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RESEARCH ARTICLE

# A hybrid HS-CSS algorithm for simultaneous analysis, design and optimization of trusses via force method

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***Abstract***

*In this paper, a hybrid heuristic method is developed using the harmony search (HS) and charged system search (CSS), called HS-CSS. In this algorithm the use of HS improves the exploitation property of the standard CSS. An energy formulation of the force method is developed and the analysis, design and optimization are performed simultaneously using the standard CSS and HS-CSS. New goal functions are introduced for minimization, and the CSS and the HS-CSS are employed for continuous optimization. An efficient method is introduced using the CSS and HS-CSS for designing structures having members of prescribed stress ratios. Finally, the minimum weight design of truss structures is formulated using the CSS and HS-CSS algorithms and applied to some benchmark problems from literature.*

***Keywords***

*harmony search · charged system search · force method of analysis · optimal design · truss structures*

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## 1 Introduction

In the last decade, many new natural evolutionary algorithms have been developed for optimization of pin-connected structures, such as genetic algorithms (GAs) [1–5], particle swarm optimizer (PSO) [6, 7], ant colony optimization (ACO) [8–10] and harmony search (HS) [11–13], charged system search (CSS) [14–16], and other metaheuristics [17–19]. These methods have attracted a great deal of attention, because of their high potential for modeling engineering problems in environments which have been resistant to solution by classic techniques. They do not require gradient information and possess better global search abilities than the conventional optimization algorithms [20]. Having in common processes of natural evolution, these algorithms share many similarities: each maintains a population of solutions which are evolved through random alterations and selection. The differences between these procedures lie in the representation technique utilized to encode the candidates, the type of alterations used to create new solutions, and the mechanism employed for selecting new patterns.

Compared to other population-based meta-heuristics, charged system search has a number of advantages that is distinguished from others. However, for improving exploitation (the fine search around a local optimum), it is hybridized with HS that utilized charged memory (CM) to speed up its convergence.

Recently simultaneous analysis and design of structures has been performed using genetic [21–23] and ant colony algorithm [24]. Here, the formulation is modified and optimization is performed using charged system search to enable the efficient solution of larger truss structures.

The present paper is organized as follows: In Section 2, we briefly introduce the CSS and HS. The new method, HS-CSS, is presented in Section 3. In Section 4, charged system search algorithm is employed for the analysis of truss and frame structures. In Section 5, a methodology is proposed for design of structures. Minimum weight design is formulated and performed using meta-heuristics optimization in Section 6. For all the above cases, the CSS and HS-CSS algorithm are found to be powerful tools. The efficiency of the HS-CSS is investigated in Section 7. Conclusions are derived in Section 8.

## 2 Introduction to CSS and HS

In order to make the paper self-explanatory, before proposing the HS-CSS algorithm for truss design optimization, the characteristics of the CSS and the HS are briefly presented in the following three sections:

### 2.1 The standard CSS

Recently an efficient optimization algorithm, known as the charged system search, has been proposed by Kaveh and Talatahari [14]. This algorithm is based on the laws from electrostatics and Newtonian mechanics.

The Coulomb and Gauss laws provide the magnitude of the electric field at a point inside and outside a charged insulating solid sphere, respectively, as follows [25]:

$$\begin{cases} E_{ij} = k_e \frac{q_i}{r_{ij}^2} & : r_{ij} \geq a \\ E_{ij} = k_e \frac{q_i}{a^3} r_{ij} & : r_{ij} < a \end{cases} \tag{1}$$

where $k_e$ is a constant known as the Coulomb constant; $r_{ij}$ is the separation of the centre of sphere and the selected point; $q_i$ is the magnitude of the charge; and "$a$" is the radius of the charged sphere. Using the principle of superposition, the resulting electric force due to $N$ charged spheres is equal to [14]:

$$F_{ij} = k_e q_j \sum_{i,i \neq j} \left( \frac{q_i}{a^3} r_{ij} i_1 + \frac{q_i}{r_{ij}^2} . i_2 \right) \frac{r_i - r_j}{\left\| r_i - r_j \right\|} \qquad \begin{cases} i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{2}$$

Also, according to the Newtonian mechanics, we have [25]:

$$\begin{aligned} \Delta r &= r_{new} - r_{old} \\ v &= \frac{r_{new} - r_{old}}{t_{new} - t_{old}} = \frac{r_{new} - r_{old}}{\Delta t} \\ a &= \frac{v_{new} - v_{old}}{\Delta t} \end{aligned} \tag{3}$$

where $r_{old}$ and $r_{new}$ are the initial and final positions of the particle, respectively; $v$ is the velocity of the particle; and $a$ is the acceleration of the particle. Combining the above equations and using the Newton's second law, the displacement of any object as a function of time is obtained as [25]:

$$r_{new} = \frac{1}{2} \frac{F}{m} . \Delta t^2 + v_{old} . \Delta t + r_{old} \tag{4}$$

Inspired by the above electrostatic and Newtonian mechanics laws, the pseudo-code of the CSS algorithm is presented as follows [14]:

#### 2.1.1 Level 1: Initialization

Step 1. Initialize the parameters of the CSS algorithm. Initialize an array of charged particles (CPs) with random positions. The initial velocities of the CPs are taken as zero. Each CP has a charge of magnitude ($q_i$) defined by considering the quality of its solution as:

$$q_i = \frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}}, \quad i = 1, 2 \ldots, N \tag{5}$$

where $fit_{best}$ and $fit_{worst}$ are the best and the worst fitness of all the particles; $fit(i)$ represents the fitness of agent $i$. The separation distance $r_{ij}$ between two charged particles is defined as:

$$r_{ij} = \frac{\left\| X_i - X_j \right\|}{\left\| (X_i + X_j)/2 - X_{best} \right\| + \varepsilon} \tag{6}$$

where $X_i$ and $X_j$ are the positions of the $i$th and $j$th CPs, respectively; $X_{best}$ is the position of the best current CP; and $\varepsilon$ is a small positive to avoid singularities.

Step 2. CP ranking. Evaluate the values of the fitness function for the CPs, compare with each other and sort them in increasing order.

Step 3. CM creation. Store the number of the first CPs equal to the size of the charged memory (CMS) and their related values of the fitness functions in the charged memory (CM).

#### 2.1.2 Level 2: Search

Step 1. Attracting force determination. Determine the probability of moving each CP toward the others considering the following probability function:

$$p_{ij} = \begin{cases} 1 & \frac{fit(i) - fitbest}{fit(j) - fit(i)} > rand \vee fit(j) > fit(i) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and calculate the attracting force vector for each CP as follows:

$$F_j = q_i \sum_{i,i \neq j} \left( \frac{q_i}{a^3} r_{ij} . i_1 + \frac{q_i}{r_{ij}^2} . i_2 \right) a r_{ij} P_{ij} (X_i - X_j) \qquad \begin{cases} j = 1, 2, ..., N \\ i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} > a \end{cases} \tag{8}$$

where $F_j$ is the resultant force affecting the $j$th CP.

Step 2. Solution construction. Move each CP to the new position and find its velocity using the following equations:

$$X_{j,new} = rand_{j1} k_a \frac{F_{j1}}{m_j} \Delta t^2 + rand_{j2} k_v V_{j,old} \Delta t + X_{j,old} \tag{9}$$

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t} \tag{10}$$

where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range (1,0); $m_j$ is the mass of the CPs, which is equal to $q_j$ in this paper. The mass concept may be useful for developing a multi-objective CSS. $\Delta t$ is the time step, and it is set to 1. $k_a$ is the acceleration coefficient; $k_v$ is the velocity coefficient to control the influence of the previous velocity. In this paper $k_v$ and $k_a$ are taken as:

$$k_v = c_1 (1 - iter/iter_{\max}) \quad k_a = c_2 (1 + iter/iter_{\max}) \tag{11}$$

where $c_1$ and $c_2$ are two constants to control the exploitation and exploration of the algorithm; iter is the iteration number and $iter_{\max}$ is the maximum number of iterations.

Step 3. CP position correction. If each CP exits from the allowable search space, correct its position using the HS-based handling as described by Kaveh and Talatahari [14, 26].

Step 4. CP ranking. Evaluate and compare the values of the fitness function for the new CPs; and sort them in an increasing order.

Step 5. CM updating. If some new CP vectors are better than the worst ones in the CM, in terms of their objective function values, include the better vectors in the CM and exclude the worst ones from the CM.

#### 2.1.3 Level 3: Controlling the terminating criterion

Repeat the search level steps until a terminating criterion is satisfied.

The CSS algorithm is illustrated in Fig. 1.

### 2.2 Harmony search algorithm

Harmony search (HS) algorithm is based on natural musical performance processes that occur when a musician searches for a better state of harmony, such as during jazz improvisation [27]. The engineers seek for a global solution as determined by an objective function, just like the musicians seek to find musically pleasing harmony as determined by an aesthetic [11].

Fig. 2 shows the optimization procedure of the HS algorithm, which consists of the following steps [11]:

Step 1: Initialize the algorithm parameters and optimization operators. The HS algorithm includes a number of optimization operators, such as the harmony memory (HM), the harmony memory size (HMS), the harmony memory considering rate (HMCR), and the pitch adjusting rate (PAR). In the HS algorithm, the HM stores the feasible vectors, which are all in the feasible space. The harmony memory size determines the number of vectors to be stored.

Step 2: Improvise a new harmony from the HM. A new harmony vector is generated from the HM, based on memory considerations, pitch adjustments, and randomization. The HMCR, varying between 0 and 1, sets the rate of choosing a value in the new vector from the historic values stored in the HM, and (1-HMCR) sets the rate of randomly choosing one value from the possible range of values. The pitch adjusting process is performed only after a value is chosen from the HM. The value (1-PAR) sets the rate of doing nothing. A PAR of 0.1 indicates that the algorithm will choose a neighboring value with 10% $\times$ HMCR probability.

Step 3: Update the HM. In Step 3, if a new harmony vector is better than the worst harmony in the HM, judged in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM.

Step 4: Repeat Steps 2 and 3 until the terminating criterion is satisfied. The computations are terminated when the terminating criterion is fulfilled. Otherwise, Steps 2 and 3 are repeated.

## 3 A heuristic harmonic charged system search optimization

The framework of heuristic harmony search and charged system search optimization (HS-CSS) algorithm is illustrated in Fig. 3. HS-CSS algorithm applies the CSS for global search, while HS works as a local search.

The application of this method is identical to standard CSS. However, after ranking the CPs in each iteration, the worst CP is excluded from the CPs and a new CP from the CM with corresponding objective function value and zero velocity is included. The following pseudo-code summarizes the HS-CSS algorithm:

#### 3.0.1 Level 1: Initialization

Step 1. Initialize the CSS algorithm parameters; Initialize an array of Charged Particles with random positions and their associated velocities.

Step 2. CP ranking. Evaluate the values of the fitness function for the CPs, compare with each other and sort increasingly.

Step 3. CM creation. Store CMS number of the first CPs and their related values of the objective function in the CM.

#### 3.0.2 Level 2: Search

Step 1. Attracting force determination. Determine the probability of moving each CP toward others, and calculate the attracting force vector for each CP.

Step 2. Solution construction. Move each CP to the new position and find the velocities.

Step 3. CP position correction. If each CP exits from the allowable search space, correct its position.

Step 4. CP ranking. Evaluate and compare the values of the objective function for the new CPs, and sort them increasingly.

Step 5. Exclude the worst CP from the CPs and include a new CP from the CM, with corresponding objective function value and zero velocity, and rank new CPs, again. At this stage, the Harmony Search is introduced to the algorithm.

Step 6. CM updating. If some new CP vectors are better than the worst ones in the CM, include the better vectors in the CM and exclude the worst ones from the CM.

#### 3.0.3 Level 3: Terminating criterion controlling

Repeat search level steps until a terminating criterion is fulfilled.

## 4 Analysis by force method and charged system search

Here, the main aim is to formulate the energy function of a structure and minimize the latter using the charged system search algorithm, while satisfying all stated compatibility conditions. The formulation is based on the minimum complementary work principle.

Suppose $p = \{p_1 p_2 \dots p_n\}^t$is the vector of nodal forces, $q = \{q_1 q_2 \dots q_{\gamma(s)}\}^t$ contains $\gamma(S)$ redundant forces, and $r = \{s_1 s_2 \dots s_m\}^t$comprises of the internal forces of the members. From equilibrium

$$\{r\} = [B_0]\{p\} + [B_1]\{q\} \tag{12}$$

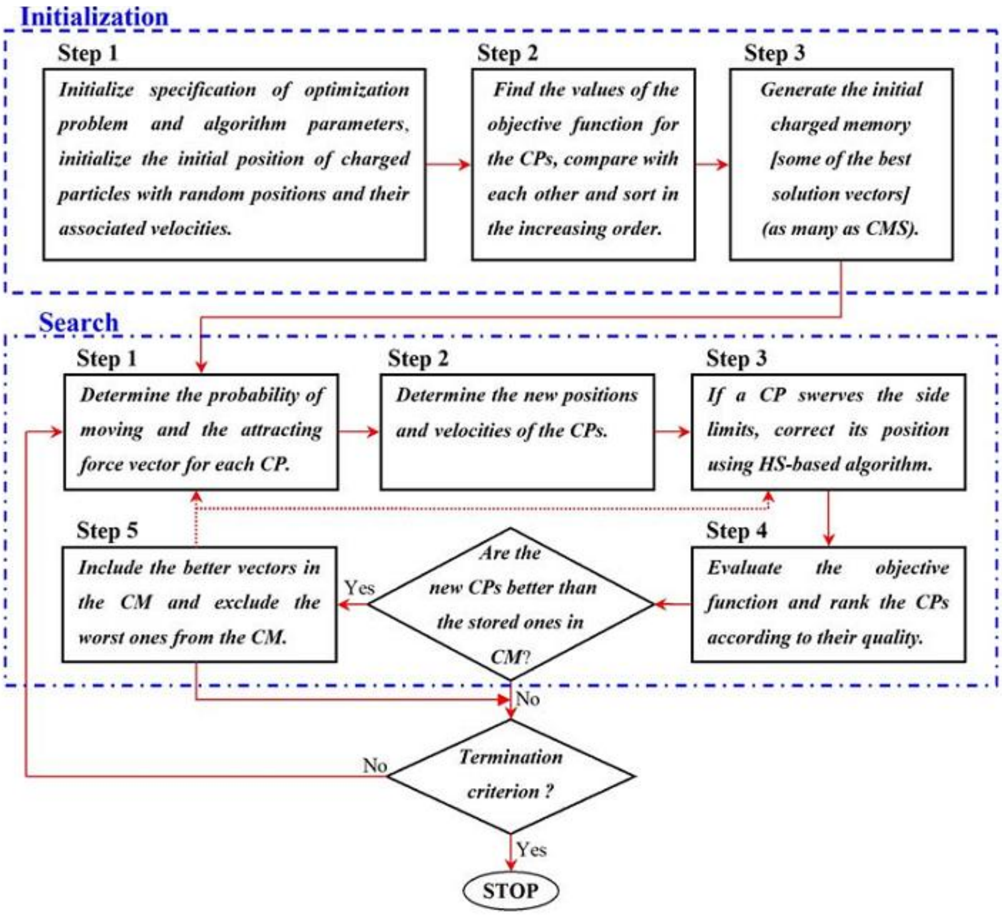



**Fig. 1.** The flowchart of the CSS

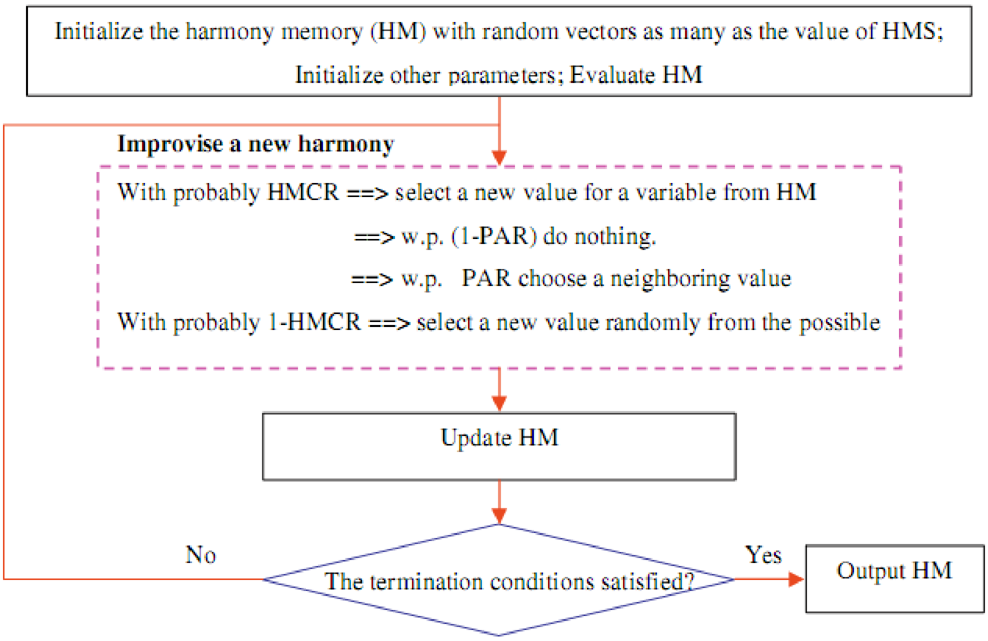



**Fig. 2.** The flowchart of the HS

and

$$U^c = \frac{1}{2} r^t F_m r \tag{13}$$

where $[F_m]$ is the unassembled flexibility matrix of the structure. Now $\{q\}$ should be calculate such that $U^c$ becomes minimum.

Substituting $\{r\}$ from Eq. (12) in Eq. (13) leads to

$$U^c = \frac{1}{2} \begin{bmatrix} p \\ q \end{bmatrix} [H] \begin{bmatrix} p & q \end{bmatrix} \tag{14}$$

where $H = [B_0 B_1]^t [F_m][B_0 B_1]$

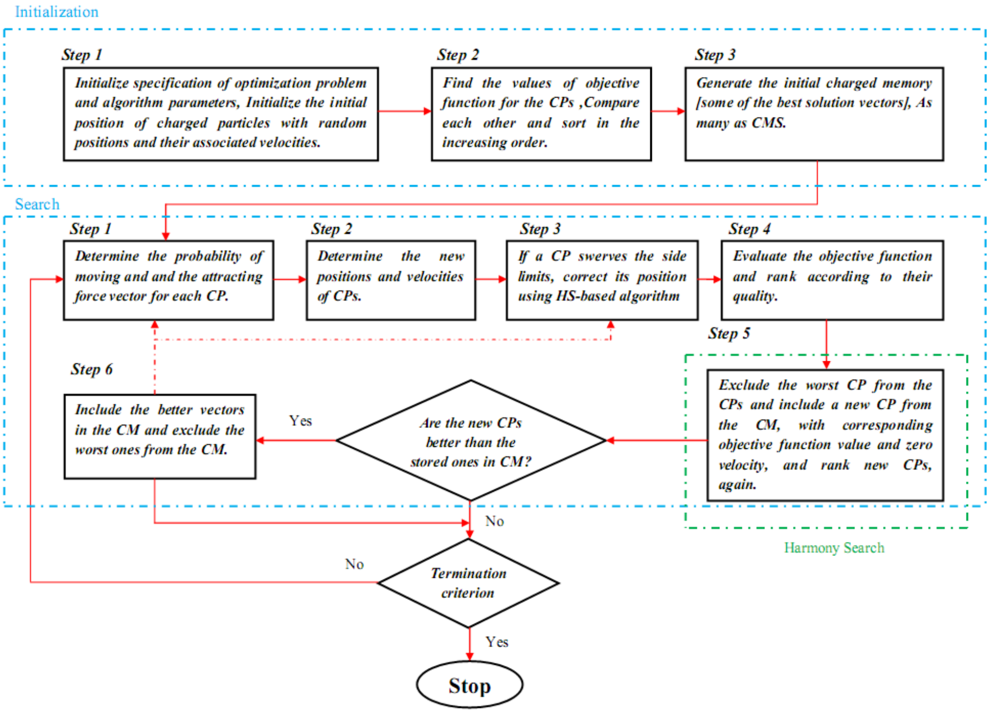



**Fig. 3.** The flowchart of the HS-CSS

Decomposing the matrix $[H]$ into four submatrices $[H_{pp}]$, $[H_{pq}]$, $[H_{qp}]$, and $[H_{qq}]$, we obtain $U^c$ as

$$U^c = \frac{1}{2}(\{p\}^t[H_{pp}]\{p\} + \{p\}^t[H_{pq}]\{q\} + \{q\}^t[H_{qp}]\{p\} + \{q\}^t[H_{qq}]\{q\}) \tag{15}$$

where

$$[H_{pp}] = B_0^t F_m B_0, \quad [H_{pq}] = B_0^t F_m B_1$$
$$[H_{qp}] = B_1^t F_m B_0, \quad [H_{qq}] = B_1^t F_m B_1 \tag{16}$$

In the classical method, the derivative of $U^c$ with respect to $\{q\}$ is found and equated to zero, leading to

$$\frac{\partial U^c}{\partial q} = 0 \Rightarrow [H_{qp}]\{p\} + [H_{qq}]\{q\} = 0$$
$$\Rightarrow \{q\} = -[H_{qq}]^{-1}[H_{qp}]\{p\} \tag{17}$$

Since $[H]$ is symmetric, therefore $[H_{qp}]^t = [H_{pq}]$, Refs. [21,24].

In the present approach, finding the inverse of $[H_{qq}]$ is not required. Instead, $U^c$ from Eq. (14) is minimized by meta-heuristic algorithms.

Previously, it has been stated that the first term of $U^c$ in Eq. (15) is constant and the second and third terms are identical. It can easily be shown that the third and fourth terms of $U^c$ are symmetric and therefore, the second and fourth terms can be omitted and the goal function can be obtained as:

$$F_u = \{q\}^t[H_{qp}]\{p\} \tag{18}$$

Since Eq. (17) holds only in a specific point of search space, and in any other point one cannot omit the second and fourth terms, therefore we use a new goal function as

$$F_u = U^c = \{p\}^t[H_{pq}]\{q\} + \{q\}^t[H_{qp}]\{p\} + \{q\}^t[H_{qq}]\{q\} \tag{19}$$

In order to introduce another goal function consider the left-hand side of the Eq. (17) that is a zero vector, and should be changed to a scalar. The best is to find its norm. If this norm is zero, all the entries should be zero. Therefore, the goal function can be written as

$$F_u = norm([H_{qp}]\{p\} + [H_{qq}]\{q\}) \tag{20}$$

In Eqs. (19) and (20), $\{p\}$, $[H]$ and its submatrices are constant; therefore the charged system search algorithms finds the results for $\{q\}$ by minimizing the complementary energy function.

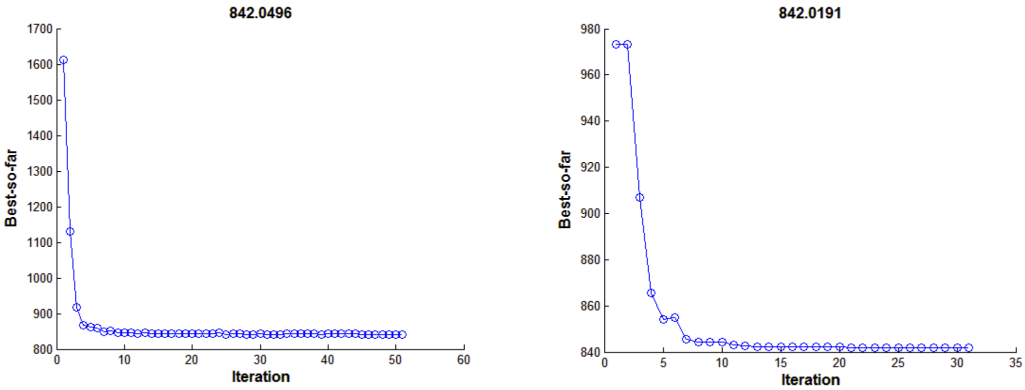
The general complementary energy function (Eq. (13)) can also be used as the goal function of minimization. In this case, there will be no need to calculate the $[H]$ matrix and its submatrices. In order to minimize $F_u$, charged system search algorithms are employed.

As the first example, consider a simple truss as shown in Fig. 4. Here, $F_u$ should be formed in terms of three unknowns. Results are provided in Table 1.

**Tab. 1.** Result of the 11-bar planar truss

| Redundant forces | Magnitude of forces | | | |
|---|---|---|---|---|
| | GA [21] | CACO [24] | Present Work CSS | Present Work HS-CSS |
| 1 | 4.6001 | 4.6846 | 4.6956 | 4.06309 |
| 2 | –3.7476 | –3.6360 | -3.7275 | -3.7794 |
| 3 | 9.8745 | 8.2832 | 8.2373 | 8.1356 |
| Min ($U^c$) | - | - | 842.0496 | 842.0191 |
| No. of analyses | 1400 | 1500 | 800 | 400 |

The number of charged particles for this example are selected as 20. The variation of $F_u$ versus the number of iterations is shown in Fig. 5.




(a) (b)

**Fig. 5.** Variation of $F_u$ versus the number of iterations: (a) for CSS (b) for HS-CSS

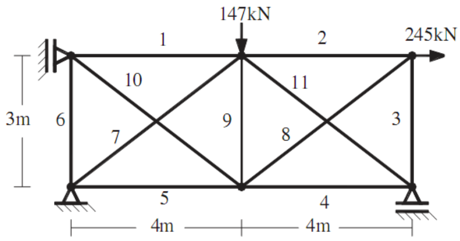



**Fig. 6.** A simple truss with pre-selected stress ratios

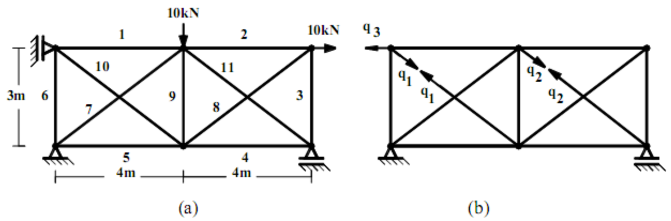



**Fig. 4.** A simple truss and the selected basic structure: (a) a planar truss; and (b) the selected basic structure

As it can be seen, using the CSS after 40 iterations, convergence is achieved but in the HS-CSS this number is only 20. However, in both the CSS and HS-CSS, we have a good results in comparison to the former results.

## 5 Charged system search for design with different member stress ratios

Consider the truss shown in Fig. 6. We want to design this truss with the constraints shown in Table 2. A basic structure similar to the one shown in Fig. 4(b) is selected, where redundants consist of two internal forces and one external reaction, denoted by $q_1$, $q_2$ and $q_3$. The complementary energy of the structure should be minimized for the analysis by the force method. If the cross sections $A_i$ $(i = 1, \cdots, m)$ are known, then the analysis can be performed using the charged system search as described in the previous section. Since the main aim is to design, one can obtain cross-sections $A$ corresponding to the selected values of $q$ (for each charged particle).

$U^c$ can be calculated as

$$U^c = \frac{1}{2}\{r\}^t[F_m]\{r\} \quad \text{where} \quad \{r\} = [\ B_0 \quad B_1\ ][\ p \quad q\ ]^t \tag{21}$$

For a truss member $F_m = L/(EA)$ and for each selected charged particle $q$, one can obtain $\{r\}$ from Eq. (21), and each $\{r\}$ corresponds to a set of cross-sectional areas $A$, the entries of which appear in the denominator of $[F_m]$. Therefore, $F_m$ is a function of $L$, $E$, $q$ and $C$ (i.e. $A$ is eliminated). $U^c$ will then be a function of $q$ and $C$ only. The pre-selected entries for $C$ may be imposed at this stage (the case $c_i = 1$ for all members, needs to be investigated). The role of $C$ in finding $A$ in terms of $q$ has thus been shown, and $U^c$ can easily be minimized by the charged system search. For simplicity in design, the cross-sections are selected as hollow squares with mean length as $h$, as shown in Fig. 7.

$U^c$should be minimized in which $[F_m]$ is a function of the unknowns $q$, $C$, $L$ and $E$ as

$$F_m = \frac{L}{EA} = \frac{L}{E\,f(r,L,C)} = g(q,C,L,E) \tag{22}$$

$$\begin{aligned} Min\ U^c = &\frac{1}{2E}\begin{bmatrix} p & q \end{bmatrix}^t \begin{bmatrix} B_0 & B_1 \end{bmatrix}^t [g(q,C,L,E)] \\ &\cdot \begin{bmatrix} B_0 & B_1 \end{bmatrix}\begin{bmatrix} p & q \end{bmatrix} \end{aligned} \tag{23}$$

Table 2 contains all the information needed for this example.

The magnitudes of $A_i$ will be determined considering the selected values for $c_i$ and design constraints (buckling,...).

A standard CSS based on the above formulation is applied to the present example and results are presented in Table 3. The results obtained from GA and CACO are also provided in this table for comparison.

For this example, 20 charged particles have been created. The convergence is achieved after 30 iterations.

It can be observed that the weight of the truss in Case 2 is reduced compared to Case 1 because of higher magnitudes of member stress ratios. The present method results in lower weight and smaller number of iterations compared to GA and CACO. Results show that all the pre-selected $c_i$ values are attained, and the convergence of the analysis/design process is guaranteed.

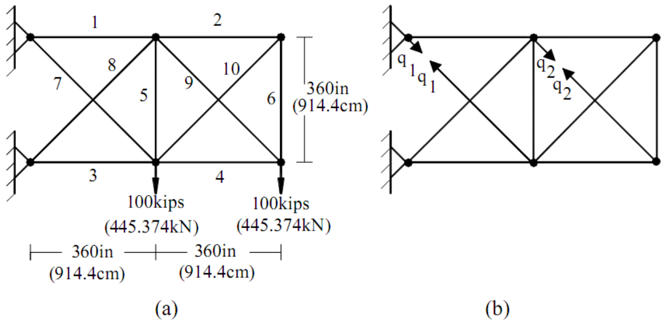



**Fig. 8.** A simple truss and the selected basic structure: (a) A planar truss; (b) The selected basic structure

## 6 Optimal design using the CSS and HS-CSS

Optimality criteria method (OCM) is one of the earliest optimization methods [28]. Fully stressed design (FSD) is a kind of OCM which leads to correct optimal for statically determinate structures under a single load condition. In the FSD all the members are supposed to be subjected to their maximal allowable stresses. To achieve such a design for an indeterminate structure with fixed geometry is not always possible. Even by changing the geometry, a FSD may not be achieved.

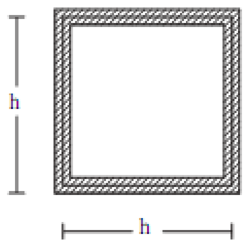



**Fig. 7.** A hollow square cross-section

Here, a CSS formulation of the FSD is presented without using direct analyses in the process of optimization. For this purpose, a truss type of structure is considered and the strain energy is written as

$$U^c = \sum \frac{p^2 L}{EA} = \sum \frac{\gamma p^2 LA}{\gamma E A^2} = \frac{1}{\gamma E} \sum \sigma_i^2 W_i \qquad (24)$$

For constant $E$ and $\gamma$, the minimum weight can be achieved only when the stresses in all the members are the same, and the corresponding term moves out of summation. One may ignore the constraint of the weight, and look for a structure which is fully stressed. The method of the previous section can be applied to perform this design.

As an example, consider the structure shown in Fig. 8 selected from Ref. [28]. Here we have a member size constraint that is provided in Table 4. This constraint leads to a design for which not all the members are fully stressed.

As an example, consider the structure shown in Fig. 8. The member size constraint provided in Table 4, leads to a design for which not all the members are fully stressed.

In this example, the internal forces in members 7 and 9 are taken as redundant forces, forming the initial charged particles of the CSS. The fitness function for Case 2 and Case 1 is the complementary energy as introduced before. Tables 5, 6, 7 contain the results of this example which are obtained using the present algorithms, ACO and GA for three cases.

In Case 3, we want to design a FS structure with the least possible weight, therefore the problem becomes more involved, since different cases may arise with the condition of FS, and one naturally wants the one with the smallest weight.

Choosing a function in the form of $F_u = W + \alpha U^c$ does not help, since the penalty functions are commonly selected as

$$f = A + \alpha B \qquad (25)$$

for which ultimately $f$ converges to A, and B approaches to zero. Therefore, $\alpha$ is often selected as a big number. The main difficulty arises when for $\alpha$, the minimum value of $f$ does not correspond to the minimum $U^c$. In this case, W is minimum while the corresponding $U^c$is not minimum, i.e. the analysis of the structure is not completed yet. Small $\alpha$ will not guarantee the $U^c$to be minimal, and a big $\alpha$ will not lead to a minimum W. Therefore, a new formulation is required.

In a formulation, we alter the second term of Eq. (25) such that its minimum value becomes zero. Then one can use a formulation similar to the common penalty function.

In this method, one does not need $U$, and optimization can be performed employing only $U^c$, where $U^c$can be written in the form of Eq. (15). For the analysis, $q$ should be selected such that Eq. (20) becomes zero. In this case, we can write

$$F(q, A) = W(1 + \alpha norm([H_{qp}]\{p\} + \ [H_{qq}]\{q\}) \qquad (26)$$

Here, the input is $\{q\}$, and having $\{q\}$ from Eq. (18), the magnitude of $F$ can be calculated and its minimum for a large value of $\alpha$ will correspond to minimum W. If a structure contains other constraints, then these should be normalized and added to the above function with a penalty coefficient. Therefore, the final formulation of the problem for the two cases of discrete and continuous cross sections are as follows:

$$\begin{gathered} Find\ q, A : A \in \{S_d or S_c\} \\ Min F(q, A) = W(A)(1 + \alpha norm([H_{qp}]\{p\} + [H_{qq}]\{q\}) \\ + \sum_{m=1}^{nc} Max[0, g_m(A)] \end{gathered} \qquad (27)$$

**Tab. 2.** Design data for the 11-bar planar truss

| **Design Variables** |
|---|
| Redundants and size variables $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; A_9; A_{10}; A_{11}$(and$q_1; q_2; q_3$) |
| ***Material property and constraint data*** |
| Elastic modulus is assumed to be constant. |
| Density of the material: $\rho = 0.00277$ kg/cm$^3$ = 0.1 lb/in$^3$ |
| $A = 0.4h^2$, $r = \sqrt{0.4A}$, thickness=0.1h |
| ***Constraints data*** |
| **Stress Ratio** |
| Case 1 C ={0.9, 0.8, 0.85, 0.8, 0.9, 0.85, 0.95, 0.9, 0.8, 0.9, 0.95} |
| Case 2 $C_i = 1;\ i = 1, \ldots, 11$ |
| **For tensile members** |
| $A > \frac{F}{0.6F_y}, \frac{L}{r} < 300; r = \sqrt{0.4A}$ |
| **For compressive members** |
| $A > \frac{F}{F_a}, F_a = \frac{(1-0.5\beta^2)F_y}{1.67+0.375+0.125\beta^3}; \beta = \frac{L\sqrt{F_y}}{6440}, \frac{L}{r} < 200;$ |
| **Stress constraints** |
| $\sigma_i < 234.43$MPa; $i = 1, 2, ...., 11$ |

**Tab. 3.** Results for the 11-bar planar truss (Cases 1–2)

| |
|---|
| **Case 1** |
| Present work (CSS). |
| q={70.98 , -45.333, 265.12}$^t$kN |
| r ={ 208.33 , 193.32 , -38.757 , 36.26 , 31.152 , -42.592 , -64.085 , 64.595 , -81.35 , 70.98, -45.33}$^t$ kN |
| A={16.45 , 17.18 , 7.43 , 4.44 , 4.44 , 8.17 , 15.625 , 6.94 , 16.58 , 6.94 , 15.63} cm$^2$ |
| W=1337.12 N |
| ACO Algorithm [21] |
| A={17.46 , 17.46 , 5.63 , 4.44 , 4.44 , 5.63 , 15.63 , 6.94 , 6.75 , 6.94 , 15.63} cm$^2$ |
| W=1238.17 N |
| Genetic Algorithm [18] |
| A={4.44 , 6.66 , 19.85 , 4.44 , 14.03 , 6.37 , 6.94 , 16.70 , 22.43 , 6.94 , 15.63} cm$^2$ |
| W=1347.30 N |
| **Case 2** |
| Present work (CSS). |
| q={73.307 , -49.675 , 245.46}$^t$kN |
| r ={186.814 , 186.505 , -40.38 , 39.74 , 34.93 , -43.98 , -50.06 , 67.3 , -84.364 , 73.307 , -49.675}$^t$ kN |
| A={13.28 , 13.26 , 6.585 , 4.44 , 4.44 , 7.173 , 15.625 , 6.94 , 13.76 , 6.94 , 15.63} cm$^2$ |
| W=1222.74 N |
| ACO Algorithm [24] |
| A={18.85 , 14.46 , 5.63 , 4.44 , 4.44 , 5.63 , 15.63 , 6.94 , 5.63 , 6.94 , 15.63} cm$^2$ |
| W=1206.65 N |
| Genetic Algorithm [21] |
| A={4.44 , 5.48 , 16.75 , 4.44 , 12.66 , 5.63 6.94 ,14.84 17.80 , 6.94 , 15.63} cm$^2$ W=1225.3 N |

**Tab. 4.** Design data of a 10-bar planar truss

| **Design Variables** |
|---|
| Size variables $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; A_9; A_{10}$ (and $q_1; q_2$) |
| ***Material property and constraint data*** |
| Elastic modulus: $E$=6.895e7 MPa =1e7 psi. |
| Density of the material: $\rho = 0.00277$ kg/cm$^3$= 0.1 lb/in$^3$ |
| For all members: $A_i \geq 0.645$ cm$^2$ (0.1 in$^2$) ; $i$=1,. . . ,10 |
| ***Stress constraints*** |
| **(a) FSD** |
| Case 1: $\lvert\sigma_i\rvert \leq$172.375 MPa (25 ksi); $i$=1,. . . ,10 |
| Case 2: $\lvert\sigma_i\rvert \leq$172.375 MPa (25 ksi); $i$=1,. . . ,8,10 and $\lvert\sigma_9\rvert \leq$344.75MPa (50 ksi); |
| **(b) Weight minimization** |
| Case 3: $\lvert\sigma_i\rvert \leq$172.375 MPa (25 ksi); $i$=1,. . . ,8,10 and $\lvert\sigma_9\rvert \leq$344.75MPa (50 ksi); |

**Tab. 5.** Results for the 10-bar planar truss (Cases 1 (FSD))

| |
|---|
| Present Work (CSS) |
| **q** ={636.39, 624.09}$^t$ kN={143.066, 140.301}$^t$ kips |
| **r** ={884.44, 3.514, -894.8, -441.26, -1.646, 3.558, 636.39, -621.73, 624.09, -4.97}$^t$ kN |
| **A**={51.313 , 0.645 , 51.91 , 25.60 , 0.645 , 0.645 , 36.92 , 36.07 , 36.20 , 0.645} cm$^2$ |
| **A**={7.953 , 0.1 , 8.046 , 3.98 , 0.1 , 0.1 , 5.722 , 5.59 , 5.61 , 0.1} in$^2$ |
| **W**= 7.1 kN (1596.46 lb) |
| Present Work (HS-CSS) |
| **q**={645.4, 615.37}$^t$ KN={145.09, 138.34}$^t$ kips |
| **r** ={878.11, 9.7, -901.18, -435.13, -1.839, 9.7, 645.4, -612.74, 615.37, -13.701}$^t$ kN |
| ={197.41, 2.18, -202.59, -97.82, -0.413, 2.18, 145.09, -137.75, 138.134, -3.08}$^t$ kips |
| **A**={50.943 , 0.645 , 52.282 , 25.24 , 0.645 , 0.645 , 37.443 , 35.548 , 35.70 , 0.795} cm$^2$ |
| **A**={7.896 , 0.1 , 8.1038 , 3.913 , 0.1 , 0.1 , 5.8 , 5.51 , 5.53 , 0.123} in$^2$ |
| **W**= 7.08 kN (1591.7 lb) |
| ACO Algorithm [24] |
| **A**={ 51.42 , 0.64 , 51.80 , 25.55 , 0.64 , 0.64 , 36.77 , 36.19 , 36.19 , 0.64} cm$^2$ |
| **A**={7.97 , 0.1 , 8.03 , 3.96 , 0.1 , 0.1 , 5.70 , 5.61 , 5.61 , 0.13} in$^2$ |
| **W**= 7.11 kN (1595.88 lb) |
| Genetic Algorithm[21] |
| **A**={ 51.16 , 0.64 , 52.06, 25.35 , 0.64 , 0.64 , 37.16 , 35.81 , 35.81 , 0.84} cm$^2$ |
| **A**={7.93 , 0.1 , 8.07 , 3.93 , 0.1 , 0.1 , 5.76 , 5.55 , 5.55 , 0.13} in$^2$ |
| **W**= 7.09 kN (1593.5 lb) |

**Tab. 6.** Results for the 10-bar planar truss (Cases 2 (FSD))

| |
|---|
| Present Work (CSS) |
| **q** ={1258.09, 0.045}$^t$ KN={282.83 , 0.01}$^t$ kips |
| **r** ={ 444.86, 444.77, -1334.42, -0.045, 0.00, 444.77, 1258.1, -0.045, 0.045, 628.98}$^t$ kN |
| **A**={ 25.83 , 25.78 , 77.40 , 0.64 , 0.64 , 25.78 , 72.96 , 0.64 , 0.64 , 36.46} cm$^2$ |
| **A**={4.00 , 3.99 , 11.99 , 0.1 , 0.1 , 3.99 , 11.31 , 0.1 , 0.1 , 5.65} in$^2$ |
| **W**= 7.76 kN (1745.3 lb) |
| Present Work (HS-CSS) |
| **q**={1212.8, 45.59}$^t$ kips |
| **r** ={476.89, 412.57, -1302.4, -32.25, 0.18, 412.57, 1212.8, -45.327, 45.59, -583.47}$^t$ KN |
| **r** ={107.21, 92.75, -292.79, -7.25, 0.04, 92.75, 272.65, -10.19, 10.25, -131.17}$^t$ kips |
| **A**={ 27.67 , 23.93 , 75.56 , 1.87 , 0.64 , 23.93 , 70.36 , 2.63 , 1.32 , 3.385} cm$^2$ |
| **A**={4.288 , 3.71 , 11.71 , 0.29 , 0.1 , 3.71 , 10.91 , 0.408 , 0.205 , 5.247} in$^2$ |
| **W**=7.61 kN (1710.7 lb) |
| ACO Algorithm [24] |
| **A**={ 26.06 , 25.35 , 77.16 , 0.64 , 0.64 , 25.35 , 72.64 , 0.64 , 0.64 , 35.87} cm$^2$ |
| **A**={4.04 , 3.93 , 11.96 , 0.1 , 0.1 , 3.93 , 11.26 , 0.1 , 0.1 , 5.56} in$^2$ |
| **W**=7.71 kN (1732.68 lb) |
| Genetic Algorithm [21] |
| **A**={ 26.58 , 25.03 , 76.64 , 0.77 , 0.64 , 25.03 , 71.87 , 1.1 , 0.64 , 35.35} cm$^2$ |
| **A**={4.12 , 3.88 , 11.88 , 0.12 , 0.1 , 3.88 , 11.14 , 0.17 , 0.1 , 5.48} in$^2$ |
| **W**=7.66 kN (1723.5 lb) |

where $S_d$ and $S_c$ are the discrete and continuous cross sections, respectively. $g_m(A)$ corresponds to violations of constraints, which include stress constraints, displacement constraints and buckling constraints. Their magnitudes can be written in the form of the absolute value of existing value to permissible value minus one.

From Tables 5, 6, 7, it is noticeable that if in a structure the maximum allowable stresses of all members are equal, then FSD leads in a minimum weight design. Otherwise, we have to add a penalty function (that contains the weight of the structure) to the goal function for the purpose of minimization using the CSS and HS-CSS algorithms.

## 7 Numerical Examples

In this section, four truss structures are optimized utilizing the CSS and the new algorithm. The final results are then compared to the solutions of other advanced heuristic methods to demonstrate the efficiency of this work. For the both CSS and HS-CSS algorithm, a population of 20 CPs is used for all the examples. The effect of the previous velocity and the resultant force affecting a CP can be decreased or increase based on the values of the $k_v$ and $k_a$ (Eq. (11)), where $c_i$ is set to 0.8.

**Tab. 7.** Results for the 10-bar planar truss (Cases 3 (weight minimization))

| |
|---|
| Present Work (CSS) |
| **q**={637.5, 612.74}$^t$ KN ={143.315 , 137.756}$^t$ kips |
| **r** ={883.68, 11.52, -895.61, -433.26, 5.58, 11.52, 637.5, -620.62, 612.74, -16.28}$^t$ KN |
| ={198.66, 2.59, -201.34, -97.4, 1.254, 2.59, 143.315, -139.52, 137.75, -3.66}$^t$ kips |
| **A**={51.267 , 0.67 , 51.96 , 25.14 , 0.645 , 0.67 , 36.98 , 36.00 , 17.75 , 0.945} cm$^2$ |
| **A**={7.946 , 0.103 , 8.053 , 3.896 , 0.1 , 0.103 , 5.732 , 5.581 , 2.755 , 0.147} in$^2$ |
| **W**=6.456 kN (1451.05 lb) |
| Present Work (HS-CSS) |
| **q**={637.21, 137.83}$^t$ KN ={143.25 , 137.83}$^t$ kips |
| **r** ={883.91, 11.3, -895.38, -433.52, 5.56, 11.3, 637.21, -620.97, 613.1, -15.97}$^t$ KN |
| **r** ={ 198.71 , 2.54 , -201.29 , -97.46 , 1.25 , 2.54 , 143.25 , -139.6 , 137.83 , -3.59}$^t$ kips |
| **A**={51.28 , 0.65 , 51.94 , 25.15 , 0.645 , 0.65 , 36.96 , 36.02 , 17.78 , 0.925} cm$^2$ |
| **A**={7.95 , 0.101 , 8.05 , 3.896 , 0.1 , 0.101 , 5.73 , 5.58 , 2.75 , 0.143} in$^2$ |
| **W**=6.45 kN (1450.9 lb) |
| ACO Algorithm [24] |
| **A**={ 51.22 , 0.71 , 51.93 , 25.10 , 0.64 , 0.71 , 36.97 , 35.93 , 17.74 , 0.97} cm$^2$ |
| **A**={7.94 , 0.11 , 8.05 , 3.89 , 0.1 , 0.11 , 5.73 , 5.57 , 2.75 , 0.15} in$^2$ |
| **W**=6.46 kN (1450.15 lb) |
| Genetic Algorithm [21] |
| **A**={ 50.26 , 1.68 , 53.03 , 24.58 , 0.64 , 1.54 , 38.52 , 35.48 , 23.16 , 2.19} cm$^2$ |
| **A**={7.79 , 0.26 , 8.22 , 3.81 , 0.1 , 0.24 , 5.97 , 5.5 , 3.59 , 0.34} in$^2$ |
| **W**=6.75 kN (1519.2 lb) |

In order to investigate the effect of the initial solution on the final result and because of the stochastic nature of the algorithm, each example is independently solved several times. The initial population in each of these runs is generated in a random manner according to Rule 2. The algorithms are coded in Matlab.

### 7.1 A ten-bar planar truss

Optimal design of 10-bar truss, as shown in Fig. 8, is considered. Table 4 contains the necessary information. In this section, a displacement constraint is added (Table 8).

In this example, two cases are considered, the first is for discrete and the second corresponds to continuous sections. In both of these *A* and *q* are design variables.

However, in the discrete case, we used a code for sections. In both cases the displacements and stresses are included. Using the formulation of the previous section and minimizing Eq. (27), we obtained the results for discrete sections as shown in Table 9. Fig. 9 provides a comparison of the convergence rates of different results.

The best result is related to the CSS and HS-CSS, however, HS-CSS is converged at iteration 395 but the standard CSS at iteration 463. Furthermore, HS-CSS at iteration 96 is resulted in 5494.163, which is near to the previous results.

Also for the continuous case, Table 10 and Fig. 10 are provided.

As it can be seen, here the best result is related to the HS-CSS algorithm which has a tiny superiority in comparison to GA (Kaveh and Rahami [21]).

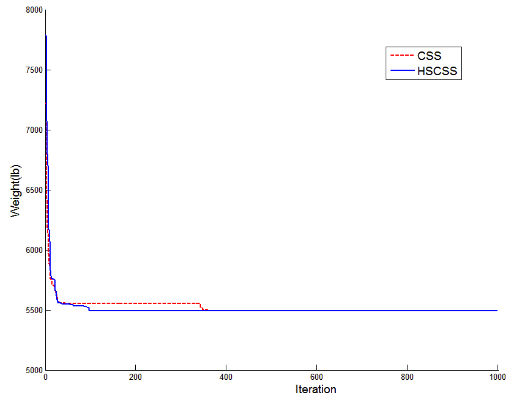



**Fig. 9.** Comparison of the convergence rates between the CSS and HS-CSS algorithms for the 10-bar planar truss structure (discrete)

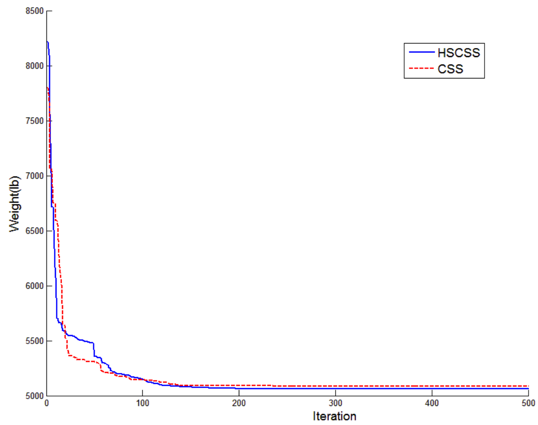



**Fig. 10.** Comparison of the convergence rates between the CSS and HS-CSS algorithms for the 10-bar planar truss structure (continuous)

**Tab. 8.** Design data for the 10-bar planar truss

| Design Variables |
| --- |
| ***Material property and constraint data*** |
| Elastic modulus: $E$= 6.895e7 MPa =1e7 psi. |
| Density of the material: $\rho =$ 0.00277 kg/cm$^3$= 0.1 lb/in$^3$ |
| **Stress constraint** $\lvert\sigma_i\rvert \leq$ 172.375 MPa (25 ksi); $i = 1, \ldots, 10$ |
| **Displacement constraint in all directions of the co-ordinate system** $\lvert\Delta_i\rvert \leq 5.08cm(2in); i = 1, \cdots, 8$ |
| **List of the available profiles** |
| **Case 1:** (Discrete sections) |
| $A_i$ = {10.4516, 11.6129, 12.8387, 13.7419, 15.3548, 16.9032, 16.9677, 18.5806, 18.9032, 19.9354, 20.1935, 21.8064, 22.3871, 22.9032, 23.4193, 24.7741, 24.9677, 25.0322, 26.9677, 27.2258, 28.9677, 29.6128, 30.9677, 32.0645, 33.0322, 37.0322, 46.5806, 51.4193, 74.1934, 87.0966, 89.6772, 91.6127, 99.9998, 103.2256, 109.0320, 121.2901, 128.3868, 141.9352, 147.7416, 170.9674, 193.5480, 216.1286}$cm^2$ |
| $A_i$ = {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5}$in^2$ |
| **Case 2:** (Continuous sections) |
| $0.0645 \leq A_i \leq 225.8960cm^2(0.1 \leq A_i \leq 35in^2); i = 1, \ldots, 10$ |

**Tab. 9.** Optimal design comparison for the 10-bar planar truss (discrete)

| Element group | Discrete sections | | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | Shih [29] | Rajeev [30] | Kaveh and Rahami[21] | Kaveh and Hassani[24] | CSS | HS-CSS |
| $A_1$ | 33.5 | 33.5 | 33.5 | 33.5 | 33.50 | 33.50 |
| $A_2$ | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| $A_3$ | 22.90 | 22.90 | 22.90 | 22.90 | 22.90 | 22.90 |
| $A_4$ | 15.50 | 15.50 | 14.2 | 14.2 | 13.90 | 13.90 |
| $A_5$ | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| $A_6$ | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| $A_7$ | 7.97 | 14.20 | 7.97 | 11.5 | 7.97 | 7.97 |
| $A_8$ | 22.00 | 19.90 | 22.90 | 22.00 | 22.90 | 22.90 |
| $A_9$ | 22.00 | 19.90 | 22.00 | 19.90 | 22.00 | 22.00 |
| $A_{10}$ | 1.62 | 2.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| Best weight: kN (lb) | 24.4271 (5491.71) | 24.9704 (5613.84) | 24.4228 (5490.738) | 24.5702 (5517.72) | 24.3747 (5479.93) | 24.3747 (5479.93) |
| No. of analyses | - | - | - | - | 9260 | 7900 |

## 7.2 A 25-bar spatial truss

The topology of a 25-bar spatial truss structure is shown in Fig. 11. In this example, designs are performed for a multiple loading case. The material density is 2767.990kg/m$^3$ (0.1 lb/in$^3$) and the modulus of elasticity is 68,950 MPa (10,000 ksi).

The structural members of this truss are arranged into eight groups, where all members in a group share the same material and cross-sectional properties, which are as follow:

(1) $A_1$, (2) $A_2$ –$A_5$, (3) $A_6$–$A_9$, (4) $A_{10}$–$A_{11}$, (5) $A_{12}$–$A_{13}$, (6) $A_{14}$–$A_{17}$, (7) $A_{18}$–$A_{21}$, and (8) $A_{22}$–$A_{25}$.

Table 11 contains the data for the design of this truss and the results for two cases.

Tables 12 and 13 list the optimal values of the eight size variables obtained by this research, and compares them with other existing results.

In the discrete case, both CSS and HS-CSS converged to the same weight. However, in the HS-CSS, this convergence oc-

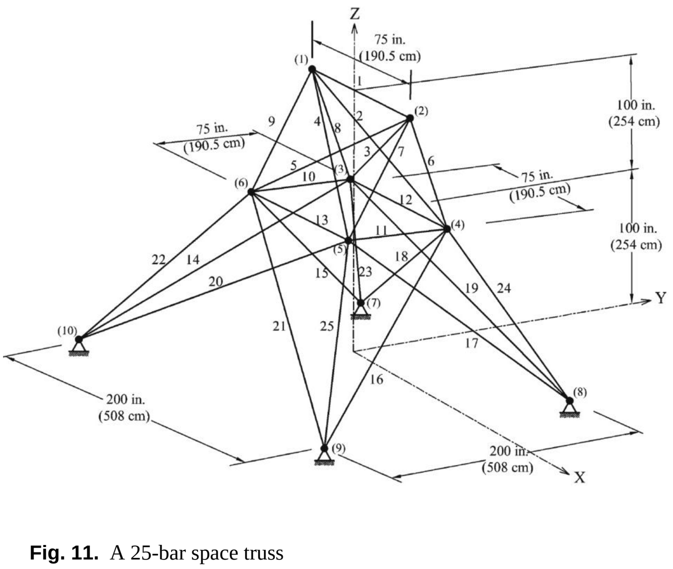



**Fig. 11.** A 25-bar space truss

**Tab. 10.** Optimal design comparison for the 10-bar planar truss (continuous)

| Element group | Continuous sections | | | | | |
|---|---|---|---|---|---|---|
| | Kaveh and Rahami [21] | Schmit and Farshi [31] | Schmit and Miura [32] | Schmit and Miura [32] | Venkayya [33] | Gellatly and Berke [34] |
| $A_1$ | 30.67 | 33.43 | 30.67 | 30.57 | 30.42 | 31.35 |
| $A_2$ | 0.1 | 0.1 | 0.1 | 0.37 | 0.13 | 0.1 |
| $A_3$ | 22.87 | 24.26 | 23.76 | 23.97 | 23.41 | 20.03 |
| $A_4$ | 15.34 | 14.26 | 14.59 | 14.73 | 14.91 | 15.6 |
| $A_5$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.14 |
| $A_6$ | 0.46 | 0.1 | 0.1 | 0.36 | 0.1 | 0.24 |
| $A_7$ | 7.48 | 8.39 | 8.59 | 8.55 | 8.7 | 8.35 |
| $A_8$ | 20.96 | 20.74 | 21.07 | 21.11 | 21.08 | 22.21 |
| $A_9$ | 21.7 | 19.69 | 20.96 | 20.77 | 21.08 | 22.06 |
| $A_{10}$ | 0.1 | 0.1 | 0.1 | 0.32 | 0.19 | 0.1 |
| Best weight:kN (lb) | 22.5153 (5061.9) | 22.6359 (5089) | 22.5818 (5076.85) | 22.7173 (5107.3) | 22.6176 (5084.9) | 22.7382 (5112.00) |
| No. of analyses | - | - | - | - | - | - |
| | Dobbs and Nelson [35] | Rozzo [36] | Khan and Willmert [37] | Kaveh and Hassani [24] | CSS | HS-CSS |
| $A_1$ | 30.5 | 30.73 | 30.98 | 30.86 | 32.86 | 31.085 |
| $A_2$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.102 | 0.10118 |
| $A_3$ | 23.29 | 23.93 | 24.17 | 23.55 | 24.303 | 23.297 |
| $A_4$ | 15.43 | 14.73 | 14.81 | 15.01 | 14.224 | 15.174 |
| $A_5$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.105 | 0.1 |
| $A_6$ | 0.21 | 0.1 | 0.41 | 0.22 | 0.11 | 0.535 |
| $A_7$ | 7.65 | 8.54 | 7.547 | 7.63 | 10.04 | 7.487 |
| $A_8$ | 20.98 | 20.95 | 21.05 | 21.65 | 20.1 | 21.13 |
| $A_9$ | 21.82 | 21.84 | 20.94 | 21.32 | 22.85 | 20.98 |
| $A_{10}$ | 0.1 | 0.1 | 0.1 | 0.1 | 33.74 | 0.1 |
| Best weight: kN (lb) | 22.5958 (5080.0) | 22.581 (5076.66) | 22.5379 (5066.98) | 2268.99 (5095.46) | 22.614 (5084.11) | 22.5118 (5061.12) |
| No. of analyses | - | - | - | - | 7660 | 8420 |

**Tab. 11.** Data for design of 25-bar spatial truss

**Design Variables**

Size variables: $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; q_1; q_2; q_3; q_4; q_5; q_6; q_7$

***Material property and constraint data***

Elastic modulus: $E$=6.895e7 MPa =1e7 psi.

Density of the material: $\rho =$ 0.00277 kg/cm$^3$= 0.1 lb/in$^3$

**Stress constraint** $|\sigma_i| \leq 275.80 MPa(40ksi); i = 1, \ldots, 25$

**Displacement constraint in the directions of X and Y in the co-ordinate system** $|\Delta_i| \leq 0.8890cm(0.35in); i = 1, 2$

**List of the available profiles**

**Case 1:** (Discrete sections)

$$A_i = \{0.1, 0.5 * I(I = 1, 2, \cdots, 76), 39.81, 40\}in^2$$

$$A_i = \{0.6452, 3.2258 * I(I = 1, 2, \ldots, 76), 256.8382, 258.0640\}cm^2$$

**Case 2:** (Continuous sections)

$$A_i \geq 0.6452cm^2(0.1in^2); i = 1, \ldots, 8$$

**Loading Data**

| Node | $P_x$: kN (kip) | $P_y$: kN (kip) | $P_z$: kN (kip) |
|---|---|---|---|
| 1 | 4.448 (1) | -44.48 (-10) | -44.48 (-10) |
| 2 | 0 | -44.48 (-10) | -44.48 (-10) |
| 3 | 2.224 (0.5) | 0 | 0 |
| 6 | 2.6688 (0.6) | 0 | 0 |

curred after only 467 iterations, but in the case of CSS this happened in 915 iterations (Fig. 12).

Also in the continuous case, although CSS has a better result, however, convergence in the HS-CSS occurred much earlier (Fig. 13).

**Tab. 12.** Optimal design comparison for the 25-bar spatial truss (discrete sections)

| Element group | | Optimal cross-sectional areas (in$^2$) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Rajeev [29] | Erbatur [38] | Kaveh and Kalatjari [39] | Kaveh and Rahami [21] | Kaveh and Hassani [24] | Present Work | |
| | | | | | | | CSS | HS-CSS |
| 1 | A1 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 2 | A2–A5 | 1.80 | 1.20 | 0.10 | 0.50 | 0.50 | 0.50 | 0.50 |
| 3 | A6–A9 | 2.30 | 3.20 | 3.50 | 3.00 | 3.50 | 3.00 | 3.00 |
| 4 | A10–A11 | 0.20 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 5 | A12–A13 | 0.10 | 1.10 | 2.0 | 2.00 | 2.00 | 2.00 | 2.00 |
| 6 | A14–A17 | 0.80 | 0.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 7 | A18–A21 | 1.80 | 0.40 | 0.10 | 0.10 | 0.1 | 0.10 | 0.10 |
| 8 | A22–A25 | 3.00 | 3.40 | 4.00 | 4.00 | 3.50 | 4.00 | 4.00 |
| Best weight: kN (lb) | | 2.428 (546.01) | 2.196 (493.80) | 2.136 (480.23) | 2.134 (479.775) | 2.11 (474.42) | 2.134 (479.775) | 2.134 (479.775) |
| No. of analyses | | - | - | - | - | - | 18300 | 9340 |

**Tab. 13.** Optimal design comparison for the twenty-five-bar spatial truss (continuous sections)

| Element group | | Optimal cross-sectional areas (in$^2$) | | | |
|---|---|---|---|---|---|
| | | Kaveh and Rahami [21] | Kaveh and Hassani [24] | Present Work | |
| | | | | CSS | HS-CSS |
| 1 | A1 | 0.10 | 0.10 | 0.10 | 0.10 |
| 2 | A2–A5 | 0.10 | 0.72 | 0.104 | 0.12 |
| 3 | A6–A9 | 3.7598 | 3.34 | 3.56 | 3.528 |
| 4 | A10–A11 | 0.10 | 0.10 | 0.10 | 0.10 |
| 5 | A12–A13 | 1.8552 | 1.82 | 1.879 | 1.871 |
| 6 | A14–A17 | 0.7755 | 0.67 | 0.796 | 0.791 |
| 7 | A18–A21 | 0.1408 | 0.32 | 0.161 | 0.152 |
| 8 | A22–A25 | 3.846 | 3.47 | 3.938 | 3.97 |
| Best weight: kN (lb) | | 2.08 (467.6293) | 2.076 (466.8) | 2.078 (467.31) | 2.08 (467.69) |
| No. of analyses | | - | - | 38640 | 15320 |

**Tab. 14.** Loading conditions for the 72-bar spatial truss

| Case | Node | $F_x$ kN (kip) | $F_y$ kN (kip) | $F_z$ kN (kip) |
|---|---|---|---|---|
| 1 | 17 | 0.0 | 0.0 | −22.25 (-5.0) |
| | 18 | 0.0 | 0.0 | −22.25 (-5.0) |
| | 19 | 0.0 | 0.0 | −22.25 (-5.0) |
| | 20 | 0.0 | 0.0 | −22.25 (-5.0) |
| 2 | 17 | 22.25 (5.0) | 22.25 (5.0) | −22.25 (-5.0) |

### 7.3 A 72-bar spatial truss with continuous sections

For the 72-bar spatial truss structure shown in Fig. 14, the material density is 2767.990 kg/m$^3$ (0.1 lb/in$^3$) and the modulus of elasticity is 68,950 MPa (10,000 ksi). The members are subjected to the stress limits of ±172.375 MPa (±25 ksi). The nodes are subjected to the displacement limits of ±0.635 cm (±0.25 in). The 72 structural members of this spatial truss are categorized as 16 groups using the symmetry:

(1) $A_1$–$A_4$, (2) $A_5$–$A_{12}$, (3) $A_{13}$–$A_{16}$, (4) $A_{17}$–$A_{18}$, (5) $A_{19}$–$A_{22}$, (6) $A_{23}$–$A_{30}$, (7) $A_{31}$–$A_{34}$, (8) $A_{35}$–$A_{36}$, (9) $A_{37}$–$A_{40}$, (10) $A_{41}$–$A_{48}$, (11) $A_{49}$–$A_{52}$, (12) $A_{53}$–$A_{54}$, (13) $A_{55}$–$A_{58}$, (14) $A_{59}$–$A_{66}$(15), $A_{67}$– $A_{70}$, and (16) $A_{71}$–$A_{72}$

The values and directions of the two load cases applied to the 72-bar spatial truss are listed in Table 14.

The results are summarized in Table 15. Also in Fig. 15, a comparison is performed between the convergence rates of the HS-CSS and CSS algorithms.

The HS-CSS algorithm can find the best design among the other existing studies. The best weight of the HS-CSS algorithm is 168.26 kg (370.96 lb), while it is 168.49 kg (371.47 lb) for the Standard CSS. The standard CSS algorithm gets the optimal solution after 14800 analyses, while it takes 13840 analyses for the HS-CSS. However, both CSS and HS-CSS have a good application in comparison to those of the previously reported methods in the literature. For example, new results are 2.1 percent lighter than those attained by the HBB-BC (the least weight

**Tab. 15.** Optimal design comparison for the 72-bar spatial truss

| Element group | | Optimal cross-sectional areas (in$^2$) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | GA [40] | ACO [41] | PSO [42] | BB–BC [43] | HBB–BC [40] | Present work: CSS | Present work: HS-CSS |
| 1 | $A_1$–$A_4$ | 1.755 | 1.948 | 1.7427 | 1.8577 | 1.9042 | 1.8135 | 1.8624 |
| 2 | $A_5$–$A_{12}$ | 0.505 | 0.508 | 0.5185 | 0.5059 | 0.5162 | 0.5219 | 0.5173 |
| 3 | $A_{13}$–$A_{16}$ | 0.105 | 0.101 | 0.1000 | 0.1000 | 0.1000 | 0.1047 | 0.1004 |
| 4 | $A_{17}$–$A_{18}$ | 0.155 | 0.102 | 0.1000 | 0.1000 | 0.1000 | 0.1015 | 0.1002 |
| 5 | $A_{19}$–$A_{22}$ | 1.155 | 1.303 | 1.3079 | 1.2476 | 1.2582 | 1.1831 | 1.1371 |
| 6 | $A_{23}$–$A_{30}$ | 0.585 | 0.511 | 0.5193 | 0.5269 | 0.5035 | 0.5259 | 0.4925 |
| 7 | $A_{31}$–$A_{34}$ | 0.100 | 0.101 | 0.1000 | 0.1000 | 0.1000 | 0.1003 | 0.1005 |
| 8 | $A_{35}$–$A_{36}$ | 0.100 | 0.100 | 0.1000 | 0.1012 | 0.1000 | 0.1 | 0.1002 |
| 9 | $A_{37}$–$A_{40}$ | 0.460 | 0.561 | 0.5142 | 0.5209 | 0.5178 | 0.4845 | 0.4758 |
| 10 | $A_{41}$–$A_{48}$ | 0.530 | 0.492 | 0.5464 | 0.5172 | 0.5214 | 0.5069 | 0.5337 |
| 11 | $A_{49}$–$A_{52}$ | 0.120 | 0.100 | 0.1000 | 0.1004 | 0.1000 | 0.1006 | 0.1014 |
| 12 | $A_{53}$–$A_{54}$ | 0.165 | 0.107 | 0.1095 | 0.1005 | 0.1007 | 0.1115 | 0.1076 |
| 13 | $A_{55}$–$A_{58}$ | 0.155 | 0.156 | 0.1615 | 0.1565 | 0.1566 | 0.1013 | 0.1002 |
| 14 | $A_{59}$–$A_{66}$ | 0.535 | 0.550 | 0.5092 | 0.5507 | 0.5421 | 0.5097 | 1.8624 |
| 15 | $A_{67}$–$A_{70}$ | 0.480 | 0.390 | 0.4967 | 0.3922 | 0.4132 | 0.3692 | 0.5173 |
| 16 | $A_{71}$–$A_{72}$ | 0.520 | 0.592 | 0.5619 | 0.5922 | 0.5756 | 0.6051 | 0.1004 |
| Weight (kN) (lb) | | 1.716 (385.76) | 1.691 (380.24) | 1.698 (381.91) | 1.689 (379.85) | 1.688 (379.66) | 1.652 (371.47) | 1.650 (370.96) |
| No. of analyses | | N/A | 18,500 | N/A | 19,621 | 13,200 | 14800 | 13840 |

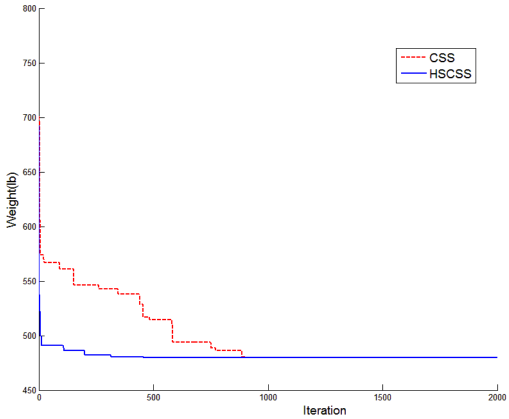



**Fig. 12.** Convergence rate comparison between the CSS and HS-CSS algorithms for the 25-bar spatial truss (discrete sections)

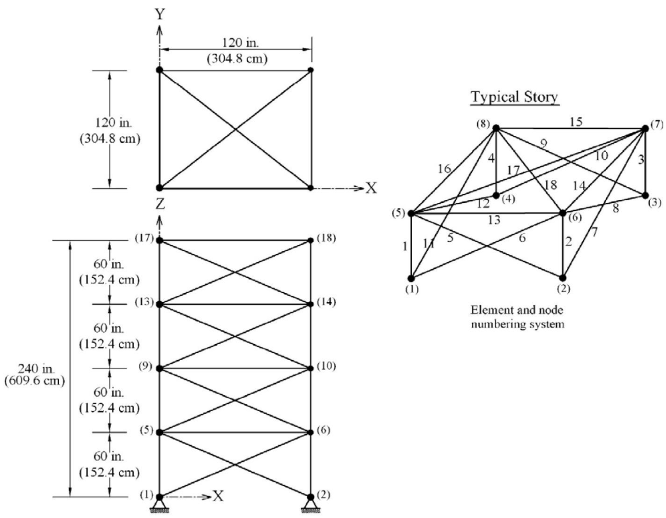



**Fig. 14.** A 72-bar spatial truss

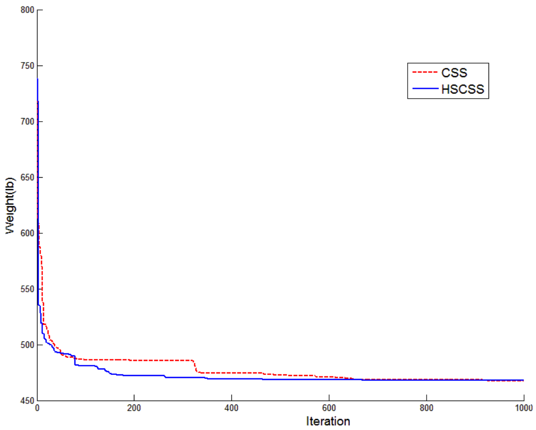



**Fig. 13.** Convergence rate comparison between the CSS and HS-CSS algorithms for the 25-bar spatial truss (continuous sections)

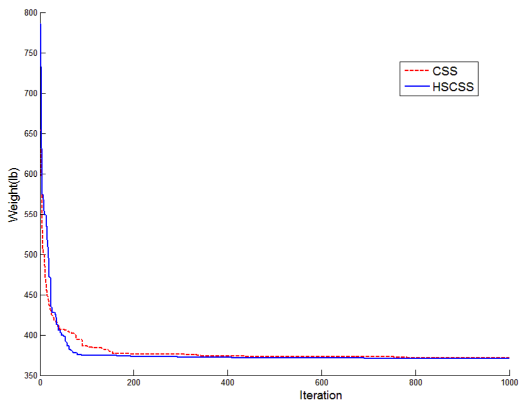



**Fig. 15.** Convergence rate comparison between the CSS and HS-CSS algorithms for the 72-bar spatial truss (continuous sections)

in the previous results).

## 8 Concluding remarks

In this article, a new heuristic algorithm is introduced by hybridizing CSS and HS algorithms (HS-CSS). Then, formulations are presented for the analysis, design and optimization of structures for use in the new algorithm. These methods employ basic ideas from energy and complementary energy and utilize the CSS and the presented algorithm and a comparative study is then performed. The CSS and HS-CSS perform analysis of structures without using classical methods which require the direct solution of the equations. Design is performed providing prescribed stress ratios for the members, and as a special case an efficient approach is suggested for fully stressed design. Formulation in terms of energy concepts permits the efficient application of the CSS and HS-CSS in optimization. The examples studied in this paper for analysis, design and optimization illustrate the capability and the accuracy of the present applications and also presented heuristic algorithm.

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