EFFECT OF STRAIN-HARDENING ON THE ELASTIC-PLASTIC BEHAVIOUR OF BEAMS

By

M. IVÁNYI

Department of Steel Structures, Budapest Technical University

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Presented by Prof. Dr. O. HALÁSZ

1. Analysis of the elastic-plastic behaviour of rolled steel I beams has to involve a number of phenomena and effects such as strain-hardening, inelastic instability, moment and shear gradient, initial imperfections. Among these, it is the strain-hardening, representing the central subject of this study, through which all other effects will be investigated.

2. In the case of mild steel, the stress-strain diagram (Fig. 1) shows after a definite yield strain-hardening to take place. Yield occurs in the so-called yield planes, and according to NÁDAI [1] yield deformation ε_y jumps over to the beginning of strain-hardening $s\varepsilon_y$. Metallurgically the phenomenon might be explained by dislocations of the polycrystalline group spreading over the crystal surfaces and assembling into a yield plane. This means that in plastic design a discontinuous stress-strain law might be applied. Up to the moment of complete strain-hardening the material may be considered heterogeneous, and only the average strain ranges between ε_y and $s\varepsilon_y$; parts of the specimen are either elastic, or strain hardened, these latter being the yield planes.

A number of researchers used similar model in their investigations [4, 6, 7, 8].

The idealized stress-strain diagram neglects several features of the real stress-strain diagram (Fig. 1, short dashed line). Yield begins with the appearance of an upper yield point σ_{yU} and to maintain the yield a dynamic yield stress σ_{yD} is needed; if the straining is stopped at this portion, stress drops to the static yield stress level σ_{yS}. The phenomenon of the upper yield stress may often miss owing to the inhomogeneity of the material, rate and character of loading.

3. Moment-deflection curves of tests conducted on steel beams with strain-hardening properties ([3, 6, 7]) describe the behaviour of beams under different loads. Beams subject to *moment gradient* (Fig. 2*b*, dashed line) are characterized by the increase of the moment beyond the plastic moment value M_p and by failure by local buckling or lateral buckling. Beams subject to *uniform moment* (Fig. 2*a*) fail by undue deformation owing to strain-harden-

ing. The effect of premature loss of stability of the compressed flange is shown by the curve OAB in Fig. 2.

4. For the plastic analysis of beams in bending a special chapter of the theory of plasticity has been widely adopted; the so-called ultimate strength theory or limit design method [2], [5]. The ultimate strength theory assumes a plastic hinge and herewith, occurrence of unlimited rotations without the

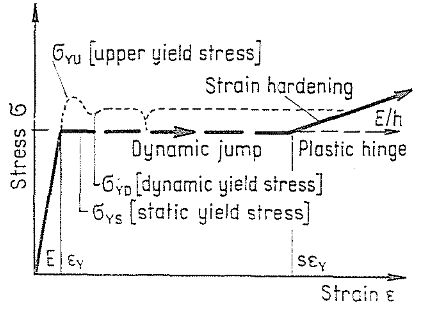

Fig. 1. Stress-strain curve for steel [6]

change of the plastic moment M_p acting on the cross-section. These rotations permit the redistribution of the moments in the whole structure. The preconditions, i. e., the limits of applicability of the plastic hinge and hereby those of the ultimate strength theory have been compiled in [5].

The relationship (Fig. 3*a*) between the moment M and the curvature $\varkappa$ may be determined [2] from the ideal elastic-plastic or rigid-plastic stress-strain diagrams for plastic hinges and corresponds to the point-hinge assumption. Let us assume a simply supported beam acted upon by a concentrated load at midspan (Fig. 3*b*); up to a moment M_p, only elastic deformations occur, so that over zero length ($\tau L = 0$) deformation will be infinite. Theoretical calculations show this deformation to reach its final value before a complete moment redistribution could take place, test results, however, are inconsistent with this statement [4].

5. Elastic-plastic behaviour of the beam may be more exactly described by strain hardening. The moment curvature diagram (Fig. 3*c*) takes the effect of strain-hardening into consideration. The $M - \varkappa$ diagram has been determined from the stress-strain relationship and from the geometry of the cross-section [2]. If the cross-section is assembled of flanges of zero thickness, the stress-strain and moment-curvature relationships are geometrically similar; from the non-zero thickness follows that in the range of strain-hardening, the slopes of the two curves will be different (the slope of the $M - \varkappa$ curve will be flatter). In the curvatures associated with the deformations outlined above, a dynamic jump takes place. In practice, this jump occurs after the complete yield of the flanges.

In the beam treated as an example (Fig. 3*d*), the maximum moment may exceed the plastic moment M_p; though a slight change in the value of the moment results in a significant change of the curvature and the resulting deformations show a good agreement with the test results obtained on beams subject to moment gradient (Fig. 2*a*, dashed line).

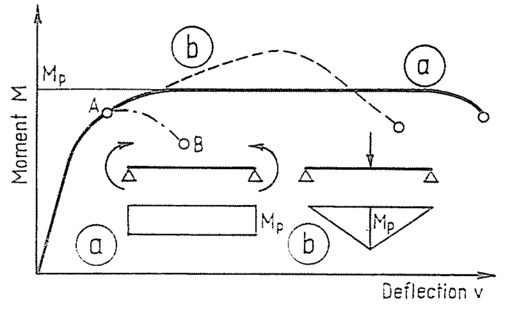

Fig. 2. Possible load-deflection curves for beams [3]

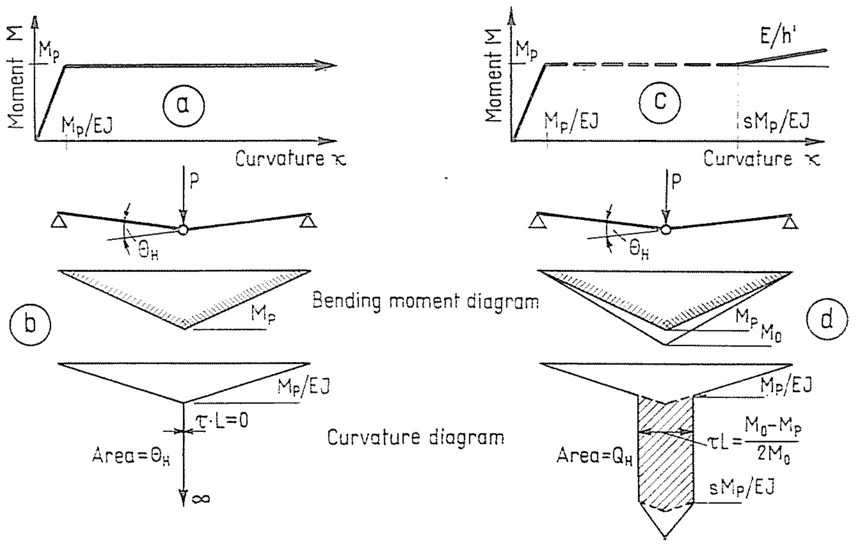

Fig. 3. Simply supported beam under a point load at midspan [4]

Application of the plastic hinge for demonstrating the actual behaviour of the beam is, in certain instances, a deceiving model. It is doubtless, however, that its use is often rather suggestive, simple, and permitting conclusions without significant error. By decomposing the diagram of curvatures (Fig. 3*d*), a method easily treated with the aid of the matrix formula, taking the strain hardening into account, may be worked out [4].

6. In a beam subject to uniform moment, this latter will not exceed M_p, but the development of a great deformation indicates the effect of strain hardening. In this case, the behaviour may be described by the relationship between the moment and the rotation of the end face; and the dynamic jump to the value associated with the strain hardening takes place at the value of the end face rotation belonging to the beginning of yield.

7. As it was mentioned already at the analysis of the tests, failure is commonly caused by the loss of stability of the compressed flange. By instability of the compressed flange two phenomena are meant: the lateral buckling, that is, deflection of the compressed flange, out of the load plane, combined with rotation and local buckling. These phenomena are common to occur simultaneously, but in special cases they might be treated also separately. Tests showed lateral buckling of a simply supported beam subject to uniform moment to begin at M_p; in case of adequate construction, however, the moment capacity in the vertical plane does not decrease (Fig. 2*a*, full line) despite the lateral displacement. Total failure takes place in the form of local buckling in a way that on the side subject to compression resulting from bending in the horizontal plane caused by lateral buckling and from bending in the vertical plane, the flange will buckle.

A simple beam loaded at midspan and laterally supported at the point of load transfer will not buckle laterally. The beam will fail by local buckling of the flange.

8. A number of researchers tried their best to analyse the phenomenon of lateral buckling; researchers of the Lehigh University [3, 4, 6, 7, 8] conducted many experiments on and theoretical investigations of strain hardening materials. They assumed that from the viewpoint of lateral buckling the compressed flange may be considered separately. According to the stress-strain diagram (Fig. 1), the material seems to lose stiffness after yield, and strain hardening begins only after a certain deformation $s\varepsilon_y$. According to the interpretation of Nádai [1], at post-yield deformation jump, parts subject to yield strain-harden; the material, though inhomogeneous, with different properties, loses its stiffness only gradually and will be homogeneous again when strain-hardening is complete. In this way the occurrence of the critical moment causing lateral buckling might be prevented even in case of finite supporting length.

The phenomenon might be discussed as a stability problem (Fig. 4*b*, full line), but reality is better approached by assuming initial inaccuracies (Fig. 4*b*, dashed line). Practically, after the complete yield of the flanges, the beam possesses again the stiffness corresponding to the modulus of strain-hardening, and this is the state for which the supporting length — commonly expressed as a multiple of the radius of gyration pertaining to the vertical axis, —which permits the development of the great deformation represented by the full line in Fig. 2*a*, can be deduced.

9. The theoretical analysis i.e. description of the local buckling of the flange, is rather intricate, because the theories of local buckling in plastic ranges are somehow contestable. An investigation in the range of strain-hardening requires, beside the modulus of strain-hardening E/h, also the shear modulus of strain hardening G' to be determined. LAY [8] determined G' in a theoretical way, assuming that shear planes fall firstly in the plane of the maximum shear stress, he determined the new shear modulus of these planes

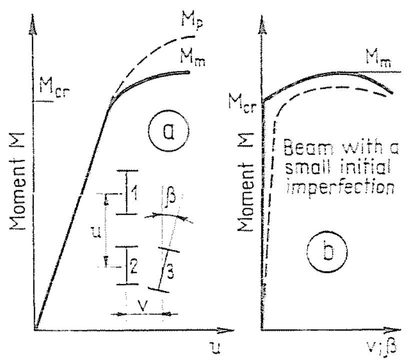

Fig. 4. Simplified moment-deformation curves [7]

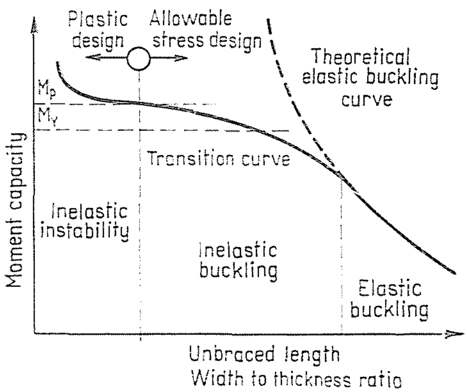

Fig. 5. Ranges of beam behaviour [3]

and owing to the inhomogeneity, he averaged the shear moduli as described above, and of the part remaining elastic. Solution of the equation of local buckling furnishes the ratio of flange thickness to flange width. In the compressed flange of the beam subject to moment gradient, the restraining effect of the elastic parts should be taken into account (the steel along the entire wave length needed for flange local buckling must yield).

In most cases, after flange local buckling the load capacity is not fully lost; lateral buckling will develop which, though could be prevented by a suitable lateral bracing, yet, owing to the implications involved in local buckling analysis, it is not worth while to endeavour to utilize load capacity of the beam to the extreme extent.

10. Plastic behaviour of beams has been studied with due regard to strain-hardening. Tentatively, three significant ranges of beam instability will be distinguished (Fig. 5):

— 1st range where the cross-section is in an entirely plastic state;
— 2nd range where instability occurs at partial yield; and
— 3rd range characterized by elastic behaviour.

Moment capacity of a beam depends on the unbraced length and on the ratio of flange width to flange thickness. In the first range flange local buckling and lateral buckling occur simultaneously in the cross-section of complete yield which, though limits deformability, yet permits the development of plastic moment; in this range, plastic design method is applied.

In the second range, neither complete moment capacity nor adequate deformability exist, in this range the design method based upon the allowable stresses is to be used.

Confines between the three ranges are somewhat arbitrary, transitions being gradual, they have only been defined with a view on easy treatment.

11. Plastic analysis of the beams has been based on the ultimate strength theory or limit design method.

In some cases requiring higher standard of accuracy, models with strain-hardening properties are needed, but in this case, further investigation on home-made mild steels is required in the ranges of yield and of strain-hardening; some basic tests should be carried out on rolled I-beams, the construction rules needed for preventing plastic instability have to be established and justified by experiments. Relevant experiments are being carried out at this Department.

Summary

Investigations have been carried out to clear up how the strain-hardening of steel could be taken into account in designing I-beams. Basis of the analysis was a discontinuous stress-strain model which described the behaviour of the beams more exactly than the ultimate strength theory or limit design method.

The in-plane behaviour of the beams subject to moment gradient and uniform moment are examined with the help of the moment-curvature relationship which is, similarly to the stress-strain diagram, characterized by discontinuity.

Attention is called upon the significance of the strain-hardening in the analysis of the phenomenon of instability (lateral buckling, local buckling), then a survey is made on the models applied for investigating this phenomenon.

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Assistant MIKLÓS IVÁNYI, Budapest XI., Műegyetem rakpart 3. Hungary.