DESIGN OF RIVETED, BOLTED AND ADHESIVE-BONDED JOINTS

By

F. Szépe

Department of Steel Structures, Budapest Technical University
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Presented by Prof. Dr. O. Halász

1. Introduction

In practical design, uniform load distribution between fasteners is assumed. This is correct, provided the plates are infinitely rigid and the fasteners behave elastically. Specifications generally allow for this assumption, some of them controlling the number of fasteners parallel to the axial load. The special literature reports of several, sometimes contradictory attempts of better approximating real load distribution and load capacity. A simple and generally valid method will here be suggested for the design of joints.

2. Notations

a — pitch or longitudinal spacing of the hole, with symbols of both adjacent fasteners in subscript;
a' and a'' are longitudinal hole spacings as affected by the load, referring to the cover plate and to the mainplate, respectively;
e — deformation (displacement) of a fastener, with serial number of the fastener in subscript;
N — fastener load, with serial number of the fastener in subscript;
C — spring constant of a fastener, load causing a deformation of 1 cm, $C = N/e$;
P — joint load;
n — number of fasteners parallel to the axial load, n_A and n_B being the required number of fasteners in elastic range and at ultimate load, respectively;
F — plate cross-sectional area, significant for the deformation, with the same subscript as for the pitch;
E — elastic modulus of the plate;
subscript A refers to the maximum load or strain where the stress-strain relationship can be considered linear from the aspect of joint forces (cf. limit of proportionality for a standard bar coupon);
subscript B refers to failure, e.g. e_B is deformation of a fastener at its ultimate load, or N_B is the ultimate bolt load;
τ — shear stress.

3. Behaviour of sheared joints

It is advisable to distinguish three ranges in joint behaviour: 1. elastic range; 2. plastic range (where fasteners behave plastically) and 3. state of failure. The conventional design method approximates range 2 but is unreliable for assessing the joint load capacity. In case of repeated loads (fatigue) or of serious deformation restrictions the design is advisably based on range 1 or elastic, and otherwise on range 3, i. e. state of failure.

4. Theory of sheared joints

Joint load P (Fig. 1) is transmitted from one plate to the other by unknown fastener forces. These forces and pitch-dependent forces produce deformations in fasteners and plates, respectively. Along the pitch between

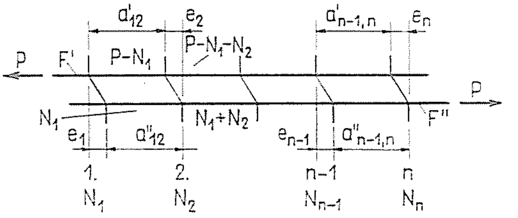

Fig. 1

e. g. fasteners 1 and 2, deformations of fasteners and of plates evidently must satisfy the equality:

$$e_1 + a''_{12} = e_2 + a'_{12}$$

arranged:

$$e_1 - e_2 = a''_{12} - a'_{12} \tag{1}$$

Statement expressed by (1) is of general validity. For each pitch an equation similar to (1) can be written, hence to n fasteners belong $n - 1$ equations. The n-th equation required for determining n unknown fastener forces expresses the equilibrium condition:

$$\Sigma N = P \tag{2}$$

5. Load distribution in the elastic range

Fastener forces can be calculated according to the elastic range, while fastener deformation can be considered proportional to the expected force (max. N_A) and that of the plate to the plate force (max. P_A). Eqs. (1) and (2) lend themselves to determine each fastener force. [1] presents a simple method to determine unknown fastener forces.

5.1. *Design principle in the elastic range*

In the elastic range, the design principle states: the required number of fasteners n_A has to be defined so that for a joint load P_A, the force N_1 acting at the fastener of maximum stress should not exceed N_A and for one fastener less, $N_1 > N_A$. Less than n_A fasteners must not be applied, number of fasteners $n > n_A$ would not increase joint load capacity. In general, the required number of fasteners is easier to determine than the force distribution, namely either by calculation or graphically.

5.2. *Calculation of the required number of fasteners*

For sake of simplicity, let the cross-sectional area $F' = F'' = F$ along the joint be constant. If, in conformity to the design principle, $N_1 = N_A$ for $P = P_A$, Eq. (1) can be written for the final pitch of the joint as:

$$e_A - e_2 = \frac{a}{EF}(P_A - N_A) - \frac{a}{EF} N_A \tag{3}$$

Here the only unknown e_2 can be calculated from (3), leading to $N_2 = e_2 C$. For the next pitch:

$$e_2 - e_3 = \frac{a}{EF}(P_A - N_A - N_2) - \frac{a}{EF}(N_A + N_2)$$

Expressing e_3 yields N_3.
The process can be continued up to $\Sigma N \geqq P$. Number of fasteners involved until this condition is met gives the number required.

5.3. *Graphical determination of the required number of fasteners*

There is a simple graphical method to determine the number of fasteners required for joints in the elastic range, provided N_A and e_A for a single fastener as well as P_A and pitch deformation Δa_A due to P_A are known.

From their knowledge, by analogy to (3):

$$e_A - e_2 = \frac{a}{EF}(P_A - N_A) - \frac{a}{EF} N_A$$

Plotting P as a function of Δa (Fig. 2a):

$$P = \frac{EF}{a} \cdot \Delta a$$

According to the initial assumption, $F' = F''$, hence the same straight line describes both mainplate and coverplate, and forces $P_A - N_A$ and N_A entrain values $\Delta a'$ and $\Delta a''$, respectively, spacings differing by $e_A - e_2$.

Plotting can further be simplified by involving the function

$$P = P_A - \frac{EF}{a} \cdot a$$

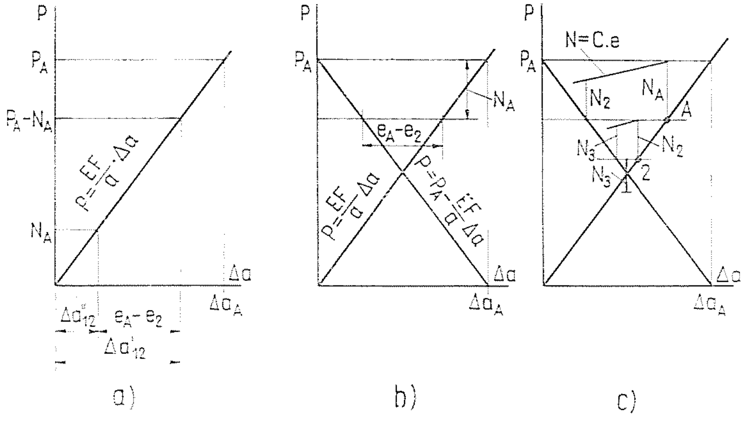

Fig. 2

a straight line connecting P_A to Δa_A (Fig. 2*b*). Namely here the two sloping straight lines representing rigidities of mainplate and coverplate cut out exactly the spacing $e_A - e_2$ of the straight line representing the relationship $P = P_A - N_A$ and parallel to the Δa axis.

Superposing diagram $N = C \cdot e$ to the distance e_A to e_2 (e_A over point A, Fig. 2*c*), N_2 can be read off directly or projected to point 2. Thereafter, superposing the rest of diagram $N = C \cdot e$ to the distance e_3 to e_2 yields N_3, etc.

As soon as the plot is at or beyond the intersection of both straight lines, the lowest number of fasteners required is obtained, namely for $F' = F''$, this intersection indicates half the force to be transferred so that the relevant number of fasteners n' allows to conclude on the required $n_A \cdot (n_A = 2n'$ or $n_A = 2n' - 1$, for particulars see [1]). For $F' \neq F''$, plotting has to be continued beyond $P_A/2$, up to P_A.

5.4. *Impotent joints*

Graphical determination of the number of fasteners lends itself to represent the case where the joint load capacity cannot be further increased by applying more fasteners of given characteristics, as shown in Fig. 3. Along the usual plotting process the fastener deformability is exhausted before reaching intersection $P/2$, it is ineffective to increase the number of fasteners, joint is an impotent one. For the elastic range, a formula for the criterion of impotency can be deduced; high-capacity fasteners with high $N_A \cdot e_A$ values are preferred.

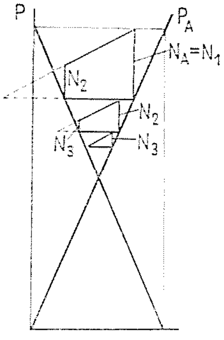

Fig. 3

6. Analysis of the failure state

In failure state 3, forces affecting both the plate and the fastener of maximum stress produce stresses beyond the limit of proportionality. To further increase the load is controlled by the failure of either the fastener under maximum stress, or of the mainplate or of both simultaneously.

Fastener spacings are invariably controlled by equations of deformation, which latter is no more proportional with the load, so that calculation methods in the elastic range are useless. The problem can be solved by iteration.

6.1. Ultimate design. Graphical determination of the required number of fasteners

The problem can be formulated as: both plate and fastener of maximum stress should fail under the same load i. e. plate load P_B should produce force N_B in the fastener of maximum stress, thus, upon ultimate plate load P_B, $N_1 \leq N_B$ for n fasteners but $N_1 > N_B$ for $n - 1$ fasteners.

The required number of fasteners may be determined graphically, similarly to the plotting method for the elastic range (Fig. 2), but deformation lines are curves.

The described graphical method can be replaced by calculation, provided the deformation curves can be described by functions, but this is of no special advantage.

6.2. Fastener categories

From load capacity aspect, multilinear joints belong to either of three categories:

The first category includes those with fasteners fully developing their individual load capacities. However favourable their physical properties are, this possibility is restricted to compact joints, below a given number of fasteners.

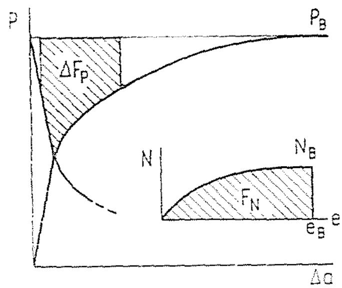

Fig. 4

In the second category, fasteners are nonuniformly loaded, so that only extreme ones are fully efficient. Joints with uniform load capacity are possible, at the cost, however, of an increased n as compared to the conventional design method ($n_B > P_B/N_B$).

The third group includes impotent joints characterized by the impossibility of uniform load capacity, irrespective of any increase of fastener number. Is there a possibility to guess the occurrence of joint impotency without the presented graphical process?

According to the graphical method, the fastener deformation diagram must overlap most of a clearly outlineable area of the plate deformation diagram. The joint is clearly not impotent where, with notations in Fig. 4:

$$F_N \geqq \Delta F_P$$

The fastener is advisably classified by the area of its deformation diagram.

7. Design of adhesive-bonded joints

Provided test results are available, the presented design method lends itself to riveted, bolted joints and to those with high-strength bolts of any material or rigidity, and even may be extended to adhesive-bonded joints. While in the design with metal fasteners, the number of fasteners required for a uniform load bearing is unknown, the design of adhesive-bonded joints consists in determining the length of overlap providing uniform load capacity.

The design with metal fasteners requires knowledge of deformation curve of the fastener and that of the connected material in one pitch. The design of adhesive bonded joints is based on the knowledge of the deformation (displacement) curves of sheared adhesive layer of length a and of a plate of the same length. Anyhow, a should be taken as short as to permit the shear stress within the adhesive layer of length a to be considered as uniformly distributed. From both deformation curves the length of overlap required can closely be approached by the graphical method as presented for the design of metal fasteners.

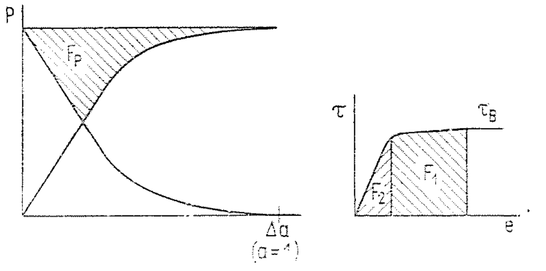

Fig. 5

From the aspect of load capacity the adhesive bonded joints belong to either of three categories, similarly to fastener joints (item 6,2). With notations in Fig. 5 (for sake of simplicity, again on the basis of case $F' = F'' = F$):

First category:

$F_1 \geq F_P$ — shear stress distribution is about uniform ($l_B = P_B/\tau_B$).

Second category:

$F_1 + F_2 > F_P$ — there is a possibility of uniform load capacity but required length of overlap is $l_B > P_B/\tau_B$ · For $F_1 + F_2 = F_P$, the required length is infinite.

Third category:

$F_1 + F_2 < F_P$ — impotent joint.

Because of the physical properties of adhesives, impotent joints are more frequent than among fastener joints.

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Summary

In practical design, joint load is assumed to be uniformly distributed between fasteners. Special literature presents several attempts to better approach real load distribution and load capacity. The design consists in determining required number of fasteners, based on the principle that the joint load (e. g. ultimate load) produces at most a predetermined load (e. g. ultimate load) in the fastener. On this basis a simple, generally valid method has been proposed to design joints and to categorize fasteners.

Reference

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Associate Professor Ferenc Szépe, Budapest XI., Műegyetem rakpart 3. Hungary.