https://doi.org/10.3311/PPci.43847 | 643


Periodica Polytechnica Civil Engineering, 70(2), pp. 643–653, 2026

# Estimation of the Hoek–Brown Constant $m_i$ from Analyzing the Uniaxial Compressive Test

Samad Narimani[1], Balázs Vásárhelyi[1*]

[1] Department of Engineering Geology and Geotechnics, Faculty of Civil Engineering, Budapest University of Technology and Economics, Műegyetem rkp. 3., H-1111 Budapest, Hungary
[*] Corresponding author, e-mail: vasarhelyi.balazs@emk.bme.hu

Received: 01 March 2026, Accepted: 27 May 2026, Published online: 15 June 2026

**Abstract**

The Hoek–Brown failure criterion is a cornerstone of rock mechanics and is widely applied in the design of underground excavations, slopes, and foundations. However, determining its intact rock constant ($m_i$) conventionally requires multiple triaxial compression tests under varying confining pressures, which are costly, time-consuming, and often infeasible when core quality or sample availability is limited. Building upon recent advances in empirical, probabilistic, and elastic-based approaches, this study develops and validates a practical method for estimating $m_i$ from standard uniaxial compressive strength (UCS) tests through analysis of the stress-dependent Poisson's ratio. The proposed framework establishes a mechanical linkage between $m_i$ and the lateral deformation behavior of intact rock, reflecting the influence of microcrack closure and brittleness. Extensive UCS data for granite, limestone, marl, sandstone, and rock salt were analyzed to evaluate the method's reliability. The estimated $m_i$ values show excellent agreement with triaxial test results for brittle lithologies and acceptable accuracy for more ductile rocks. Monte Carlo uncertainty analysis confirms the robustness of the approach, particularly for crystalline and well-cemented formations. The method offers a cost-effective and theoretically grounded alternative for preliminary design and rock characterization where triaxial testing is impractical, thereby enhancing the applicability of the Hoek–Brown criterion in routine engineering practice.

**Keywords**

rock failure envelope, Hoek–Brown material constant, Poisson's ratio, laboratory test, uncertainty analysis

## 1 Introduction

One of the most commonly used failure criterion in the rock mechanical and rock engineering practice is the Hoek–Brown failure criteria, which can be expressed as follow for intact rock [1]:

$$\sigma_1 = \sigma_3 + \sigma_c \left( m_i \frac{\sigma_3}{\sigma_c} + 1 \right)^{0.5}, \quad (1)$$

where $\sigma_1$ and $\sigma_3$ are the major and minor main stresses, respectively, $\sigma_c$ is the uniaxial compressive strength of the intact rock and $m_i$ is the Hoek–Brown (HB) material constant, which depends on the type of the rock, among the others.

The Hoek–Brown failure criterion is widely implemented in numerical models and engineering design guidelines for underground excavations, tunnels, slopes, and mining. The parameter $m_i$ strongly influences predicted rock mass strength, deformation, and failure behavior. Therefore, reliable estimation of $m_i$ is essential for safe and economical design. The Hoek–Brown material constant, $m_i$ can be determined with triaxial tests. Minimum 5 intact rock specimens are required for these measurements – the laboratory investigations are time-consuming and expensive. To reduce the number of specimens, Kovári and Tisa [2] and Kovári et al. [3] proposed a new laboratory measurement method for determining the failure envelope on a specimen, which is also included in the proposals of ISRM [4]. Recently, Narimani et al. [5] presents the advantage and disadvantages of this 50 years-old procedure.

In recent decades, several researchers have proposed alternative techniques to estimate the Hoek–Brown material constant $m_i$ using indirect parameters and statistical correlations. Marinos and Hoek [6] emphasized the empirical relationships between geological strength index (GSI), lithology, and intact strength parameters. [7, 8] further developed classification-based estimation charts for $m_i$. More recently, Aladejare and Wang [9] incorporated

**Cite this article as:** Narimani, S., Vásárhelyi, B. "Estimation of the Hoek–Brown Constant $m_i$ from Analyzing the Uniaxial Compressive Test", Periodica Polytechnica Civil Engineering, 70(2), pp. 643–653, 2026. https://doi.org/10.3311/PPci.43847

probabilistic regression approaches, while Arshadnejad and Nick [10] and Tsiambaos and Sabatakakis [11] investigated empirical correlations between $m_i$ and uniaxial compressive strength (UCS) for various lithologies. These studies generally agree that $m_i$ is not constant even within a lithological group but varies with microstructural features such as grain size, cementation, and degree of fracturing. However, these correlations remain largely empirical and often require extensive datasets for calibration, which limit their universal applicability.

In addition to empirical and probabilistic approaches, several studies emphasized the fundamental role of tensile cracking and crack initiation processes in controlling brittle rock failure and the Hoek–Brown material constant. Diederichs [12] demonstrated that damage initiation, spalling, and brittle failure around deep underground excavations are strongly governed by the relationship between compressive and tensile strength parameters. Later, Perras and Diederichs [13] provided a comprehensive review of tensile strength concepts and testing methods for rocks, highlighting the importance of tensile cracking in brittle failure processes. These studies suggest that indirect relationships between uniaxial compressive strength (UCS), tensile strength, crack initiation stress, and HB parameter $m_i$ may provide a physically meaningful framework for estimating rock strength properties from standard laboratory tests.

## 2 Limitations of triaxial testing

Triaxial testing has long been regarded as a standard approach for estimating intact rock strength parameters, including the Hoek–Brown constant $m_i$, because it subjects specimens to controlled confining pressures that better resemble *in situ* stress states than uniaxial tests. However, despite this advantage, conventional triaxial testing has several practical and methodological limitations that can constrain its routine application in engineering practice.

First, triaxial tests are time-consuming and costly. Preparing high-quality cylindrical specimens, applying multiple confining pressure levels, and conducting full strength envelopes require significant laboratory time and resources. For a meaningful estimation of intact strength parameters such as $\sigma_{ci}$ and $m_i$, a series of triaxial tests at different confining pressures often must be performed, which increases both testing time and variability in results due to specimen heterogeneity and sampling errors [9, 14].

Second, sample preparation and quality control present challenges. Triaxial specimens must meet strict geometric tolerances and surface finishes to ensure uniform stress transmission and avoid eccentric loading, which can distort results. In practice, especially for poorer quality or fractured rocks, obtaining enough intact cores that satisfy these requirements is difficult, leading to weak statistical confidence in the derived strength envelope.

Third, conventional triaxial tests ignore three-dimensional stress states. Most laboratory triaxial cells apply an equal confining pressure in the two minor principal stress directions ($\sigma_2 = \sigma_3$), whereas in the field the intermediate principal stress $\sigma_2$ can differ significantly from $\sigma_3$. True triaxial tests that vary all three principal stresses more realistically are still uncommon due to the complexity and cost of the apparatus, which limits the understanding of stress path effects on rock strength [15, 16].

Fourth, variability and scale effects can influence test outcomes. Rock mechanical properties, including strength parameters, may depend on specimen size and inherent heterogeneity, resulting in scale effects that make the extrapolation of laboratory results to field conditions uncertain. Smaller cores can exhibit higher variability compared to larger blocks, affecting regression fits to failure criteria like Hoek–Brown [17].

Fifth, completing a full triaxial test program for each rock type is not always feasible during early project phases. Many engineering projects begin with only routine uniaxial compressive strength (UCS) data available, while triaxial testing may be deferred due to budgetary or schedule constraints. Moreover, tensile strength tests, which are also relevant for Hoek–Brown parameter estimation, are rarely conducted as standard laboratory procedures because of specimen preparation challenges.

Taken together, these limitations create a practical need for alternative or complementary approaches that can reliably estimate key Hoek–Brown parameters such as $m_i$ using more readily available laboratory data. The present study addresses this gap by proposing a method to infer $m_i$ from stress-dependent Poisson's ratio and standard UCS results, offering a cost-effective and time-efficient alternative to extensive triaxial testing.

## 3 Theoretical frameworks: relationship between $m_i$, UCS, tensile strength, and Poisson's ratio

Several studies have attempted to relate the Hoek–Brown constant to measurable elastic parameters. Hoek and Diederichs [18] proposed that Poisson's ratio and Young's modulus could serve as proxies for estimating rock deformability and, indirectly, $m_i$. Singh and Goel [19] explored the use of elastic constants for empirical estimation of the

Hoek–Brown parameters, emphasizing their dependence on lithology and confinement. More recently, Wang and Shen [20] and He et al. [15] studied the coupling between triaxial strength, brittleness, and deformation parameters, suggesting that Poisson's ratio captures essential information on microcrack evolution. Despite these advancements, a generalized and practical model that directly links $m_{\mathrm{i}}$ to the stress-dependent Poisson's ratio from standard UCS tests remains lacking. This motivates the present study.

The Hoek–Brown material constant $m_{\mathrm{i}}$ is widely recognized as a key parameter describing the intrinsic strength and brittleness of intact rock [1]. Physically, $m_{\mathrm{i}}$ reflects how rapidly rock strength increases with confining pressure and is therefore closely related to the rock's internal microstructure, crack density, grain interlocking, and frictional resistance along potential failure planes. Despite its importance, $m_{\mathrm{i}}$ is traditionally determined from multiple triaxial compression tests, which are labor-intensive, costly, and often impractical in routine engineering investigations. This has motivated several researchers to explore indirect relationships between $m_{\mathrm{i}}$ and more easily measurable rock properties.

One of the earliest and most practical links between $m_{\mathrm{i}}$ and basic strength properties was proposed by Cai [21], who demonstrated that for brittle rocks the ratio between uniaxial compressive strength $\sigma_c$ and tensile strength $\sigma_t$ correlates closely with the Hoek–Brown constant when this ratio exceeds approximately 8. This observation suggests that $m_{\mathrm{i}}$ is fundamentally related to rock brittleness, since a high $\sigma_c/\sigma_t$ ratio typically characterizes brittle materials, whereas lower ratios indicate more ductile behavior. Rocks with higher brittleness tend to exhibit higher $m_{\mathrm{i}}$ values, reflecting a steeper increase in strength with confinement.

The relationship between compressive strength, tensile strength, and the Hoek–Brown constant has also been discussed in the context of brittle crack initiation and spalling phenomena. Following the concepts summarized by [12, 13], the Hoek–Brown material constant may be approximated from the ratio between uniaxial compressive strength and tensile strength:

$$m_{\mathrm{i}} \approx \mathrm{UCS}/\sigma_t , \tag{2}$$

where UCS is the uniaxial compressive strength and $\sigma_t$ is the tensile strength of the intact rock. This relationship reflects the brittle character of the rock material because highly brittle rocks generally exhibit high UCS/$\sigma_t$ ratios and correspondingly high $m_{\mathrm{i}}$ values.

The reviewer also highlighted the relationship between tensile strength and crack initiation stress (CI), expressed as:

$$\sigma_t \approx \mathrm{CI}/\beta , \tag{3}$$

where $\beta$ is commonly taken as 8 according to Griffith [22] and approximately 12 according to Murrell [23]. Combining these relations yields:

$$m_i \approx \left(\mathrm{UCS} \times \beta\right)/\mathrm{CI}. \tag{4}$$

These formulations further support the interpretation that the Hoek–Brown constant is fundamentally linked to brittle crack initiation and tensile failure mechanisms.

At the same time, Poisson's ratio ($v$) is a key elastic parameter that describes the ratio between lateral and axial deformation under loading. Physically, Poisson's ratio is sensitive to the internal crack structure of the rock [24]:

- At low stress levels, pre-existing microcracks remain open, leading to lower lateral deformation and thus lower Poisson's ratio.
- As axial stress increases, microcracks progressively close, resulting in higher lateral strain and an increase in Poisson's ratio.

This stress-dependent evolution of Poisson's ratio implies that $v$ is not merely an elastic constant but also an indicator of damage and crack closure processes within the rock matrix.

Lógó and Vásárhelyi [25] showed that Poisson's ratio is systematically related to the $\sigma_c/\sigma_t$ ratio, meaning that rocks with higher compressive-to-tensile strength ratios tend to exhibit different lateral deformation characteristics than more ductile rocks. Since both $m_{\mathrm{i}}$ and $v$ are influenced by rock brittleness and microcrack behavior, a mechanical link between these parameters is physically plausible.

By combining the empirical relationship proposed by Cai [21] between $m_{\mathrm{i}}$ and $\sigma_c/\sigma_t$ with the formulation of Lógó and Vásárhelyi [25] linking $v$ to $\sigma_c/\sigma_t$, it becomes possible to express $m_{\mathrm{i}}$ as an implicit function of Poisson's ratio. This establishes a theoretical basis for estimating the Hoek–Brown constant from standard uniaxial compression tests that include lateral strain measurements.

From a mechanical perspective, this relationship can be interpreted as follows:

- Rocks with low Poisson's ratio generally possess more open or compliant microcrack networks, which also correspond to higher brittleness and higher $m_{\mathrm{i}}$.

- Rocks with higher Poisson's ratio tend to be more ductile or crack-resistant, typically associated with lower $m_i$ values.

Consequently, Poisson's ratio measured during a uniaxial compressive strength test contains embedded information about the rock's failure behavior under confinement, even though the test itself is performed without lateral pressure.

The present study builds upon this theoretical framework by analyzing stress-dependent Poisson's ratio obtained from conventional UCS tests and using it to estimate the Hoek–Brown constant $m_i$. This approach provides a practical alternative to full triaxial testing while retaining a sound mechanical interpretation rooted in rock microstructure and deformation behavior.

In Eq. (1), accurate knowledge of $m_i$ is important, as even a small deviation can cause significant differences in design in the rock environment.

Cai [21] provided that the ratio of uniaxial compressive strength ($\sigma_c$) and tensile strength ($\sigma_t$) corresponds within the error limit to its Hoek–Brown material constant if its value is greater than 8, i.e.,:

$$m_i \approx \frac{\sigma_c}{|\sigma_t|}. \tag{5}$$

According to the estimation of Lógó and Vásárhelyi [25], the Poisson's ratio of the intact rock ($v$) can be calculated by knowing the ratio of the uniaxial compressive strength ($\sigma_c$) and the tensile strength ($\sigma_t$):

$$v = \frac{1}{\sqrt{\frac{\sigma_c}{|\sigma_t|}} + 1}. \tag{6}$$

Applying the theory of Cai [21] and using Eq. (5), expression Eq. (6) can be written in the following form:

$$v = \frac{1}{\sqrt{m_i} + 1}. \tag{7}$$

That is, it can be stated that an exact relationship can be given between the Poisson's ratio of intact rock and the Hoek–Brown material constant.

The literature thus demonstrates two major trends:

1. Reliance on empirical regression models requiring extensive experimental databases;
2. Attempts to incorporate microstructural or elastic indicators into strength parameter estimation.

However, few studies have exploited the stress-dependent evolution of Poisson's ratio, which physically reflects microcrack closure and damage progression. By leveraging this parameter, the present method provides a novel bridge between elastic deformation behavior and strength characterization, offering both theoretical and practical advantages.

## 4 Methodology

The aim is to estimate the Hoek–Brown intact rock constant $m_i$ using only standard uniaxial compressive strength (UCS) tests combined with stress-dependent Poisson's ratio evaluation. The procedure consists of the following steps.

Cylindrical intact rock specimens were prepared in accordance with ISRM suggested methods. The height-to-diameter ratio was maintained between 2.0 and 2.5 to ensure uniform stress distribution during loading. The specimen ends were ground to achieve planarity and parallelism, minimizing eccentric loading effects.

Uniaxial compressive strength tests were conducted under displacement-controlled loading conditions until peak failure. The axial load and corresponding axial deformation were continuously recorded. The peak stress obtained during the test was defined as the uniaxial compressive strength $\sigma_c$.

During the UCS test, both axial strain ($\varepsilon_a$) and lateral strain ($\varepsilon_l$) were measured simultaneously. Axial strain was obtained using axial extensometers or strain gauges mounted along the specimen axis, while lateral strain was recorded using circumferential extensometers or lateral strain gauges positioned at mid-height of the specimen (Fig. 1).

The simultaneous measurement of axial and lateral strains enables continuous calculation of stress-dependent Poisson's ratio throughout the loading process.

The Poisson's ratio, which is the ratio of the lateral ($\varepsilon_l$) to axial ($\varepsilon_a$) strain can be calculated from the stress-strain relationship measured by standard uniaxial compressive strength testing. Depending on the method of calculation,

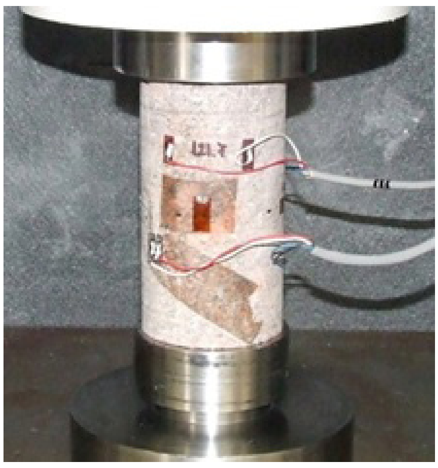

**Fig. 1** A prepared sample at the beginning of the UCS test

the following Poisson's rations can be distinguished, according to Fig. 2:

- *Secant* ($v_s$): it is determined from the value between the examined deformation state and the origin (i.e., the slope of the line drawn from the origin to the examined point):

$$v_s = \frac{\varepsilon_l}{\varepsilon_a}. \tag{8}$$

- *Tangent* ($v_t$): it is derivative of the axial and lateral deformation curve, i.e., we calculate the tangent slope fitted to the given point under examination:

$$v_t = \frac{\Delta\varepsilon_l}{\Delta\varepsilon_a}. \tag{9}$$

- *Average* ($v_{av}$): which can be calculated from the slope of a straight line drawn on a section under examination (arbitrarily selected between points *a* and *b*). In practice, a value of ±5% is usually taken into account:

$$v_{av} = \frac{\varepsilon_{la} - \varepsilon_{lb}}{\varepsilon_{aa} - \varepsilon_{ab}}. \tag{10}$$

As can be clearly seen in Fig. 3, since lateral deformation does not depend linearly on axial deformation, the Poisson's ratio values defined above vary continuously as a function of compressive stress. It follows that the Poisson's ratio cannot be constant, but must be a stress-dependent value [26]. This change is shown in Fig. 3 for secant, average and tangent Poisson's rate values as a function of normalized value of uniaxial compressive strength ($\sigma/\sigma_c$) (based on the analysis of granitic rock from Bataapáti radioactive waste repository).

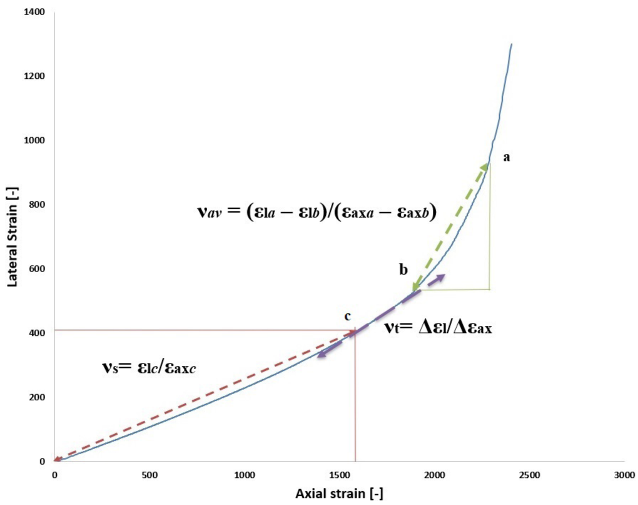


**Fig. 2** Determination of different Poisson's ratios based on axial and lateral strains

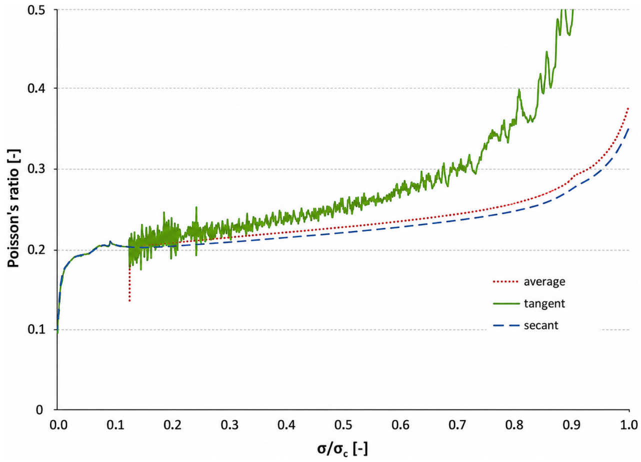


**Fig. 3** Poisson's ratio of the intact rock as a function of stress normalized by uniaxial compressive strength of granitic rock sample [26]

After analyzing the measured results in Fig. 4, Narimani et al. [26] suggested the following analytical solution. It can be used to a good approximation between 0.1 $\sigma/\sigma_c$ and 0.8 $\sigma/\sigma_c$:

$$v_i = A_i + \frac{\tan\left(160\frac{\sigma}{\sigma_c} - 80\right)}{B_i}, \tag{11}$$

where $A_i$ and $B_i$ are rock dependent material constants ($i$: secant, tangent or average). The material constant $A_i$ is equal to the value of the Poisson's ratio taken at 50%, i.e., the value recommended by ISRM [4]. The value of $B_i$ indicates the rock's sensitivity to stress: the higher the count, the less sensitive Poisson's ratio is to changes in stress.

According to ISRM suggested methods, elastic parameters such as Young's modulus and Poisson's ratio should be evaluated at intermediate stress levels, typically around 50% of the peak strength, where the rock still behaves

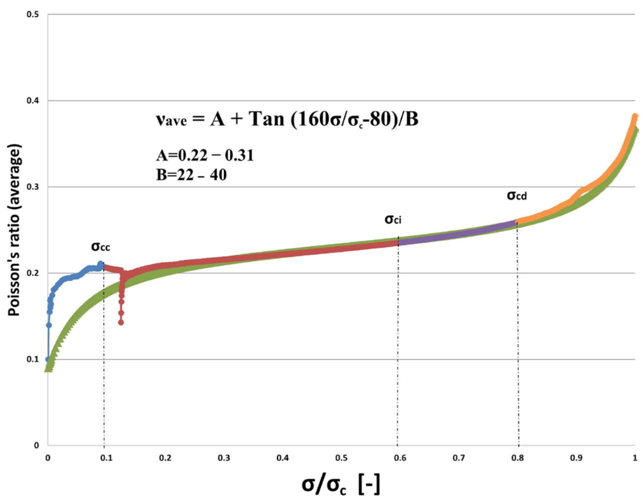


**Fig. 4** Average Poisson's ratio – $\sigma/\sigma_c$ curve for granitic rock specimen [26]

predominantly in an elastic manner and major damage or unstable crack propagation has not yet developed. At lower stress levels (< 30% $\sigma_c$), the measured Poisson's ratio may be influenced by the closure of pre-existing microcracks, surface irregularities of the specimen, and seating effects in the testing machine, which can lead to non-representative deformation measurements. Conversely, at stress levels above approximately 70–80% of $\sigma_c$, the rock enters a damage-dominated regime characterized by progressive crack coalescence, strain localization, and nonlinear deformation, which can distort the true elastic response of the material.

The stress level of 50% $\sigma_c$ therefore represents a compromise between these two extremes: it is sufficiently high to minimize the influence of initial crack closure, yet low enough to avoid significant pre-failure damage. At this stage of loading, the measured Poisson's ratio can be considered a representative indicator of the intact rock's intrinsic deformational behavior rather than a reflection of progressive failure processes.

Furthermore, using a consistent reference stress level across different rock types ensures comparability of results when estimating $m_\mathrm{i}$. This standardization is particularly important when applying the proposed method to diverse lithologies such as granite, limestone, sandstone, marl, and salt rock, each of which exhibits different stress-dependent deformation characteristics.

For these reasons, the value of Poisson's ratio at 50% of the uniaxial compressive strength is adopted in this study as the most suitable and reliable basis for estimating the Hoek–Brown material constant.

Using the theoretical relationship established in Eqs. (8), (9) and (10), the Hoek–Brown material constant $m_\mathrm{i}$ can be expressed as a function of Poisson's ratio:

$$m_i = f\left(\nu\right). \tag{12}$$

The constant $A$ in Eq. (11) is equal to the value of the Poisson's ratio taken at 50% of the strength, and assuming that the Poisson's ratio ($\nu$) is a function of the Hoek–Brown material constant ($m_\mathrm{i}$), applying Eq. (7) we obtain the following form:

$$m_\mathrm{i} = \left(\frac{1}{A} - 1\right)^2. \tag{13}$$

The proposed procedure provides a systematic and reproducible framework for estimating the Hoek–Brown constant from conventional laboratory tests that are widely available in engineering practice.

## 5 Measured results

Table 1 summarizes the results of a large number of uniaxial compressive strength tests performed on various rocks: granite, limestone, marl, sandstone, and rock salt [27]. Both the "$A$" constants and the calculated "$m_\mathrm{i}$" HB material constants are summarized in case of secant, average and tangent values. The measured HB constants are also shown. One can see, the measured $m_\mathrm{i}$ values are near to the calculated values.

The comparison between measured and calculated Hoek–Brown constants reveals a clear lithology-dependent trend in the accuracy of the proposed method. The agreement between estimated and measured $m_\mathrm{i}$ values varies systematically with rock type, which can be explained by differences in mineral composition, microstructure, brittleness, and deformation mechanisms.

Granite exhibits high measured $m_\mathrm{i}$ values (14.4), and the proposed method using the secant Poisson's ratio provides an almost identical estimate (14). This very good agreement can be attributed to the highly brittle nature of granite. As a crystalline igneous rock with strong grain interlocking and low clay content, granite typically shows a high compressive-to-tensile strength ratio and pronounced brittle crack propagation. Its relatively low Poisson's ratio at intermediate stress levels reflects limited lateral deformation prior to failure. Since the theoretical framework links brittleness with both low Poisson's ratio and high $m_\mathrm{i}$, the method performs particularly well for this rock type.

On the other hand, sandstone also shows high $m_\mathrm{i}$ values (17 measured, 16 estimated). The good agreement is likely due to the predominantly brittle intergranular failure mechanism in well-cemented sandstones. The lateral deformation characteristics captured through Poisson's ratio are consistent with its frictional and grain-controlled strength behavior. The similarity to granite in terms of brittleness explains why the estimation method performs reliably for this lithology.

**Table 1** The "$A$" constants of different type of investigated rocks and the calculated and measured HB constants

| Rock | $A$ | | | $m_\mathrm{i}$ | | | |
|---|---|---|---|---|---|---|---|
| | sec | av | tan | sec | av | tan | meas. |
| Granite | 0.21 | 0.22 | 0.22 | 14 | 13 | 13 | 14.4 |
| Limestone | 0.26 | 0.27 | 0.27 | 8 | 7 | 7 | 8 |
| Marl | 0.31 | 0.29 | 0.30 | 5 | 6 | 5 | 7 |
| Sandstone | 0.20 | 0.20 | 0.22 | 16 | 16 | 13 | 17 |
| Salt rock | 0.35 | 0.36 | 0.37 | 3 | 3 | 3 | 4 |

Moreover, limestone represents an intermediate case, with moderate $m_i$ values (8 measured, 8 estimated). The excellent agreement suggests that the method remains valid for carbonate rocks, provided that they are relatively intact and not strongly weathered. Limestone generally exhibits moderate brittleness and some degree of microcrack development, which is adequately captured through stress-dependent Poisson's ratio measurements at 50% of UCS.

Also, for marl, the difference between measured (7) and estimated (5) $m_i$ values is more noticeable. Marl typically contains a significant clay fraction, which introduces ductile deformation components and nonlinear stress–strain behavior even at moderate stress levels. The presence of clay minerals enhances lateral deformability and increases Poisson's ratio, which may lead to a slight underestimation of brittleness and consequently of $m_i$. This indicates that the proposed method may require calibration factors for clay-rich or transitional sedimentary rocks.

Finally, salt rock shows the lowest $m_i$ values (4 measured, 3 estimated) and a moderate deviation between measured and calculated results. Rock salt behaves in a ductile manner and exhibits time-dependent plastic flow. Its high Poisson's ratio and significant lateral deformation reflect a fundamentally different failure mechanism compared to brittle rocks. Since the Hoek–Brown criterion was originally developed for brittle rock materials, the estimation approach based on elastic Poisson's ratio becomes less precise for materials exhibiting plastic or viscoplastic behavior.

To further interpret the lithology-dependent performance of the proposed estimation method, the mechanical characteristics of each investigated rock type are summarized in Table 2.

Table 2 compares measured and estimated Hoek–Brown constants together with qualitative indicators of brittleness, deformation behavior, and agreement quality. This comparison allows a clearer understanding of how rock microstructure and failure mechanism influence the reliability of the proposed UCS-based approach.

**Table 2** Rock-type dependent interpretation of $m_i$ estimation accuracy

| Rock type | Measured $m_i$ | Estimated $m_i$ | Rel. error (%) |
|---|---|---|---|
| Granite | 14.4 | 14.0 | −2.8 |
| Sandstone | 17 | 16 | −5.9 |
| Limestone | 8 | 8 | 0.0 |
| Marl | 7 | 5 | −28.6 |
| Salt rock | 4 | 3 | −25.0 |
| **Rock type** | **Brittleness level** | **Deformation behavior** | **Agreement quality** |
| Granite | High | Low $v$, brittle crack propagation | Very good |
| Sandstone | High | Intergranular brittle failure | Very good |
| Limestone | Moderate | Moderate lateral deformation | Excellent |
| Marl | Moderate–Low | Clay influence, nonlinear behavior | Moderate |
| Salt rock | Low | Ductile, plastic flow tendency | Moderate |

## 6 Uncertainty analyses

Although the proposed method provides close agreement between estimated and measured Hoek–Brown constants, we quantified the uncertainty of the estimated $m_i$ values to assess robustness and practical usability.

The principal sources of uncertainty considered here are:

- Measurement uncertainty in axial and lateral strains during UCS tests (affecting Poisson's ratio at 50% UCS, $v_{50}$);
- Regression/model uncertainty arising from using a fitted empirical relation between $v_{50}$ and $m_i$;
- Specimen-to-specimen variability due to natural heterogeneity of the rock and sample preparation;
- Model applicability error when applying an empirical relation calibrated on a limited lithologic dataset to different rock types (notably ductile or clay-rich rocks).

To propagate these uncertainties to the predicted $m_i$ values we performed a Monte Carlo analysis as follows:

1. The secant Poisson's ratio at 50% UCS ($v_{50}$) and the corresponding secant-based $m_i$ estimates from Table 1 were used to fit a simple linear model:

$$m_i = a + bv_{50}. \tag{14}$$

The coefficients $a$ and $b$ in Eq. (14) were determined empirically using linear regression analysis based on the measured dataset summarized in Table 1. The secant Poisson's ratio values measured at 50% of UCS ($A$ values) were used as the independent variable, while the experimentally measured Hoek–Brown constants $m_i$ obtained from triaxial testing were used as the dependent variable. A least-squares regression procedure was applied to establish the best-fitting linear relationship between $v$ and $m_i$ for the investigated lithologies (granite, sandstone, limestone, marl, and salt rock). The resulting regression equation was:

$$m_i = a + bv, \tag{15}$$

where the fitted coefficients were determined as: $a = 31.5$ and $b = -84.6$.

The negative slope reflects the inverse relationship between Poisson's ratio and rock brittleness. Rocks with lower Poisson's ratio generally exhibit more brittle behavior and consequently higher Hoek–Brown constants. Conversely, ductile rocks tend to show higher lateral deformation, larger $v$ values, and lower $m_i$ values.

The regression coefficients were not assumed theoretically but derived directly from the experimental dataset analyzed in this study. Because the available dataset covers only a limited range of lithologies, the obtained coefficients should presently be interpreted as calibration parameters valid primarily for intact isotropic rocks exhibiting brittle to semi-brittle mechanical behavior.

The residual standard deviation of the regression was subsequently used in the Monte Carlo simulations to propagate model uncertainty into the predicted $m_i$ confidence intervals.

The choice of a linear regression form in Eq. (14) was motivated by the approximately monotonic inverse trend observed between Poisson's ratio and measured Hoek–Brown constant across the investigated rocks. Preliminary analysis indicated that a simple linear model provides stable fitting behavior and avoids overparameterization given the relatively limited number of lithologies available in the present database. Although nonlinear formulations may further improve predictive accuracy for ductile rocks, the linear form was considered appropriate for establishing a first-order engineering relationship suitable for uncertainty propagation and practical application.

2. For each rock type the $v_{50}$ measurement was treated as a normal random variable centered at the tabulated secant value ($A$) with a realistic laboratory uncertainty (standard deviation). We adopted conservative, lithology-dependent standard deviations for $v_{50}$: Granite: $\sigma_v = 0.008$; Sandstone: $\sigma_v = 0.008$; Limestone: $\sigma_v = 0.01$; Marl: $\sigma_v = 0.02$; and Salt rock: $\sigma_v = 0.02$. These values reflect plausible laboratory measurement variability (higher for clay-rich or ductile materials).
3. For each rock type, Monte Carlo realizations were generated and from the generated ensemble, the mean, median, standard deviation, and the empirical 2.5%–97.5% (95%) confidence interval for predicted $m_i$ for each lithology were derived.

The Monte Carlo results are summarized in Table 3 and shown in Fig. 5 (relation between $v_{50}$ and predicted $m_i$ with 95% prediction band) and Fig. 6 (predicted $m_i$ with 95% confidence intervals).

According to the uncertainty analysis, for brittle rocks (granite and sandstone) the relative uncertainty in predicted $m_i$ is low (≈ a few percent), indicating robust performance of the method in these lithologies. And for intermediate rocks (limestone) uncertainties are moderate. Moreover, for marl and rock salt (clay-rich or ductile materials), predicted $m_i$ intervals are wide and, in the case of rock salt, include nonphysical negative bounds when the

**Table 3** Monte Carlo-derived uncertainty of predicted Hoek–Brown constant $m_i$ based on secant Poisson's ratio at 50% UCS

| Rock type | $v_{50}$ (mean) | St.dev $v_{50}$ | Point estimate $m_i$ |
|---|---|---|---|
| Granite | 0.21 | 0.008 | 14 |
| Sandstone | 0.20 | 0.008 | 16 |
| Limestone | 0.26 | 0.01 | 8 |
| Marl | 0.31 | 0.02 | 5 |
| Salt rock | 0.35 | 0.02 | 3 |

| Rock type | Monte Carlo mean $m_i$ | 95% confidence interval | St.dev | Relative uncertainty (%) |
|---|---|---|---|---|
| Granite | 14 | 13.0–16.7 | 1.1 | ≈ 4% |
| Sandstone | 15 | 14.0–17.5 | 1.1 | ≈ 5% |
| Limestone | 9.8 | 7.7–12.3 | 1.2 | ≈ 6% |
| Marl | 5.5 | 3.6–9.6 | 1.6 | ≈ 13% |
| Salt rock | 2 | –2.2–6.0 | 1.8 | ≈ 17% |

Notes:
1. Each confidence interval corresponds to the $2.5^{th}$–$97.5^{th}$ percentiles from 10,000 Monte Carlo realizations per lithology.
2. Negative lower bounds for salt rock reflect extrapolation of the linear model beyond its calibrated range and are interpreted as model uncertainty rather than physical values.
3. Relative uncertainty = 1/2 (CI width / mean) × 100%.

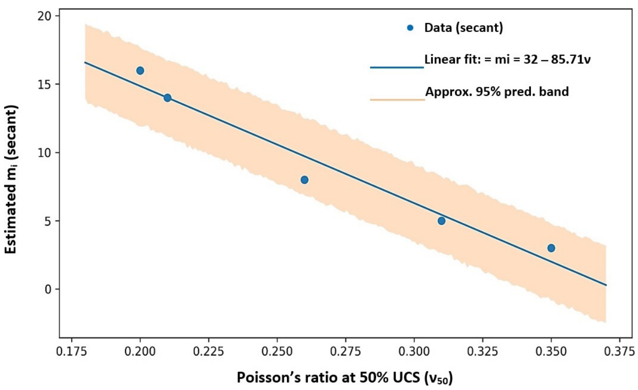


**Fig. 5** Relation between secant Poisson's ratio at 50% UCS ($v_{50}$) and estimated $m_i$ with approximate 95% prediction band (Monte Carlo propagation)

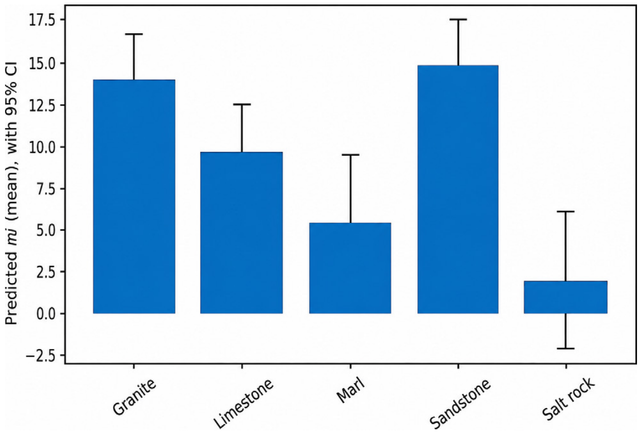


**Fig. 6** Predicted $m_i$ (mean) with 95% confidence intervals per rock type

linear model is extrapolated, this highlights poor model reliability for such materials.

## 7 Discussion

The results presented in this study demonstrate that the Hoek–Brown intact rock constant ($m_i$) can be reliably estimated from uniaxial compressive strength (UCS) testing by incorporating the stress-dependent evolution of Poisson's ratio. This finding has significant implications for both theoretical understanding and practical applications in rock engineering.

Traditional determination of $m_i$ relies on triaxial testing programs involving multiple confining pressures, which provide direct but expensive and time-consuming data. Several researchers have sought to reduce this testing burden by developing empirical correlations between $m_i$ and other measurable rock properties such as UCS, tensile strength, or geological strength index (GSI) [6–8, 10, 11]. Although these approaches offer convenience, they are often purely statistical, limited to specific lithologies, and insensitive to the actual deformation behavior of rock under load. In contrast, the present method captures the intrinsic mechanical response through Poisson's ratio, a parameter physically linked to microcrack closure and lateral strain, thereby introducing a mechanistic rather than purely empirical basis for $m_i$ estimation.

The observed correlation between $m_i$ and Poisson's ratio at 50% of UCS is consistent with the theoretical expectation that rocks with lower Poisson's ratio exhibit more brittle characteristics and a steeper strength increase under confinement. Brittle crystalline rocks such as granite and sandstone displayed low $v$ values and high $m_i$, while ductile rocks like marl and rock salt exhibited higher $v$ and lower $m_i$. This behavior aligns with previous findings that Poisson's ratio is sensitive to microstructural integrity, crack density, and mineral composition [18–20]. Hence, the proposed relationship provides an interpretable measure of brittleness that links micro-mechanical deformation processes with macroscopic strength parameters.

The present results are also consistent with the brittle failure concepts proposed by Diederichs [12] and reviewed by Perras and Diederichs [13], where crack initiation and tensile damage propagation govern the onset of brittle rock failure. In brittle lithologies such as granite and sandstone, tensile crack growth and limited lateral deformation produce relatively low Poisson's ratio values together with high Hoek–Brown constants. This explains why the proposed $v$–$m_i$ relationship performs particularly well for these rocks. Conversely, in marl and salt rock, ductile deformation and plastic flow mechanisms weaken the direct relationship between tensile cracking and confinement sensitivity, resulting in larger deviations between measured and estimated $m_i$ values.

The comparison between measured and estimated $m_i$ values reveals strong lithological control on method accuracy. For brittle lithologies (granite, sandstone, and limestone), the difference between estimated and measured $m_i$ values remained below 6%, confirming that stress-dependent Poisson's ratio effectively captures the deformation mechanism up to failure. For more ductile materials (marl and rock salt), underestimation occurred due to the increasing influence of plastic or viscoplastic deformation that decouples lateral strain from confining-pressure sensitivity. This highlights the need for lithology-specific calibration factors when extending the model to clay-rich or evaporitic rocks.

Monte Carlo uncertainty analysis revealed that the predictive reliability of $m_i$ estimation depends primarily on the precision of lateral strain measurement and the variability of Poisson's ratio at 50% UCS. For brittle rocks, relative uncertainty remained within 5%, comparable to the variability typically observed among triaxial test results. For ductile rocks, uncertainty increased substantially, reflecting the limitations of applying the Hoek–Brown criterion which was developed for brittle failure to materials governed by time-dependent creep or plastic flow. Nevertheless, the proposed linear $v$–$m_i$ model provided consistent central estimates, confirming that measurement noise does not significantly bias results within its calibrated domain.

It should be emphasized that the regression coefficients used in Eq. (14) are dataset-dependent calibration parameters derived from the presently available experimental database. Consequently, their numerical values may evolve as additional lithologies and larger datasets become available. Nevertheless, the observed inverse $v$–$m_i$ relationship remained physically consistent across all investigated rocks and agrees with the expected transition from brittle to ductile deformation behavior. Future studies should therefore investigate whether lithology-specific or nonlinear regression formulations can further improve predictive capability.

From a practical standpoint, this approach enables engineers to estimate $m_i$ using only standard UCS equipment augmented with lateral strain gauges, thereby avoiding complex triaxial testing. This is particularly beneficial in early-stage projects, remote field campaigns, or when core recovery limits sample size. The method's theoretical grounding also facilitates integration into numerical modeling and design workflows where both elastic and strength parameters are needed. Moreover, by linking $m_i$ to Poisson's ratio, a routinely measured elastic constant, the approach promotes greater internal consistency among rock mechanical parameters used in stability and deformation analyses.

Despite its advantages, several limitations merit further study. First, the empirical constants in the $v$–$m_i$ relationship were derived from a limited dataset; expanding this to a broader lithological spectrum would improve generalizability. Second, the current formulation assumes isotropic and homogeneous intact rock behavior, whereas anisotropy and fabric effects may alter the $v$–$m_i$ correlation. Third, time-dependent deformation and post-peak strain softening were not considered but could be relevant for weak or weathered rocks. Future work should combine this method with digital image correlation or acoustic emission monitoring to capture microcrack evolution and refine the mechanical interpretation. Additionally, incorporating machine learning regression frameworks could help automatically calibrate the relationship for diverse rock types using large-scale laboratory databases.

## 8 Conclusions

This study introduced and validated a practical methodology for estimating the Hoek–Brown intact rock constant ($m_i$) from standard uniaxial compressive strength (UCS) tests by analyzing the stress-dependent behavior of Poisson's ratio. The proposed approach establishes a physically meaningful linkage between $m_i$ and the lateral deformation characteristics of intact rock, reflecting the underlying mechanisms of microcrack closure and brittleness. Analysis of extensive UCS data from granite, sandstone, limestone, marl, and rock salt confirmed that the method yields $m_i$ values in excellent agreement with those obtained from conventional triaxial testing for brittle lithologies, while maintaining acceptable accuracy for ductile materials.

Uncertainty analysis demonstrated that, for crystalline and well-cemented rocks, the relative error of estimated $m_i$ remains within 5%, highlighting the robustness and reproducibility of the proposed relationship. The results confirm that the secant Poisson's ratio measured at 50% of UCS represents an optimal indicator of the intact rock's deformation state and provides a reliable basis for $m_i$ estimation.

The developed framework offers a cost-effective and time-efficient alternative to triaxial testing, particularly valuable during early-stage investigations or when sample quantity is limited. By linking strength and deformation parameters through measurable laboratory quantities, this method enhances the integration of elastic and failure criteria in rock mechanical design. Future work should expand the database to encompass anisotropic and weakly consolidated rocks and investigate nonlinear or machine-learning extensions to further refine predictive performance.

## References

[1] Hoek, E., Brown, E. T. "The Hoek-Brown Failure criterion and GSI – 2018 edition", Journal of Rock Mechanics and Geotechnical Engineering, 11(3), pp. 445–463, 2019. https://doi.org/10.1016/j.jrmge.2018.08.001

[2] Kovári, K., Tisa, A. "Multiple Failure State and Strain Controlled Triaxial tests", Rock Mechanics, 7(1), pp. 17–33, 1975. https://doi.org/10.1007/BF01239232

[3] Kovári, K., Tisa, A., Attinger, R. O. "The concept of "continuous failure state" triaxial tests", Rock Mechanics and Rock Engineering, 16(2), pp. 117–131, 1983. https://doi.org/10.1007/BF01032794

[4] Ulusay, R., Hudson, J. A. (eds.) "The complete ISRM Suggested Methods for Rock Characterization, Testing and Monitoring: 1974-2006", International Society for Rock Mechanics, 2007. ISBN 978-975-93675-4-1

[5] Narimani, S., Davarpanah, S. M., Vásárhelyi, B. "Advances and challenges in multiple failure state triaxial testing of rocks – 50 years' experience", In: EUROCK 2026, Skopje, North Macedonia, in press. (Accepted for publication: 23 May 2026])

[6] Marinos, P., Hoek, E. "GSI: A geologically friendly tool for rock mass strength estimation", In: Proceedings International Conference Geotechnical and Geological Engineering (GeoEng 2000), Melbourne, Australia, 2000. ISBN 978-1-58716-068-4 [online] Available at: https://www.rocscience.com/assets/resources/learning/hoek/2000-GSI-A-Geologically-Friendly-Tool-for-Rock-Mass-Strength-Estimation.pdf [Accessed: 01 March 2026]

[7] Sonmez, H., Ulusay, R. "Modifications to the geological strength index (GSI) and their applicability to stability of slopes", International Journal of Rock Mechanics and Mining Sciences, 36(6), pp. 743–760, 1999.
https://doi.org/10.1016/S0148-9062(99)00043-1

[8] Hoek, E., Marinos, P., Benissi, M. "Applicability of the geological strength index (GSI) classification for very weak and sheared rock masses. The case of the Athens schist formation", Bulletin of Engineering Geology and the Environment, 57(2), pp. 151–160, 1998.
https://doi.org/10.1007/s100640050031

[9] Aladejare, A. E., Wang, Y. "Probabilistic Characterization of Hoek–Brown Constant mi of Rock Using Hoek's Guideline Chart, Regression Model and Uniaxial Compression Test", Geotechnical and Geological Engineering, 37(6), pp. 5045–5060, 2019.
https://doi.org/10.1007/s10706-019-00961-7

[10] Arshadnejad, S., Nick, N. "Empirical models to evaluate of "$m_i$" as an intact rock constant in the Hoek-Brown rock failure criterion", In: 19th Southeast Asian Geotechnical Conference & 2nd AGSSEA Conference, Kuala Lumpur, Malaysia, 2016, pp. 943–948. ISBN 978-93-80813-44-8 [online] Available at: https://www.researchgate.net/publication/308207697_Empirical_models_to_evaluate_of_mi_as_an_intact_rock_constant_in_the_Hoek-Brown_rock_failure_criterion [Accessed: 01 March 2026]

[11] Tsiambaos, G., Sabatakakis, N. "Considerations on strength of intact sedimentary rocks", Engineering Geology, 72(3–4), pp. 261–273, 2004.
https://doi.org/10.1016/j.enggeo.2003.10.001

[12] Diederichs, M. S. "The 2003 Canadian Geotechnical Colloquium: Mechanistic Interpretation and Practical Application of Damage and Spalling Prediction Criteria for Deep Tunnelling", Canadian Geotechnical Journal, 44(9), pp. 1082–1116, 2007.
https://doi.org/10.1139/T07-033

[13] Perras, M. A., Diederichs, M. S. "A Review of the Tensile Strength of Rock: Concepts and Testing", Geotechnical and Geological Engineering, 32(2), pp. 525–546, 2014.
https://doi.org/10.1007/s10706-014-9732-0

[14] Hoek, E. "Practical Rock Engineering", [pdf] ROCSCIENCE, 2023. Available at: https://static.rocscience.cloud/assets/resources/learning/hoek/Practical-Rock-Engineering-E.Hoek-2023.pdf [Accessed: 01 March 2026]

[15] He, S., Cheng, H., Cheng, L., Yuan, F., Zhang, M. "Comparison and analysis for prediction accuracy of true triaxial rock strength criterion", Frontiers in Earth Science, 12, 1416979, 2024.
https://doi.org/10.3389/feart.2024.1416979

[16] Minaeian, V., Rasouli, V., Dewhurst, D. N. "True Triaxial Strength Testing of Sandstones", In: 75th EAGE Conference & Exhibition incorporating SPE EUROPEC 2013, London, UK, 2013. ISBN 978-90-73834-48-4
https://doi.org/10.3997/2214-4609.20130562

[17] Omar, T., Sadrekarimi, A. "Effect of triaxial specimen size on engineering design and analysis", International Journal of Geo-Engineering, 6(1), 5, 2015.
https://doi.org/10.1186/s40703-015-0006-3

[18] Hoek, E., Diederichs, M. S. "Empirical estimation of rock mass modulus", International Journal of Rock Mechanics and Mining Sciences, 43(2), pp. 203–215, 2006.
https://doi.org/10.1016/j.ijrmms.2005.06.005

[19] Singh, B., Goel, R. K. "Engineering Rock Mass Classification: Tunneling, Foundations, and Landslides", Butterworth-Heinemann, 2011. ISBN 978-0-12-385878-8

[20] Wang, W., Shen, J. "Comparison of existing methods and a new tensile strength based model in estimating the Hoek-Brown constant $m_i$ for intact rocks", Engineering Geology, 224, pp. 87–96, 2017.
https://doi.org/10.1016/j.enggeo.2017.05.008

[21] Cai, M. "Practical Estimates of Tensile Strength and Hoek–Brown Strength Parameter mi of Brittle Rocks", Rock Mechanics and Rock Engineering, 43(2), pp. 167–184, 2010.
https://doi.org/10.1007/s00603-009-0053-1

[22] Griffith, A. A. "The Theory of Rupture", In: Proceedings of the First International Congress for Applied Mechanics Delft, Delft, The Netherlands, 1924, pp. 55–63.

[23] Murrell, S. A. F. "A criterion for brittle fracture of rocks and concrete under triaxial stress, and the effect of pore pressure on the criterion", In: Proceedings of the Fifth Symposium on Rock Mechanics, Minneapolis, MN, USA, 1963, pp. 643–659.

[24] Jaeger, J. C., Cook, N. G. W., Zimmerman, R. W. "Fundamentals of Rock Mechanics", Wiley-Blackwell, 2007. ISBN 978-0-632-05759-7

[25] Lógó, B. A., Vásárhelyi, B. "Estimation of the Poisson's Rate of the Intact Rock in the Function of the Rigidity", Periodica Polytechnica Civil Engineering, 63(4), pp. 1030–1037, 2019.
https://doi.org/10.3311/PPci.14946

[26] Narimani, S., Kovács, L., Vásárhelyi, B. "An innovative method to determine the stress-dependency of Poisson's ratio of granitic rocks", Scientific Reports, 15(1), 16111, 2025.
https://doi.org/10.1038/s41598-024-75892-2

[27] Narimani, S., Vásárhelyi, B. "New Insights into Poisson's Ratio Variability in Rocks under Load", International Journal for Numerical and Analytical Methods in Geomechanics, 50(2), pp. 854–869, 2026.
https://doi.org/10.1002/nag.70146