periodica polytechnica

Civil Engineering
56/1 (2012) 115–122
doi: 10.3311/pp.ci.2012-1.13
web: http://www.pp.bme.hu/ci

EDUCATIONAL PAPER

Hundred years after the first triaxial test

Ferenc Deák / *Péter* Ván / *Balázs* Vásárhelyi

Received 2011-11-17, accepted 2012-02-27

Abstract

It was more than hundred years ago, when Theodore von Kármán (born as Tódor Kármán) designed and developed the first triaxial cell for investigation of brittle rocks behavior. His first research was based on Carrara marble and Mutenberg sandstone with varying confining pressures up to 600 MPa, demonstrating changes from brittle stages to ductile behavior in addition to hardening. The target of this paper is to give tribute to his development and work on this topic, having inspired and influenced many people in rock engineering and geophysical sciences, among others.

After a short historical overview of this research the published data are recalculated using different empirical failure criteria which are widely used in the rock mechanics and rock engineering practice. For the recalculation and description of Kármán's triaxial tests the original Hungarian paper was used.

Keywords

Kármán · rock mechanics · brittle · ductile · deformation · triaxial test

Acknowledgement

P. Ván aknowledges the financial support of the OTKA K81161 and TT 10-1-2011-0061/ZA-15-2009 for this research.

Ferenc Deák
RockStudy Ltd., Pécs, Hungary

Péter Ván
Dept. of Theoretical Physics, Wigner FK, RMI, and Dept. of Energy Engineering, BME and Montavid Thermodynamic Research Group, Budapest, Hungary

Balázs Vásárhelyi
Dept. of Structural Engng, Pollack Mihály Faculty of Engng, Univ. of Pécs, Pécs, Hungary

1 A short biography of von Kármán

Theodore von Kármán (his original Hungarian name: Kármán Tódor) (Budapest, May 11, 1881 – Aachen, May 7, 1963) was a Hungarian-American engineer and physicist who was primarily active in the fields of aeronautics and astronautics. His family got the nobility from Franz Joseph, king of Hungary. Kármán was strict about the correct usage of his name regarding the pronunciation and writing. However, one exception was Hungary where it is not common to use the nobility prefix "von".

Kármán graduated as mechanical engineer at the Royal Joseph University (now Budapest University of Technology and Economics) in 1902. Later on he worked as a mechanical engineer in Hungary before moving to Germany, where he got a job as an assistant of Ludwig Prandtl at the University of Göttingen. In 1912 he accepted a position as the director of the Aeronautical Institute at Aachen, one of the country's leading universities. His time in Aachen was interrupted by military service in the Austro-Hungarian Army 1915–1918, where he designed an early stage helicopter, after this period he turned back to Aachen.

In 1930 he left Europe and had emigrated into the United States of America, where he had obtained his greatest results in the fields of aeronautics and astronautics.

A hundred years ago Tódor Kármán published his results on triaxial tests of brittle rocks [17, 19]. He first presented his results at the meeting of the machinery, mining, etc. divisions of the Hungarian Society of Engineers and Architects (Magyar Mérnök- és építész-Egylet), held on October 8, 1910 and repeated it at the meeting of the III division of the Hungarian Academy of Sciences nine days later. First his results were published in Hungarian in the Journal of Hungarian Engineers and Architects in 1910 [8] and a year later it was published in German [9], as well. The title of the Hungarian and the German versions are different. While the Hungarian title emphasizes the conceptual question: "What influences the strength of the material?" ("Mitől függ az anyag igénybevétele?" [8]) the German title describes the method: "Strength experiments under pressure from all sides" ("Festigkeits Versuche unter allseitigem Druck" [9]).

2 “The Bomb”

During that time E. Heyn, being the professor of geology in Heidelberg, pointed out that, contrary to logical expectations, mountains which consist of brittle rocks are deformed as plastic material [10]. Kármán got interested in this problem, therefore he designed the first triaxial test chamber (named “Bomb”, by Kármán) which has been produced by the company Krupp in Essen.

The schematic drawing of this cell is shown in Fig. 1. Height of this triaxial cell should be approximately 1 meter (we should make just assumption on the scale of this cell, because we have not specifications). The first triaxial samples had a diameter of 40 mm and a height of approximately 100 mm (i.e. diameter/height ratio was around 1:2.5), to avoid bending. This ratio is similar to the one suggested by ISRM [7]. This cell was able to operate up to a confining pressure of 6,000 atm (= 608 MPa). To prevent the contact between sample and glycerine (which ensured the confining pressure), a very thin (0.1 mm) brass membrane was applied. Force and deformation was measured by means of manometer and micrometer gauges, respectively. Sensitivity of the micrometer screws were 1/100 mm.

The experimental machine was designed to enable an independent manipulation of the axial and confining pressures. The confining pressure was generated by a hand pump compressing the *a* space filled with water (Fig. 1). That was multiplied by the D_1 piston (approx. 1:24) and it is transmitted to the *b* space. The sample was fixed at *c* and the *b*, *c* spaces were filled with glycerine, and connected through the drillhole indicated by *B*, therefore, the sample was submitted to the pressure generated by piston D_1.

The glycerine is a favorable hydraulic fluid of high pressure experiments, because its compressibility is half of water, and because it has relatively high density which acting as a sealing liquid too.

The longitudinal force is transmitted through the D_2 piston which penetrates into the *c* space and directly compresses the rock sample. Therefore the axial compressive force is independent of the pressure in the *c* space and from the friction, which exists at the lining, but the compressive force is the same that we can measure at normal compressive strength measurements.

The manometers were installed at the hydraulic cylinder (base of the D_2 piston), and in the *a* space, respectively. Both measurements were influenced by the friction between the pistons and inlays. Kármán analyzed this question and concluded that, the friction forces can be determined and considered very precisely. To determine the friction forces, the cells *b* and *c* were filled with glycerine and the manometers were compared at different motions of the pistons.

1 D_1 pushes the D_2 piston,

2 D_2 pushes the D_1 piston,

3 both pistons moves forward (the high pressure results in a large volumetric change of the fluid),

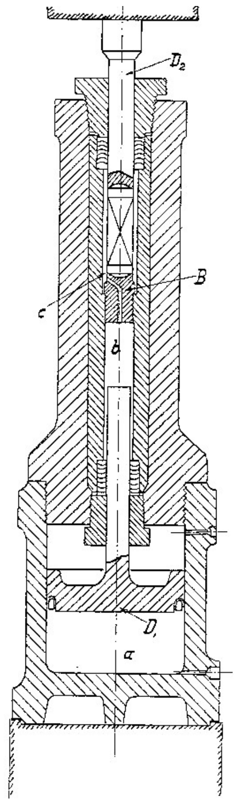


Fig. 1. “The Bomb”, the triaxial cell of Kármán [8]

4 both pistons are slowly released.

The diagram presented in Fig. 2 summarizes the results of the friction force measurements in case of these motions. This diagram has been used to determine the suitable correction, and was applied in the experiments.

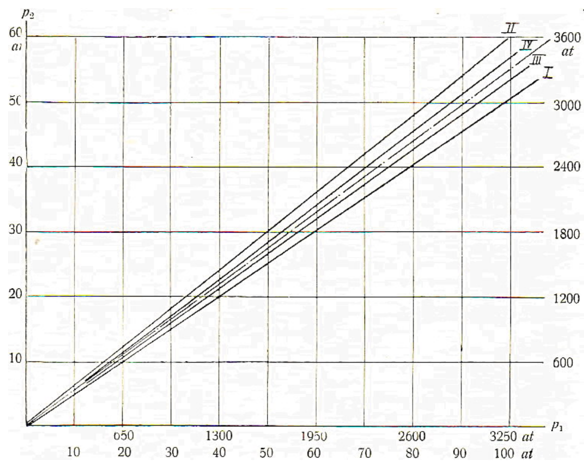


Fig. 2. Diagram for defining the friction forces [8]

The above presented first triaxial cell is similar to a Hoek cell [4] and the measuring method is near to the ISRM suggested method [7].

3 “What influences the strength of a material?”

The question of Kármán, which he put forward in the first sentence of his paper, was theoretical: what is the correct quantity to measure the strength material? More specifically he wanted to test the two hypotheses of Mohr:

1 The limit of the elastic behavior is independent at the value of the intermediate main stress.

2 The limit of the elasticity and fracture is determined by the following formula $\tau = f(\sigma)$ for any kind of load. Here τ is the tangential and σ is the normal stress, respectively.

Former experiments have shown that these hypotheses are valid for plastic and elastic materials, but are questionable for brittle ones.

Initially, Kármán analyzed Carrara marble and Mutenberg sandstone at different confining pressures. He selected these rock types because he was looking for relatively homogenous and isotropic rocks. His paper mentions that results on sandstones highly depend on water content, therefore, Kármán carried out his research using dry samples.

After several attempts, he performed 10 successful experiments on marble and 6 on sandstones. Fig. 3A and 3B illustrate published effective stress-strain data concerning marble and sandstone, respectively. According to his results, both brittle materials become plastic due to the increasing hydrostatic pressure. This phenomenon is documented on Fig. 4. Based on these results, Kármán demonstrated the mechanical behavior of brittle materials caused by different confining pressures. This phenomenon became fundamental in geophysical-, rock engineering- and rock mechanical knowledge.

To point out, he was not the first being interested in triaxial behaviors of rock materials. The first confirmed experiment was performed by Kick in 1892 [11] in Prague, using a completely different method. This testing was the first experimental confirmation of brittle-ductile transition, however it was only qualitative. Quantitative experimental work was first carried out by Kármán [8, 9].

Notable, that Mogi's [13] widely used brittle-ductile transition limit (i.e. $\sigma_1 = 4.4\ \sigma_3$) can be applied for these rocks: for the marble it is $\sigma_3 = 115\,\text{MPa}$ confining pressure between V. and VI. lines (Fig. 3A) and for the sandstone it is between the III. and IV. lines ($\sigma_3 = 85\,\text{MPa}$) (Fig. 3B).

Criteria of elasticity and failure as the function of the confining pressure: from the point of view of the Mohr hypothesis one should determine those σ_1 and $\sigma_2 = \sigma_3$ values, that correspond to the limit of elastic behaviour and failure. Regarding the *elastic limit* Kármán mentions that the appearance of permanent deformation depends on the precision of the measurement, therefore it is uncertain. The practical engineering definition – where the elastic limit is characterized by the given ratio of the permanent and elastic deformations - is not suitable for conceptual purposes. Therefore he considers the yield stress instead of the limit stress of elasticity.

There are other kind of problems determining *the failure limit*, where the failed material can be easily identified, but before that the material is in instable equilibrium, as a consequence the determination of the exact stresses leading to failure are uncertain, more over the whole failure process will dependent on the structure of the machine, on the method and the exact conditions of the experiment. According to Kármán's opinion, the failure conditions are characterized by the maximal stresses before the failure and not the actual stress state at the failure.

He took several photos of the crystals before and after the deformations which were also published in his papers in [8, 9]. Analyzing the photos he realized that the deformation appears between crystals (rigid material), or inside the crystals (plastic material).

3.1 Permanent deformation and related phenomena

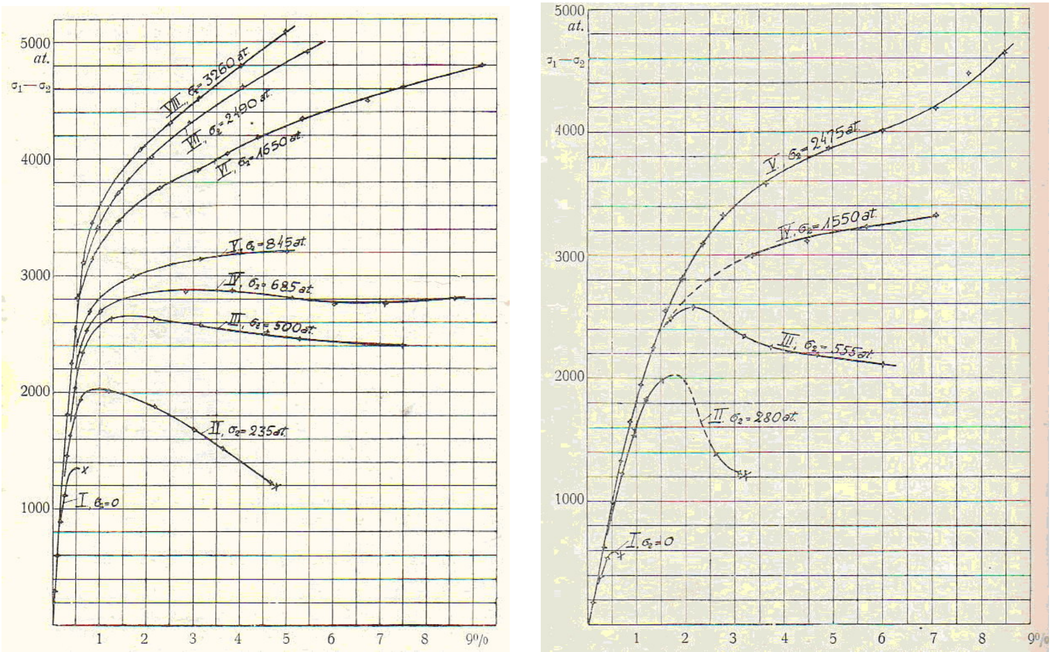
It was not a new fact, that the marble and other brittle materials have permanent deformation under high pressure. This is apparent investigation on samples from the Earth's mantle, and it was also demonstrated by a series of experiments by Kick [12]. He put the samples in stearin and pressed with a piston. However, in his experiments the pressure transmitted by the stearin was inhomogeneous, therefore the samples suffered slight permanent deformations [12]. Of course with this adjustment it was impossible to determine the exact stresses. The main advantage of the experiments of Kármán was the knowledge of the stress condition in every state of the permanent deformation. This permanent deformation took place without volume change, and especially in case of marble at high confining pressure also without the loss of material coherence.

Kármán shaped some samples after the high pressure experiments and brought them under normal uniaxial compressive test. The previous permanent axial deformation of these samples had been 10-12%, however their strength decreased only by 15-20%. The observation of the surface of the shaped, permanently deformed samples showed that the marble became more white, and less clear.

3.2 Conditions of failure

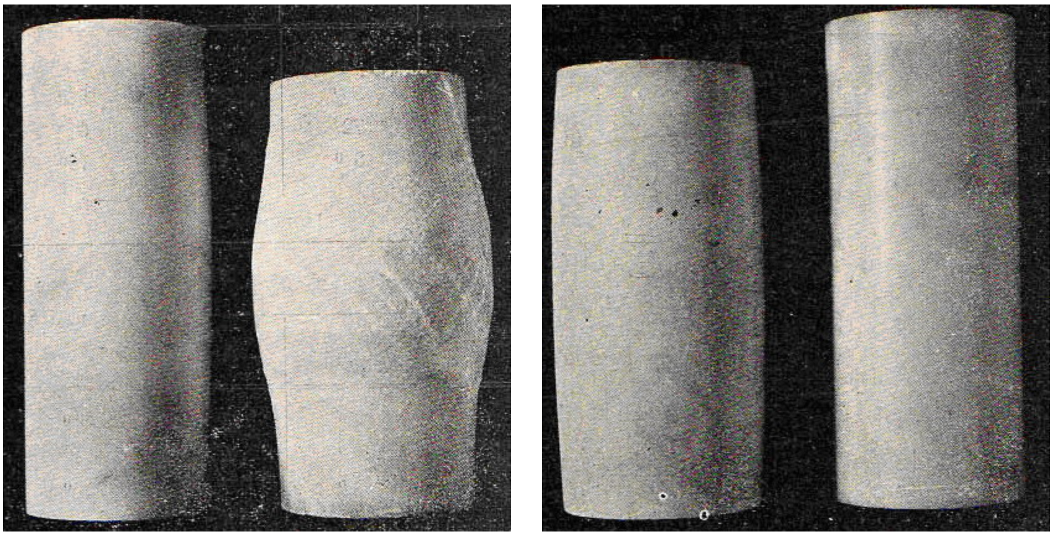
Kármán emphasized the difficulties to identify the conditions of failure by the properties of the stress strain curve. He remarked that if the sample failed, that happened by decreasing loading and decreasing deformation. Therefore he proposed, that the failure is determined by the instability of the whole system, including the machine. However, it is not easy to separate reasons of the apparent instability indicated by the decreasing loading. It may come from the interaction of the sample and the elastic properties of the machine, and also from the weakening role of the increasing cracks in the material. As the speed of the process plays an important role here, this separation requires more refined and extended experiments.

He distinguished two types of cracking. According to his experiments the failure started with more or less regular sliding, because of the Mohr hypothesis. According to Mohr the vertical cracks are due to the cracking arising from the primary sliding. The difference between the primary sliding and secondary cracked surfaces was visible in the experiments.



A B

Fig. 3. The effective stress vs. deformation of (A) Carrara marble and (B) Mutenberg sandstone rock samples in case of different confining pressures. (1 atm = 0.101325 MPa) [8]



A B

Fig. 4. The original (left on *A*, and right on *B*) and due to low (*A*) and high (*B*) confining pressure deformed marble sample (right on *A*, and left on *B*) [8]

On Fig. 5 one can see the usual failure on the sandstone cylinder, where the primary sliding surface is a regular cone, and the penetration of this cone caused a range of vertical cracks in the material.

3.3 Microscopic observations of the marble

Introducing this part Kármán asked the following important questions:

- How is it possible, that materials, which are rigid in uniaxial compressive strength measurements, under high pressure can deform without cracking and failure and behave like plastic or ductile materials?
- How is it possible, that the elastic limit initially increases proportionally with the confining pressure, but later approaches a constant value?

To find the answer, Kármán investigated the deformations of the material's structure with mineralogical investigation using optical microscopy. He worked mainly with the marble.

Marble is composed of closely fitted calcite crystals, with minimal matrix elements. In calcite macles can appear easily, therefore he assumed, that this phenomena is responsible for the ductile behavior. This hypothesis was confirmed by the microscopic investigation. He observed that the number of macles (see the parallel lines that run through some crystal grains) increased considerably on the microscopic samples taken from

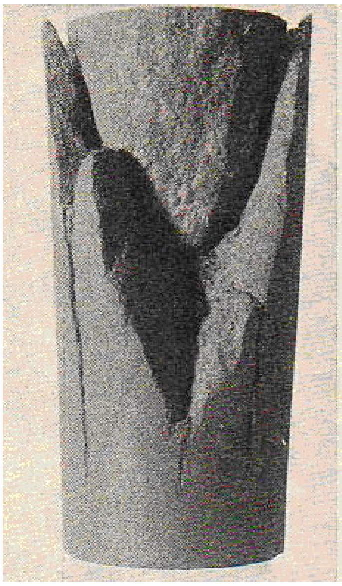


Fig. 5. Cracking of a sandstone sample after a normal uniaxial strength testing [8]

compressed, permanently deformed rock (Fig. 6 and 7). Comparing also the intact and the uniaxially compressed samples one can see, that the number of macles did not increase considerably (Fig. 6 and 8).

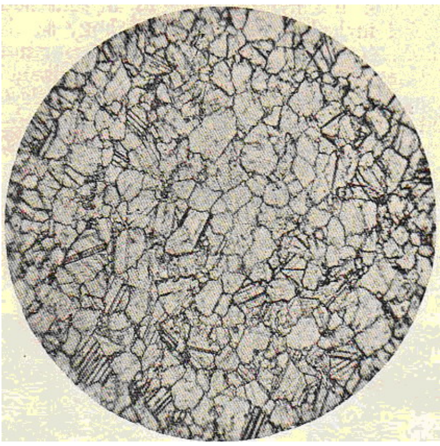


Fig. 6. The microscopic picture of the marble before deformation (magnification 50X) [8]

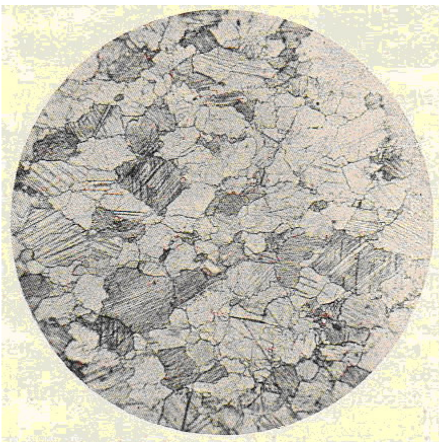


Fig. 7. The microscopic picture of the marble after 9% permanent foreandaft changing under a 2500 atm or 253 MPa pressure (magnification 50X) [8]

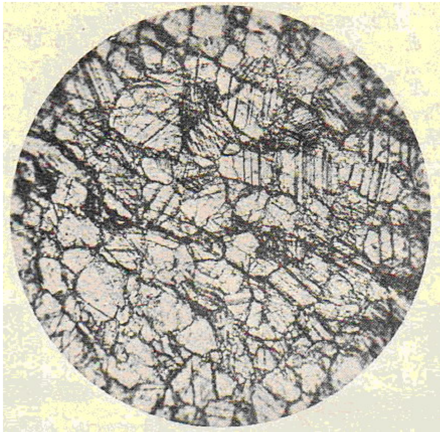


Fig. 8. A marble thin sections photo made after a common uniaxial compressive strength test (magnification 50X) [8]

3.4 Two types of deformation

Kármán could distinguish two main deformation types after the microscopic investigation: when the deformation took place inside the crystals, and the other type is the relative slipping of grains (intragranular and intergranular types). These two types are the extremes, a transition between the two basic types is more characteristic (an example is shown on Fig. 9).

The intergranular deformation is typical at low confining pressure, because the higher pressures can prevent the relative motion of the grains. Therefore the elastic limit increases with the confining pressure.

On the other hand, the intragranular deformation appears, if the confining pressure is high enough to completely prevent the relative slipping of the grains. In that case the confining pressure does not seem to influence the elastic limit.

The two typical modes of deformation are seen on Fig. 10–12 with a magnification higher than on the previous pictures.

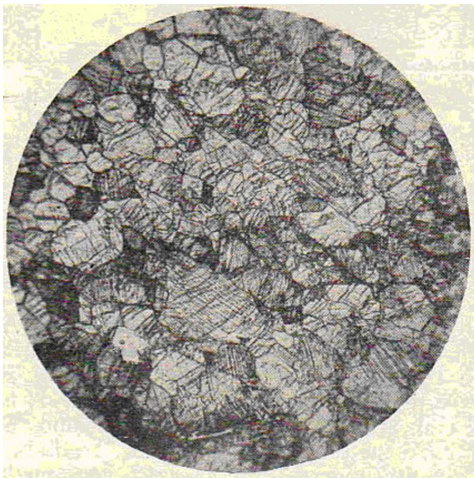


Fig. 9. The microscopic picture of the marble after 13% permanent fore and aft changing under an 500 atm, or 50,7 MPa pressure (magnification 50X) [8]

3.5 Microscopic analyses of the sandstone:

While comparison to the marble – the sandstone is composed of more than one mineral and therefore the composition is of

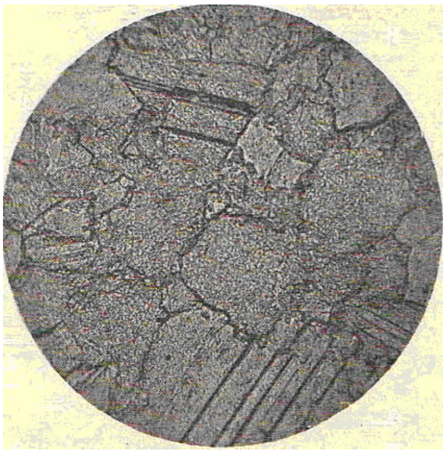


Fig. 10. The microscopic picture of the marble before deformation (magnification 175X) [8]

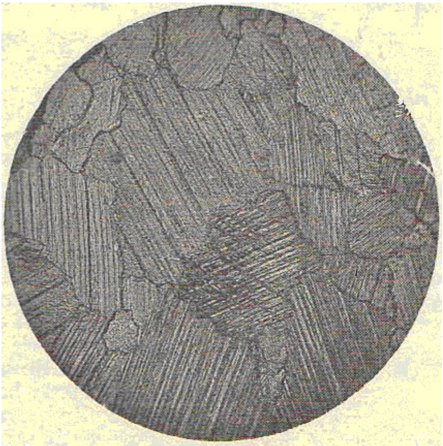


Fig. 11. The typical view of intragranular deformation (magnification 175X) [8]

Fig. 12. The microscopic picture of the intergranular deformation (magnification 175X) [8]

higher importance. It is composed of several crystals, with very different mechanical properties. Therefore the above mentioned two types of deformation appear together.

The overall purpose of Kármán's experimental investigation was to test the hypothesis of Mohr for brittle materials. He claimed that the previously observed failure mechanisms, the shear band formation and cleavage fractures cannot be related by a Mohr-type criteria, or by a single and unique relation of the normal and tangential stresses. He wanted to explore the conditions of these mentioned different failure modes. We do not know whether he had performed the second part of the planned experiments, testing the tensile strength of brittle materials or not.

4 Recalculating the results of Kármán

Kármán [8, 9] published his measured failure limits as functions of the confining pressure. We had to read the data from the figures and re-calculated them into MPa – they are collected in Tables 1 and 2, respectively.

Tab. 1. The measured points of failure at the stress space for the marble [8] (recalculated values)

No. sample	Confining pressure	Axial pressure
	$\sigma_2 = \sigma_3$ [MPa]	σ_1 [MPa]
I	0	138
II	24	237
III	51	319
IV	69	361
V	86	411
VI	167	Min. 654
VII	252	Min. 759
VIII	330	Min. 837

Tab. 2. The measured points of failure at the stress space for the sandstone [8](recalculated values)

No. sample	Confining pressure	Failure limit
	$\sigma_2 = \sigma_3$ [MPa]	σ_1 [MPa]
I	0	70
II	28	235
III	56	318
IV	157	491
V	251	Min. 717

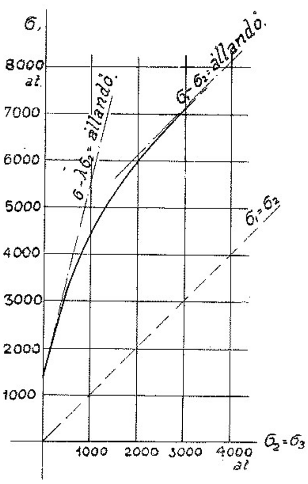
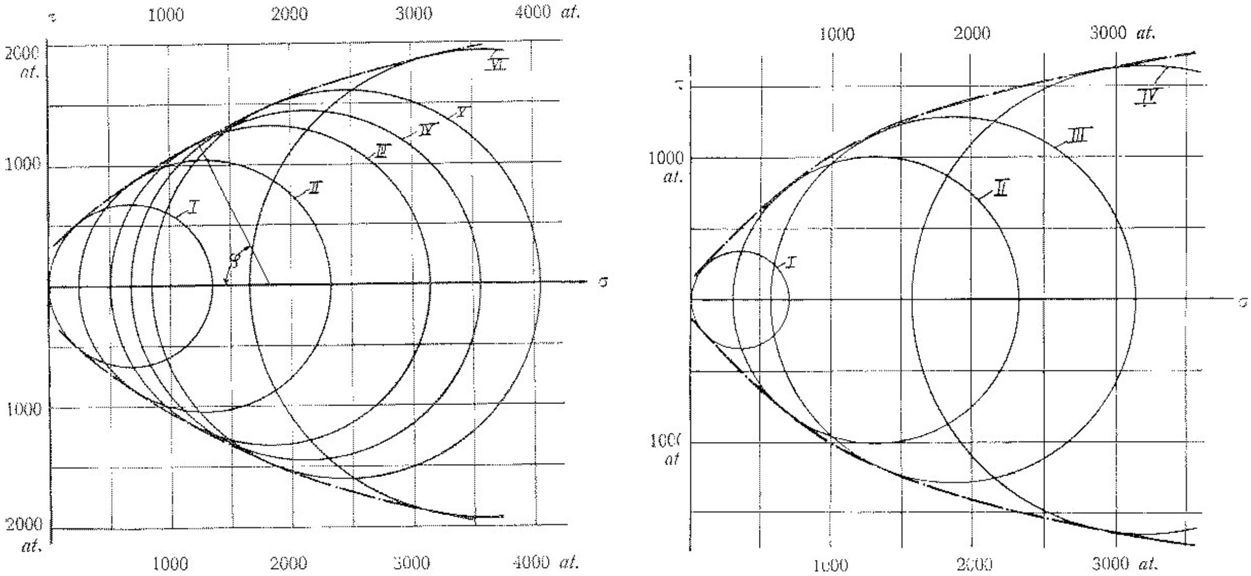


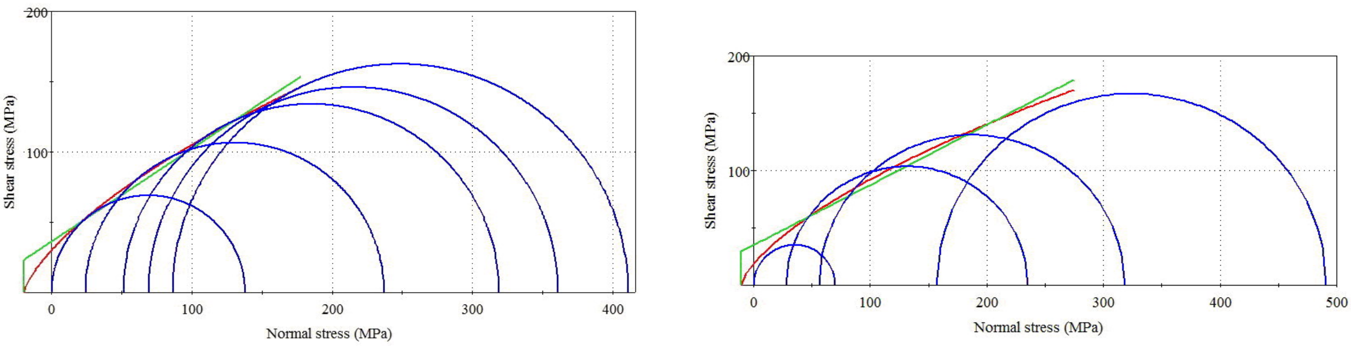
Fig. 15. Relationship between the main stresses ("állandó" means constant) according to Kármán [8]

Kármán, using the Mohr circles, plotted his results but at the time there was no theory for determining the failure envelope



A B

Fig. 13. The limit line of the (A) marble and (B) sandstone using the Mohr theory, according to Kármán [8]



A B

Fig. 14. The recalculated limit lines of the (A) marble and (B) sandstone (the overlaid envelopes: Mohr-Coulomb with green and Hoek-Brown with red color)

of the material. These circles for the marble and sandstone are shown in Figs. 13A and 13B. Our recalculated circles in MPa dimensions are shown in Figs. 14A and 14B.

Plotting the σ_1 and the σ_3 main stresses he realized that the $\sigma_1 - \sigma_3$ curve from $\sigma_1 - \lambda\sigma_3$ = constant trends to $\sigma_1 - \sigma_3$ = constant line (see Fig. 15) corresponding to the mentioned two dominant failure modes. These λ values were not calculated by Kármán – it is 5.2 for the marble and and 6.8 the sandstone after our recalculations from his data.

Up to now several empirical formulas have been developed for the failure envelope of rocks. We have calculated the parameters of some of these non-linear empirical equations for the marble and the sandstone. The asymptotic standard errors of the parameters are given, too.

- Equation of Murrell [14]:

$$\sigma_1 = \sigma_c + a\sigma_3^b \tag{1}$$

	A	b
Marble	8.0 ± 0.8	0.79 ± 0.02
Sandstone	28.6 ± 1.8	0.53 ± 0.01

- Equation of Hobbs [5]:

$$\sigma_1 = \sigma_c + \sigma_3 + a\sigma_3^b \tag{2}$$

	A	b
Marble	7.9 ± 1.0	0.71 ± 0.03
Sandstone	41.5 ± 7.1	0.37 ± 0.04

- Equation of Franklin [3]:

$$\sigma_1 = \sigma_c + a(\sigma_1 + \sigma_3)^b \tag{3}$$

	A	b
Marble	5.34 ± 0.33	0.66 ± 0.01
Sandstone	6.5 ± 3.3	0.61 ± 0.08

- Equation of Hoek and Brown [6] for intact rock:

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + \sigma_c^2)^{1/2} \tag{4}$$

	m
Marble	7.28 ± 0.14
Sandstone	11.9 ± 2.0

- Equation of Yoshida et al. [20]:

$$\sigma_1 = \sigma_3 + a\sigma_c \left(\frac{\sigma_3}{\sigma_c} + s\right)^b \quad (5)$$

	A	b
Marble	14.3 ±2.6	0.43 ± 0.04
Sandstone	0.46 ± 0.18	0.27 ± 0.02

In this formula $s = a^{-1/b}$, because $\sigma_1(\sigma_3 = 0) = \sigma_c$.

- Equation of Bieniawski [1]:

$$\sigma_1 = \sigma_c + a\sigma_c \left(\frac{\sigma_3}{\sigma_c}\right)^b \quad (6)$$

	A	b
Marble	2.85 ± 0.05	0,79 ± 0.02
Sandstone	3.92 ± 0.04	0,53 ± 0.01

Most of these simple two parameter criteria fits well within the data of Kármán experiments. The one parameter Hoek-Brown results in a good correlation (acceptable for sandstone), too. Fig. 14A–B show that for the data of Kármán the performance of the simplest criteria is acceptable, too. The fitted parameters for the sandstone data show high asymptotic standard errors for the criteria of Hobbs, Franklin and Yoshida (parameter a in every cases). This indicates that the different criteria may be different from the point of view of parameter sensitivity (see also [16]).

5 Conclusion

100 years ago Kármán started to investigate the strength of materials with a new experimental method. His method became a standard, recently basically the same technique is used for investigating the influence of the confining pressure to the strength of the rock. His investigations initiated further research and today we know more about brittle ductile transitions of rocks. For example the concept of damage sheds a new light to the failure mechanisms [12, 15], the rheological concepts, role of the loading speed is also far more elaborated today [2]. We know, that brittle-ductile transition of rocks is not connected exclusively to triaxial loading conditions, it can appear e.g. in case of two point bending tests, too [18].

However, looking back sincerely to the title of Kármán's paper "What influences the strength of the material?", the question is still not answered. The emphasis and the concepts may be different, but there are several important practical and theoretical details that we do not know yet. On the other hand in some cases we cannot be sure whether these details are really details, or they are essential. For example: we may observe that the above mentioned criteria are all empirical. According to our knowledge there are no *simple theoretical* criteria with only few parameters that could explain the most important observations of the experiments of Kármán. Therefore it seems to us that the real understanding of the Kármán experiments, especially considering the distinction of the different failure modes (tensile and compressive failure) in the complete three dimensional stress space, is still missing.

References

1 **Bieniawski Z T**, *Estimating the strength of rock materials*, J. S. Afr. Inst. Min. Metall. **74** (1974), 312–320.

2 **Cristescu N D, Hunsche U**, *Time Effects in Rock Mechanics*, Wiley Series in Materials, Modelling and Computation, John Wiley and Sons, 1998.

3 **Franklin J A**, *Triaxial strength of rock material*, Rock Mech. Rock Eng. **3** (1971), 86–98.

4 **Franklin J A, Hoek E**, *Developments in triaxial testing technique*, Rock Mech. Rock Eng. **2** (1970), 223–228.

5 **Hobbs D W**, *The strength and the stress-strain characteristic of coal in triaxial compression*, J. Geol. **72** (1964), 214–231.

6 **Hoek E, Brown E T**, *Underground excavations in rock*, Inst. Min. Metall., London, 1980.

7 **ISRM**, *Commission on Standardization of Lab and Field Tests, Suggested Methods for Determining the Strength of Rock Materials in Triaxial Compression*, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr. **15** (1978), 47–51.

8 **Kármán T**, *Mitől függ az anyag igénybevétele? (What influences the strength of the material?)*, Magyar Mérnök- és építész-Egylet Közlönye **10** (1910), 212–226. in Hungarian.

9 **Kármán von Th**, *Festigkeits Versuche unter allseitigem Druck*, Z. Verhandl. Deut. Ingr. **55** (1911), 1749–1759. in German.

10 **Kármán von Th, Edison L.**, *The wind and beyond – Theodore von Kármán: the Pioneer in aviation and pathfinder in space*, Little, Brown and Comp. Inc., 1967.

11 **Kick F**, *Die Principien der mechanischen Technologie und die Festigkeitslehr, Zweite Abhandlung*, Z. Verhandl. Deut. Ingr. **36** (1892), 919–933. in German.

12 **Martin C D, Chandler N A**, *The progressive fracture of Lac du Bonnet granite*, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr. **31** (1994), 643–659.

13 **Mogi K**, *Pressure dependence of rock strength and transition from brittle fracture to ductile flow*, Bull. Earthquake Res. Inst. (Tokyo Univ.) **44** (1966), 215–232.

14 **Murrel S A K**, *A criterion for brittle fracture of rocks and concrete under triaxial stress and the effect of pore pressure on the criteria*, 5. US Rock Mech. Symp, Balkema, 1965, pp. 563–577.

15 **Ván P, Vásárhelyi B**, *Second Law of thermodynamics and the failure of rock materials*, 38 US Rock Mech. Symp (Washington D.C.), DC Rocks (Elsworth D, Tinucci J P, Heasley K A, eds.), Balkema, 2001, pp. 767–773.

16 ______, *Sensitivity analysis of the Hoek-Diederichs rock mass deformation modulus estimating formula*, Proceedings of the 11 ISRM Symp. (Lisbon), The Second Half Century of Rock Mechanics (Soussa L R, Ollala C, Grossmann N F, eds.), Taylor & Francis, 2007, pp. 411–414.

17 ______, *Centenary of the first triaxial test - recalculation of the results of Kármán*, Eurock'2010 (Laussane), Rock Mechanics in Civil and Environmental Engineering (Zhao J, Labiouse V, Dudt J-P, Mathier J-F, eds.), Taylor & Francis, 2010, pp. 59–62.

18 **Vásárhelyi B**, *Investigation of crack propagation with different hydrostatic pressure on anisotropic gneiss*, Eurock'2001 (Espoo), Rock Mechanics – a Challenge for Society (Säkkä P, Eloranta P, eds.), Taylor & Francis, 2001, pp. 187–190.

19 ______, *Tribute to the first triaxial test performed in 1910*, Acta Geod. et Geoph. Hung. **45** (2010), 227–230, DOI 10.1556/AGeod.45.2010.2.7.

20 **Yoshida N, Morgenstein N R, Chan D H**, *Failure criteria for stiff sils and rocks exhibiting softening*, Can. Geotechn. J. **27** (1990), 195–202.