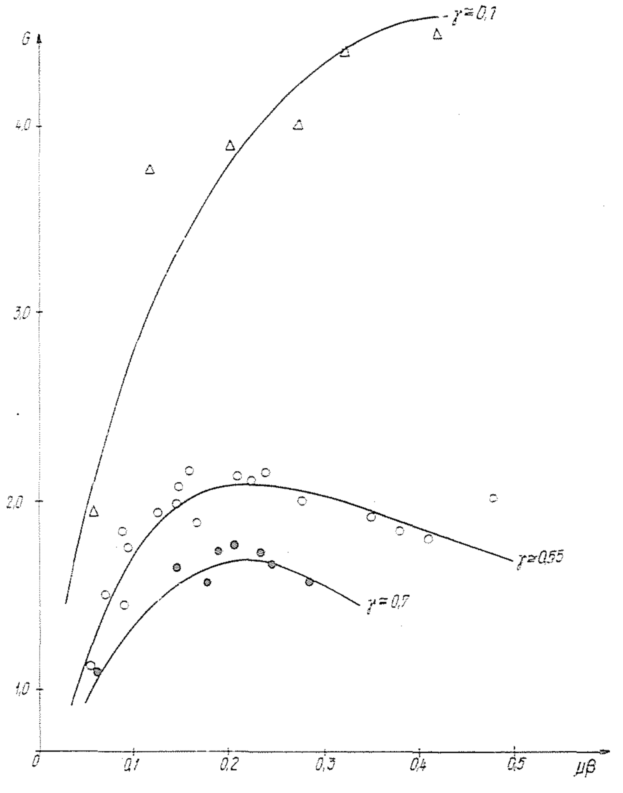

Fig. 7

Effect of eccentricity

From the form of the G ratio (15a) it is evident that prestressing wires are advisably placed as eccentrically as possible, namely with decreasing η factor the ratio of ultimate to cracking moment is reduced. Interaction between eccentricity and specific steel percentage affects also the ultimate deformation.

Effect of bending-tensile strength of concrete

Coefficient α i.e. ratio of bending-tensile to cube strength of concrete affects only the cracking moment value. For low $\nu\gamma$ and low η values, G is much affected by the bending-tensile strength. In conformity with actual design methods, from the aspect of untimely prediction of beam failure, it is not advisable to apply a variety of increased bending-tensile strengths of a given concrete grade.

Effect of the accuracy of an approximate stress-strain diagram of the wires

Stress-strain diagrams combined of two straight lines are accurate enough only for stresses exceeding the nominal yield point. Notice that calculated values are to the detriment of safety especially about the break point. Of course, the most exact may be the stress-strain diagram of the steel material to be built in.

Effect of prestress and losses

Since coefficients γ and ν affect the G ratio identically, they will be jointly considered.

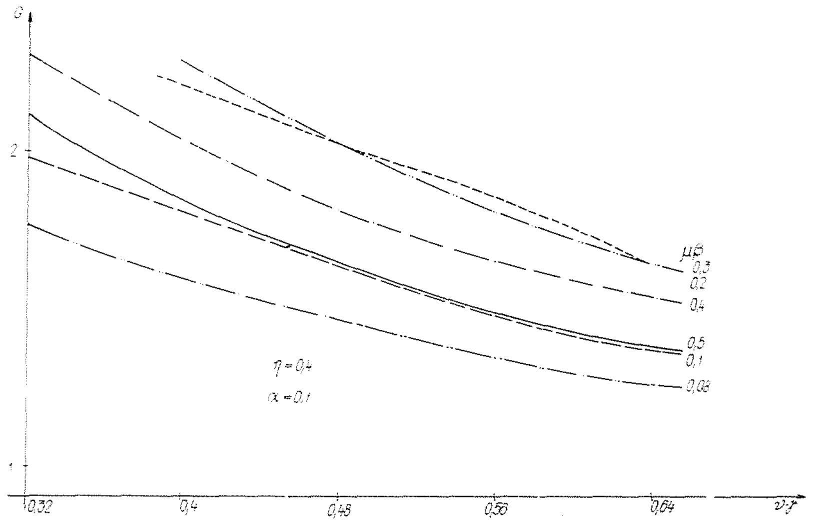

Fig. 8

For low $\mu\beta$ values, where wires are subject to plastic strain at failure, the joint $\nu\gamma$ coefficient hardly affects else than the cracking moment value, its increase reduces the load bearing range of the beam predicting an imminent failure.

For high $\mu\beta$ values, the ultimate moment decreases at a lower rate as a function of $\nu\gamma$, hence for decreasing $\nu\gamma$ the G value slowly increases. Fig. 8 shows the variation of G vs. $\nu\gamma$. Decrease of $\nu\gamma$ is seen to increase the ratio, while for identical $\nu\gamma$ values, the increase of $\mu\beta$ is concomitant with the decrease of G ratios.

Let us refer here to a previous statement that there is little failure indication by high $\mu\beta$ beams because of their relatively small deflection. Fig. 9 shows specific load-deflection curves for three beams with differently tensioned wires, from among those presented in [1]. Different beam behaviours from the aspects of both loads and deflections are clearly visible.

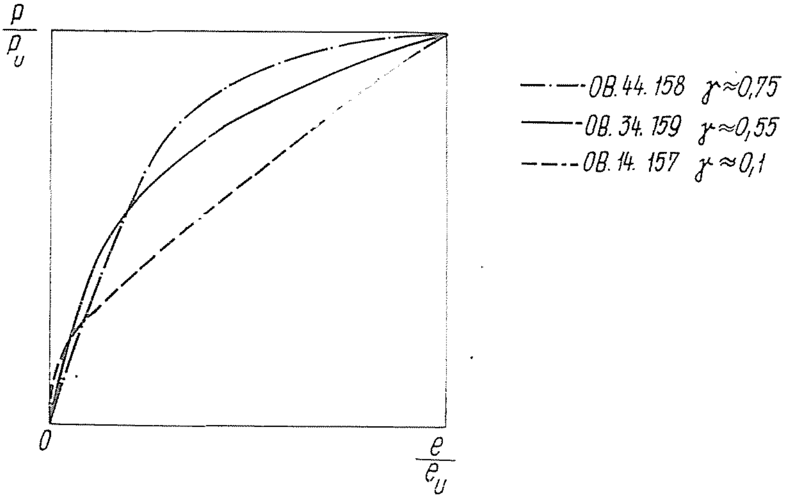

Fig. 9

Effect of the accuracy of the approximate stress-strain diagram of concrete

The approximate form of the stress-strain diagram of concrete has little effect on the G ratio. The diagram of two straight lines may simplify the calculation of ultimate moment, though it is harmless for the margin of safety of the structure. According to data in [1], the ultimate compression value of higher grade concretes exceeds at a slight safety the standard values. Significantly higher ultimate compressive strains have been observed for lower grade concretes ($5-6^0/_{00}$). (Notice that observed compression values significantly depend on the basis length of the instrument.) Higher ε_{cu} values could be specified for these concretes, though to the detriment of load bearing reserve. Let us mention the correlation between ultimate strain and optimum exploitation of the flexural cross-section.

Effect of specified ultimate tendon elongation due to outer loads

Specification of the allowed excess strain of steel ε_{cu} may have several objects, interpreted differently by codes. DIN 4227 tolerates 5‰ of excess elongation due to outer loads, together with cracks of the width and number as controlled by the "ultimate condition". At the same time, this value defines the degree of exploitability of the concrete cross-section. To restrict significant plastic deformations, the FIP—CEB recommendation (R 4,211—31 in [4]) sets a limit of 10‰ to total elongations.

Effect of ordinary reinforcement

This problem is of importance because in design practice often the entire tension represented by the tensile concrete flange is absorbed by the ordinary reinforcement, as customary for the design of reinforced concrete structures. This little affects the cracking moment, increases the ultimate moment, but greatly reduces both number and width of beam cracks and ultimate deflection. Hence, from this aspect it may induce a brittle failure of the beam.

Ordinary steel in the compression zone has little effect: it may increase the ultimate moment, reduce the necessary concrete depth (hence the dead weight) and increase thereby the beam toughness [5].

Thereafter only the part of tensile, ordinary reinforcement with a centre of gravity coincident with that of prestressing steel will be considered.

Let us write the Mörsch ultimate moment for the median of steels in the cross-section; assume ultimate strain to develop in the extreme compressed fibre of concrete:

$$M_u = \frac{3\varepsilon_{cu} - \varepsilon_{cc}}{3\varepsilon_{cu}} \, bh^2 \sigma_{cu} \, \xi(1 - 0{,}39\xi) \tag{16}$$

$$\xi = \frac{3\varepsilon_{cu}}{3\varepsilon_{cu} - \varepsilon_{cc}} \, (\mu_s \beta_s + \mu\beta\psi_1) \tag{17}$$

Pertaining ξ and ψ_1 values can be found by trial-error method in several steps. $\mu_s\beta_s$ values are seen in (17) to affect the ξ value, thus, for given $\mu\beta$ and $\nu\gamma$, a given percentage of ordinary reinforcement may reduce the prestressing steel stress at beam failure below the nominal yield point, to the detriment of beam toughness.

Substitute (16) into the numerator of the fraction (1) and simplify:

— for a cross-section in general:

$$G = \frac{\dfrac{3\varepsilon_{cu} - \varepsilon_{cc}}{3\varepsilon_{cu}} \vartheta^2 \xi (1 - 0{,}39\xi)}{\varkappa\varrho \dfrac{1+\Delta}{\Delta} \left[\dfrac{\alpha}{\lambda} + \dfrac{\mu\beta}{\varkappa} \nu\gamma \left(1 + \dfrac{\eta}{\varrho} \dfrac{\Delta}{1+\Delta} \right) \right]}; \tag{18a}$$

— for a rectangular cross-section:

$$G = \frac{2 \dfrac{\varepsilon_{cu} - \varepsilon_{cc}}{\varepsilon_{cu}} \vartheta^2 \xi (1 - 0{,}39\xi)}{\dfrac{\alpha}{\lambda} + \mu\beta\nu\gamma(1 + 6\eta)}. \tag{18b}$$

On the basis of (18*b*), let us consider the variation of G as a function of $\mu_s\beta_s$, for various $\mu\beta$ values, making use of the quoted data in [3]. From Fig. 10 it appears that for a prestress factor $\mu\beta > 0.3$, use of ordinary tensile reinforcement does not significantly increase the ultimate moment value. (For instance, quadruplation of the ordinary reinforcement percentage increased the G value from 1.6 to 1.8 and from 1.9 to 3.3 for $\mu\beta = 0.4$ and for $\mu\beta = 0.1$, respectively.)

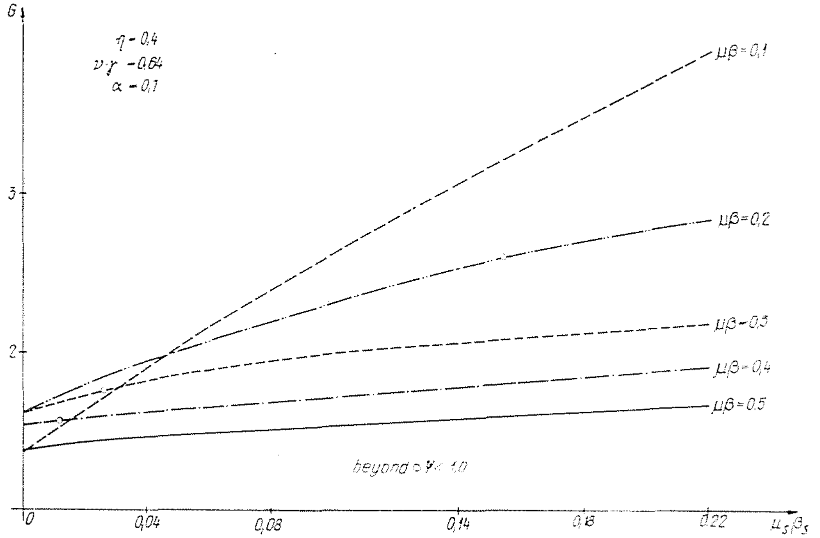

Fig. 10

In the same figure, on each $G = f(\mu_s\beta_s)$ curve belonging to any $\mu\beta$ value, the point was marked, indicating a $\mu_s\beta_s$ value beyond which $\psi_1 < 1.0$, prestressing wires do not yield at beam failure in bending. At the same time, beam deformations vs. load vary about linearly, imminent failure in bending is not signalled by an important plastic deflection. It is also seen that for $\mu\beta = 0.5$, wire strains remain below the nominal yield strain for even $\mu_s\beta_s = 0$, this is the case of overreinforcement. Values in Fig. 10 refer to

$$\eta = 0.4, \quad \nu\gamma = 0.64, \quad \varkappa = 0.1.$$

Obviously, $\nu\gamma$ i.e. the specific value of effective prestressing strain at time $t = \infty$ markedly affects both the steel strain at beam failure in bending and the deflection.

Beam design has to attempt maintenance of a prestress value in the tendons, sufficient for the wire stress at beam failure to exceed the nominal yield point with the maximum strain increase taken into account.

Conclusions

Design of bonded-wire prestressed beams has to comprise an analysis for the safety to brittle failure.

Possibility of under- or overreinforced beams has to be eliminated, either by specifying lower and upper limits for $\mu\beta$ and $\mu\beta + \mu_s\beta_s$ (according to the ACI Standard), or by specifying a lower limit for the ratio of ultimate to cracking moment of the cross-section [6], but in either case the designer must warrant that at beam failure, both prestressed and ordinary reinforcement stresses exceed either the nominal or the actual yield stress. It should be considered whether the complementary ordinary reinforcement intended to safely absorb tensions acting on the flexural concrete cross-section causes over-reinforcement of the beam.

Brittle failure can also be avoided by setting an upper limit to the mean compressive stress due to prestress in the cross-section. This requirement, together with setting a lower limit to the prestress for crack control or absence of tension in the tensile extreme fibre under outer load ([8]) imposes restrictions to the permitted steel percentage. These requirements are dependent on the cross-section shape, hence limits should be different for each type. By now, the recent Hungarian Highway Bridge Specification [6] contains — though a single — upper limit for mean prestressing stresses.

Provided the beam design does not meet requirements for safety to brittle failure, the safety factor of the beam — quotient of ultimate by working load — has to be increased. Recent Hungarian codes ([6], [7]) already contain such stipulations.

This study left the safety to beam failure in shear unconsidered. Failure in shear is known to be generally of brittle character, highly important to examine, and to our opinion, analysis of fictitious tensile stresses is insufficient.

Summary

After a definition of the concept of brittle failure, typical states of deformation of underreinforced, normally reinforced and overreinforced bonded-wire prestressed concrete beams under load have been presented. General expressions have been given for the determination of the Mörsch ultimate moment, the cracking moment and their ratio.

Examining the phenomenon of brittle flexural failure as the variation of the ratio of ultimate to cracking moment of beams, effect of various properties (geometry, strength) of the beam cross-section on this ratio has been presented.

An evaluation is given of the efficiency of this type of code specifications and modifications are proposed to specification stipulations for safety to brittle failure.

References

1. WARWARUK, J.—SOZEN, M. A.—SIESS, CH. P.: Investigation of Prestressed Reinforced Concrete for Highway Bridges. Part II. "Strength and Behaviour in Flexure of Prestressed Concrete Beams", University of Illinois, Engineering Experiment Station, Bulletin No. 464, 1962.

2. OLADAPO, I.: Relationship Between Moment Capacity of Flexural Cracking and at Ultimate in Prestressed Concrete Beams. ACI Journal, Proceedings V. 65. No. 10. Oct. 1968.
3. TASSI, G.—KLATSMÁNYI, T.—WINDISCH, A.: Study of some stipulations in Chapter G: Prestressed Concrete Structures of the Highway Bridge Specification 1968.* (Manuscript).
4. Recommandations Pratiques pour le Calcul et l'Exécution des Ouvrages en Béton Précontraint. Comité FIP—CEB. Bulletin d'Information No. 54. Mars 1966.
5. GURFINKEL, G.—KHACHATURIAN, N.: Ultimate Design of Prestressed Concrete Beams. University of Illinois. Engineering Experiment Station. Bulletin 478. 1965.
6. KPM HI/1—68 Highway Bridge Specification.* Part II. Közlekedési Dokumentációs Vállalat, Budapest, 1968.
7. Prestressed Concrete Structures.* Hungarian Standard Tentative MSz 15022/p. 2.
8. KLATSMÁNYI, T.—TASSI, G.: Contribution to the Analysis of Stress in Prestressed Concrete Beams. Periodica Polytechnica C. E. **14** (1970).

* In Hungarian.

Assistant Andor WINDISCH, Budapest XI., Sztoczek u. 2, Hungary