ΨP periodica polytechnica

Civil Engineering
56/2 (2012) 213–219
doi: 10.3311/pp.ci.2012-2.07
web: http://www.pp.bme.hu/ci

RESEARCH ARTICLE

SIMP type topology optimization procedure considering uncertain load position

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Received 2012-04-22, accepted 2012-09-14

Abstract

In this paper a new type of probabilistic optimal topology design method is elaborated for continuum type of structures where the points of application of the loads are given randomly. In the proposed probabilistic topology optimization method the minimum penalized weight design of the discretized structure is subjected to compliance constraint and side constraints. The compliance expression is probabilistic one. By the use of an appropriate stochastic upperbound theorem, the original stochastic mathematical programming problem is substituted by a deterministic one. The numerical procedure is based on iterative formula which is formed by the use of the first order optimality condition of the Lagrangian function. The application is illustrated by numerical example.

Keywords

topology optimization · probability · stochastic loading · optimality criteria method · optimal design · robust design

Acknowledgement

The present study was supported by the Hungarian National Scientific and Research Foundation (OTKA) (grant K 81185).

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1 Introduction

The topology optimization has more than 100 years of history and still it is an expanding field in structural optimization. The numerical procedure for FE (finite element) based topology optimization of continuum type of structures was elaborated first by Rossow and Taylor [19] in 1973, but the real expansion started at the end of 80-s [4, 20]. The majority of the papers still deal with deterministic problems. During these years several optimal topologies were numerically calculated but the analytical confirmations – which have come recently (Rozvany [21], Sokółet. al [22]) – are mostly missing. Until the end of the last century almost one could not find any publication on topology optimization considering uncertainties.

During the last years before the millennium almost there were no publications in the topic of probability based topology optimization. The stochastic optimization works of Marti and Stöckl [16, 17] provide early information about this topic. The paper of Duan et al. [5] is among the very first publications in the field of uncertainty based topology optimization. This work presents an entropy-based topological optimization method for truss structures by the use of iteration technique. Also a truss topology optimization (layout optimization) of the object of the paper of Alvarez and Carrasco [1] in case stochastic loading. They showed mathematically that a problem of finding the truss of minimum expected compliance (stability of the members are not considered) under stochastic loading conditions is equivalent to the dual of a special convex minimax problem. Dunning et al. [6, 7] introduce an efficient and accurate approach to robust structural topology optimization for continuum type of problems. The objective is to minimize expected compliance with uncertainty in loading magnitude and applied direction, where uncertainties are assumed normally distributed and statistically independent. This approach is analogous to a multiple load case problem where load cases and weights are derived analytically to accurately and efficiently compute expected compliance and sensitivities. Illustrative examples using a level-set-based topology optimization method are then used to demonstrate the proposed approach.

Topology optimization with uncertainty in the magnitude and locations of the applied loads and with small uncertainty in the locations of the structural nodes is the object of the paper of Guest and Igusa [8]. Their method is based on the assumption that the loading uncertainties are taken into consideration as "safety factors" of the deterministic load cases in the load combination. The effects of geometric uncertainty were estimated using second order stochastic perturbation and uncertainties in the stiffness of the structure were transformed into a mathematically equivalent system of auxiliary loads. This technique is extended for nonlinear effects of global instability [9] and material property uncertainties [2], to put more control on the variability of the ?nal design via including variance of the compliance [3]. Asadpoure et al. [3] present a computational strategy that combines deterministic topology optimization techniques with a perturbation method for the quantification of uncertainties associated with structural stiffness, such as uncertain material properties and/or structure geometry. The applied technique leads to significant computational savings when compared with Monte Carlo-based optimization algorithms. Jalalpour et al. [9] extend the perturbation based topology optimization procedure [8] to approximate the effect of random geometric imperfections on the second order response of trusses. Monte Carlo simulation together with second-order elastic analysis is used to verify that solutions offer improved performance in the presence of geometric uncertainties.

Lógó [13] and Lógó et al. [12, 14] elaborated a rather powerful method for the stochastic topology optimization where the magnitude of the loads or the compliance bounds are given by their mean values, covariances and distribution functions. By the use of direct integration technique for the calculation of the uncertain bounds or applying an appropriate approximation for the loading uncertainties the stochastic expressions are substituted by an equivalent deterministic ones to make the optimization problem robust. The loading positions, as uncertain data in the topology optimization problem, is considered in [15]. Here two computational models and the corresponding algorithms are elaborated. Both models use simple transformations to substitute the original load position problem with uncertain loading magnitude ones. This work is a continuation of the above cited papers.

In this paper the uncertainties of the load positions are considered and the goal is to provide a SIMP type algorithm to solve a continuum type topology optimization problem. By the use of a simple simulation technique and the stochastic upperbound theorem of Kataoka [10] a generalized compliance design problem is elaborated. The uncertain quantities are substituted by their generalized statistical measures. To solve this constrained mathematical programming problem an iterative solution technique is derived by the use of the optimality criteria method. To demonstrate the applicability of the algorithm numerical examples are presented and compared.

2 Mathematical and mechanical background

2.1 Approximation of a Probabilistic Expression

According to the approximation theory of Kataoka [10] a stochastic expression can be upperbounded by a convex deterministic one. From the literature the generalization of this theory is known by Prekopa [18]. The outline of this method can be explained as follow: if $\xi_1, \xi_2, ..., \xi_n$ have a joint normal distribution, then the set of $\mathbf{x} \in \Re^n$ vectors satisfying

$$P(x_1\xi_1 + x_2\xi_2 + ... + x_n\xi_n \le 0) \ge q \tag{1}$$

is the same as those satisfying

$$\sum_{i=1}^{n} x_i\mu_i + \Phi^{-1}(q)\ \sqrt{\mathbf{x}^T\mathbf{K}_{ov}\mathbf{x}} \le 0 \tag{2}$$

where $\mu_i = E(\xi_i)$, $(i = 1, 2, ..., n)$ is the mean value of the randomly given element ξ_i, $\mathbf{K}_{ov}$ is the covariance matrix of the random vector $\boldsymbol{\xi}^T = (\xi_1, \xi_2, ..., \xi_n)$, q is a fixed probability and $0\langle q\langle 1$, $\Phi^{-1}(q)$ is the inverse cumulative distribution function (so called probit function) of the normal distribution. Expression (2) is convex, the proof can be found in Prekopa [18]. According the original approximation theory of Kataoka the probit function is substituted by an appropriate constant and the Gaussian distribution is not a requirement.

In the following the above theory of Prekopa is applied.

2.2 Probabilistic Compliance Design

The deterministic compliance design procedure of a linearly elastic 2D structure (disk) in plane stress is known from literature (e.g. (Rozvany[20]). This topology optimization problem is given as follows:

$$W = \sum_{g=1}^{G} \gamma_g A_g t_g^{\frac{1}{p}} = \min! \tag{3a}$$

subject to

$$\begin{cases} \mathbf{u}^T\mathbf{F} - C \le 0; \\ -t_g + t_{\min} \le 0; \quad (\text{for} \quad g = 1, ..., G), \\ t_g - t_{\max} \le 0; \quad (\text{for} \quad g = 1, ..., G). \end{cases} \tag{3b-d}$$

Here the ground element thicknesses t_g are the design variables with lower bound $t_{\min}$ and upper bound $t_{\max}$, respectively. By the use of the FE (finite elements) discretization, each ground element $(g = 1, \ldots, G)$ contains several sub-elements $(e = 1, \ldots, E_s)$, whose stiffness coefficients are linear homogeneous functions of the ground element thickness t_g. Furthermore γ_g is the specific weight and $A_g$the area of the ground element g. $\mathbf{u}^T$ is the nodal displacement vector associated with the loading $\mathbf{F}$. The displacements $\mathbf{u}$ can be calculated from $\mathbf{Ku} = \mathbf{F}$, where $\mathbf{K}$ is the system stiffness matrix. p is the penalty parameter $(p \ge 1)$ and the given compliance value is denoted by C. The above constrained mathematical programming problem can be solved by the use of an appropriate SIMP algorithm (e.g. Lógó [11]).

Let us suppose that the structure (the design domain) and the boundary conditions (supports and loadings) are given (Fig. 1). The material is linearly elastic and isotropic. The loading is given by deterministic (magnitude and direction) and probabilistic (point of application) data. It is very important to note that the elaborated method will be suitable to find the optimal topology of the continuum type structures in the case when only the point of applications are stochastically given (the magnitudes and the directions are not stochastic). It means that practically the loading domain is given and either the structural layout itself can carry the variation of the loadings or a secondary structure is provided to transfer this loading domain to the optimized structure. The different uncertain locations are given by x_i, $(i = 1, .., n)$ where the external loads $\mathbf{F}^T = [\mathbf{f}_1, \mathbf{f}_2, ..., \mathbf{f}_i, ..., \mathbf{f}_n]$ act with given magnitude $f_i = |\mathbf{f}_i|\,(i = 1, ..., n)$. The distance of the load vector $\mathbf{f}_i$ indicated by x_i as point of application (Fig. 1) follows a given distribution – for sake of simplicity here all the stochastic data are Gaussian ones-. Because the precise value of x_i is not known, x_i is given by its mean value $\overline{x_i}$ and standard deviation σ_i. Using a simple calculation the probability of the position of a force being at a certain location can be determined or it is given in advance (see in Section 3, Fig. 2). Due to the stochastic nature of the point of application of the loads the compliance calculation is difficult and the topology optimization can not be elaborated easily.

As it is known, the compliance value can be calculated as:

$$\mathbf{u}^T\mathbf{F} = u_1 f_1 + u_2 f_2 + \ldots + u_n f_n \tag{4}$$

where the displacements $(u_i, i = 1, ..., n)$ are obtained from $\mathbf{Ku} = \mathbf{F}$ linear system and denote the displacement under the force $\mathbf{f}_i\,(i = 1, ..., n)$ in the direction of this load. The magnitude $f_i = |\mathbf{f}_i|\,(i = 1, ..., n)$ and the direction are deterministic values and due to the stochastic nature of the point of applications x_i $(i = 1, \ldots, n)$ the displacements $u_i(i = 1, ..., n)$ are probabilistic. Also due to the linear theory of the mechanical model they can follow a Gaussian distribution. (For sake of simplicity it is assumed, otherwise instead of Prekopa's theorem the original Kataoka-theorem is used.)

By the use of a generalized compliance design concept (Lógó [13]) a new constraint

$$P\left(\mathbf{u}^T\mathbf{F} - C \le 0\right) \ge q \tag{5}$$

can be introduced instead of eq. (3b). Here $0\langle q\langle 1$ is a given expected probability value what gives information about the possibility of a failure. Following the upperbound theorem of Kataoka [9] and the generalization theorem of Prekopa [18] introduced above eq.(5) can be substituted by the following deterministic expression which is convex:

$$\sum_{i=1}^{n} f_i\overline{u_i} - C + \Phi^{-1}(q)\,\sqrt{\mathbf{b}^T\mathbf{K}_{ov}\mathbf{b}} \le 0. \tag{6}$$

Here $\overline{u_i} = E(u_i), i = 1, ..., n$ is the expected value of the displacement under the force $\mathbf{f}_i\,(i = 1, ..., n)$ in the direction of this load, $\mathbf{b}^T = [f_1, f_2, ..., f_i, ..., f_n]$, $\mathbf{K}_{ov}$ is the covariance matrix of these displacements. The expected displacement value $\overline{u_i} = E(u_i)(i = 1, ..., n)$ and the corresponding elements $\kappa_{i,j}$ $(i = 1, ..., n;\ j = 1, ..., n)$ of the covariance matrix $\mathbf{K}_{ov}$ can be computed as the result of a certain type of simulation.

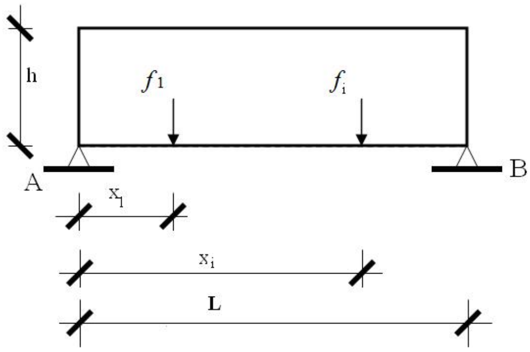


Fig. 1. The design domain with the boundary conditions

Then the penalized minimum weight problem subjected to probabilistic compliance constraint has the form:

$$W = \sum_{g=1}^{G} \gamma_g A_g t_g^{\frac{1}{p}} = \min! \tag{7a}$$

subject to

$$\begin{cases} \sum_{i=1}^{n} f_i\overline{u_i} - C + \Phi^{-1}(q)\,\sqrt{\mathbf{b}^T\mathbf{K}_{ov}\mathbf{b}} \le 0; \\ -t_g + t_{\min} \le 0; \quad (\text{for} \quad g = 1, ..., G), \\ t_g - t_{\max} \le 0; \quad (\text{for} \quad g = 1, ..., G) \end{cases} \tag{7b}$$

This type of constrained mathematical programming problem can be solved by using an appropriate optimality criteria algorithm (see e.g. Lógó [13]).

3 Probabilistic Compliance Design in the Case of Uncertain Loading Positions: Simplified Simulation

Let us consider the design problem given in Fig. 1. Since the loading positions are not known precisely an equivalent loading system should be also created around the expected location $\overline{x_i}$ of each force $\mathbf{f}_i$ to perform the simulation. According to the original distribution assumption, the mean value and the standard deviation of the point application are determined by the force system $f_{ij}(j = 1, \ldots, k)$ with the original magnitude f_i - for sake of simplicity here seven points – as "base" points are used with symmetrical adjustment $(f_{i1}, f_{i2}, f_{i3}, f_{i4})$. Each load is independent and a well-defined probability value $w_{ij}(j = 1, \ldots, k = 7)$ is assigned to them (in practice it can take as design information). The determination of this probability value $w_{ij}(j = 1, \ldots, k)$ is based on the original distribution and it can be calculated with a simple computation. In this way the loading is given by these doubled parameters -$w_{ij}(j = 1, \ldots, k = 7)$, $(f_{i1}, f_{i2}, f_{i3}, f_{i4})$- and applied as independent load cases. The modified topology design problem is given in Fig. 2.

Applying these forces at these "base" points as loads the stochastic design problem becomes a deterministic one after

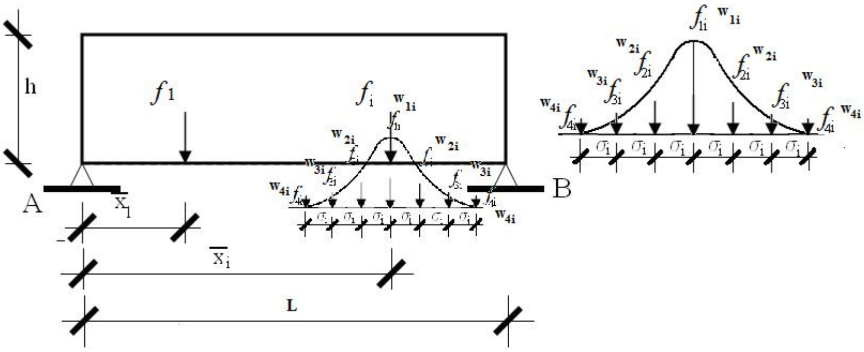


Fig. 2. The design domain with the modified loadings and the corresponding probabilities

this transformation. By the use of the element $f_{ij}(j = 1, ..., k)$ of these force system one by one, the displacement vectors $\mathbf{u}_{ij}(j = 1, ..., k)$ can be calculated from the $\mathbf{Ku}_{ij} = \mathbf{f}_{ij}$ linear equations. Since the material is linearly elastic the additive properties of the displacements and the *reciprocity theorem* can be applied. Using these vectors and the assigned probability values $w_{ij}(j = 1, ..., k)$ the expected displacement $\bar{u}_i$ and its variation $D^2(\bar{u}_i)$ can be calculated in the following form:

$$\bar{\mathbf{u}}_i = \sum_{j=1}^{k} \mathbf{u}_{ij} w_{ij}; \tag{8a}$$

$$D_i^2(\bar{u}_i) = \sum_{j=1}^{k} \left(u_{ij}\right)^2 w_{ij} - \bar{u}_i^2. \tag{8b}$$

These computed values are used to compose the element of the mathematical programming problem eq. (7). Due to the nature of this type of loading the covariance matrix is diagonal.

$$\mathbf{K}_{ov} = \left\langle D_1^2(\bar{u}_1), D_2^2(\bar{u}_2), \ldots, D_n^2(\bar{u}_n) \right\rangle \tag{9}$$

Interchanging the expected compliance calculation by the generalized expected strain energy formulation the penalized minimum weight problem subjected to probabilistic compliance constraint has the form:

$$W = \sum_{g=1}^{G} \gamma_g A_g t_g^{\frac{1}{p}} = \min! \tag{10a}$$

subject to

$$\left\{ \begin{array}{l} \sum\limits_{i=1}^{n} \bar{\mathbf{u}}_i^T \mathbf{K} \mathbf{u}_i - C + \Phi^{-1}(q)\ \sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}} \leq 0 \\ -t_g + t_{\min} \leq 0; \quad (\text{for} \quad g = 1, ..., G) \\ t_g - t_{\max} \leq 0; \quad (\text{for} \quad g = 1, ..., G) \end{array} \right. \tag{10b-d}$$

Since the mathematical nature of the problem (10) is similar to a classical topology optimization problem all the mathematical statements concerning convexity and differentiability are valid too (Rozvany [19], Lógó [11, 13]). The penalization of the ground element thicknesses t_g results in a more distinct material distribution indicating material or no material. Due to this penalization the optimization problem is non-unique in some sense, but the method is widely applied in engineering optimization.

The above constrained mathematical programming problem (eq. (10b-d)) can be solved by the use of a modified SIMP algorithm (Lógó [13]). The iterative algorithm is derived from the first order optimality conditions.

Neglecting the details, one can obtain

$$\frac{1}{p}\gamma_g A_g t_g^{\frac{1-p}{p}} + \nu \left(\sum_{i=1}^{n} \left(\frac{\partial \bar{\mathbf{u}}_i^T}{\partial t_g} \mathbf{K} \bar{u}_i + \mathbf{u}_i^T \frac{\partial \mathbf{K}}{\partial t_g} \bar{\mathbf{u}}_i + \mathbf{u}_i^T \mathbf{K} \frac{\partial \bar{\mathbf{u}}_i}{\partial t_g} \right) + \right.$$
$$\left. + \Phi^{-1}(q) \frac{\partial \left(\sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}} \right)}{\partial t_g} \right) - \alpha_g + \beta_g = 0, \quad (g = 1, ..., G). \tag{11a}$$

To evaluate the derivation $\frac{\partial \left(\sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}} \right)}{\partial t_g}$ one can write that

$$\frac{\partial \left(\sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}} \right)}{\partial t_g} = -\frac{\sum\limits_{i=1}^{n} \sum\limits_{e=1}^{E_g} \left(\sum\limits_{j=1}^{k} \left(\mathbf{u}_{ije}^T \tilde{\mathbf{K}}_e \mathbf{u}_{ie} \right)^2 w_j \right) - \left(\mathbf{u}_{ie}^T \tilde{\mathbf{K}}_e \bar{\mathbf{u}}_{ie} \right)^2}{\sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}}} = -\frac{VAR_g}{VAR_s}, \tag{11b}$$

where VAR_g is a ground element based compliance expression and VAR_s is the whole structure based one. Introducing the following notations

$$R_g = t_g^2 \sum_{e=1}^{E_g} \mathbf{u}_{ge}^T \tilde{\mathbf{K}}_{ge} \bar{\mathbf{u}}_{ge} \quad \text{and} \quad \mathrm{B_g} = \mathrm{t_g^2} \Phi^{-1}(\mathrm{q}) \frac{\mathrm{VAR_g}}{\mathrm{VAR_s}} \tag{11c}$$

the eq.(11a) becomes very simple

$$\frac{1}{p}\gamma_g A_g t_g^{\frac{1-p}{p}} - \nu \frac{R_g + B_g}{t_g^2} - \alpha_g + \beta_g = 0; \quad (g = 1, ..., G). \tag{11d}$$

Because the eq. (11d) is created on the base of the extension of the classical topology optimization problem (1) with expression $\Phi^{-1}(q)\ \sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}}$ the fulfilment of the regularity conditions of the problem above are equivalent with the regularity conditions of the original optimization problem (10).

As it is in optimality criteria type methods one can define two sets of the thicknesses: a set of active ($\mathcal{A}$) and a set of passive ($\mathcal{P}$) thicknesses [15].

If $t_{\min} < t_g < t_{\max}$ (or by other words, the ground element is "active", $g \in \mathcal{A}$) by the use of eq. (11c) the following formula can be obtained

$$t_g = \left(\frac{\nu p \left(R_g + B_g \right)}{A_g \gamma_g} \right)^{\frac{p}{p+1}}. \tag{12}$$

All other case either $t_g = t_{\min}$ or $t_g = t_{\max}$ are applied ([10]). If $t_g = t_{\min}$ or $t_g = t_{\max}$ we call the ground element "passive" ($g \in \mathcal{P}$).

In order to keep the number and layout of ground elements constant and avoid the ill-conditioned stiffness matrix, one can replace the zero element thickness ($t_{\min}$) with a small but finite value (e.g. $t_{\min} = 10^{-6}$). If the probabilistic compliance constraint is active in problem (10a-d) (e.g. satisfies the equality sign) the following form holds

$$\sum_{i=1}^{n} \mathbf{u}_i^T \mathbf{K} \bar{\mathbf{u}}_i + \Phi^{-1}(q)\, VAR_s - C = 0. \tag{13}$$

Because the compliance value of the g-th ground element is computed by the addition of

$$R_g = t_g^2 \sum_{e=1}^{E_g} \bar{\mathbf{u}}_{ge}^T \tilde{\mathbf{K}}_{ge} \bar{\mathbf{u}}_{ge}$$

and

$$\Phi^{-1}(q)\, VAR_s = $$
$$= \Phi^{-1}(q) \sqrt{\left(\sum_{g=1}^{G} t_g^2 \sum_{e=1}^{E_g} \sum_{i=1}^{n} \sum_{j=1}^{k} \left(\mathbf{u}_{ij}^T \tilde{\mathbf{K}}_e \mathbf{u}_i \right)^2 w_{ij} - \left(\mathbf{u}_i^T \tilde{\mathbf{K}}_e \bar{\mathbf{u}}_i \right)^2 \right)}$$

the total compliance value of the structure is the summation of the ground elements (active and the unit thickness passive) compliances.

$$C - \Phi^{-1}(q) \sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}} = \sum_{g \in \mathcal{P}} \frac{R_g}{t_g} + \sum_{g \in \mathcal{A}} \frac{R_g}{t_g}. \tag{14}$$

The compliance values of the zero thickness passive elements can be neglected. By the use of the formulation of thickness calculation (eq. (12)) of the active elements the former compliance calculation is

$$\begin{aligned} & C - \sum_{g \in \mathcal{P}} R_g \\ & -\Phi^{-1}(q) \sqrt{\left(\sum_{g \in \mathcal{P}} t_g^2 \sum_{e=1}^{E_g} \sum_{i=1}^{n} \sum_{j=1}^{k} \left(\mathbf{u}_{ij}^T \tilde{\mathbf{K}}_e \mathbf{u}_i \right)^2 w_j - \left(\mathbf{u}_i^T \tilde{\mathbf{K}}_e \bar{\mathbf{u}}_i \right)^2 \right)} \\ & = \frac{\sum_{g \in \mathcal{A}} R_g + \Phi^{-1}(q) \sqrt{\left(\sum_{g \in \mathcal{A}} t_g^2 \sum_{e=1}^{E_g} \sum_{i=1}^{n} \sum_{j=1}^{k} \left(\mathbf{u}_{ij}^T \tilde{\mathbf{K}}_e \mathbf{u}_i \right)^2 w_j - \left(\mathbf{u}_i^T \tilde{\mathbf{K}}_e \bar{\mathbf{u}}_i \right)^2 \right)}}{\left(\frac{\nu p (R_g + B_g)}{A_g \gamma_g} \right)^{\frac{p}{p+1}}} \\ & = \frac{\sum_{g \in \mathcal{A}} \left(R_g + GVAR \right)}{\left(\frac{\nu p (R_g + B_g)}{A_g \gamma_g} \right)^{\frac{p}{p+1}}}. \end{aligned} \tag{15}$$

The Lagrange multiplier ν as a step length can be formulated similarly to the deterministic solution procedure. Since the thickness value for passive elements ($g \in \mathcal{P}$) is given and for active elements ($g \in \mathcal{A}$) the formulation is elaborated, it can be calculated by the use of eq. (15). The calculated value of the Lagrange-multiplier ν belonging to the active elements can be given as follow

$$\nu = \left(\frac{\sum_{g \in \mathcal{A}} \left(\frac{A_g \gamma_g}{p (R_g + B_g)} \right)^{\frac{p}{p+1}} \left(R_g + GVAR \right)}{C - \sum_{g \in \mathcal{P}} \left(\frac{R_g}{t_g} + \Phi^{-1}(q) \sqrt{\mathbf{b}^T \mathbf{K}_{ov} \mathbf{b}} \right)} \right)^{\frac{p+1}{p}}, \quad \text{for} \quad \mathcal{A} \neq 0). \tag{16}$$

The optimal solution can be obtained by evaluating iteratively the thickness values t_g and the Lagrange-multiplier ν from (11) and (16).

4 Numerical examples

To demonstrate the application of the method and the algorithm elaborated above three small examples are calculated. The deterministic design (here the load is located at the position of its expected value) and the stochastic design are presented. In addition of these examples the assumed analytical solution of the deterministic designs are also introduced.

4.1 Deterministic Design

In this problem a dimensionless 40x40 units square type ground structure is the object of the design (Fig. 3 a.–c.).

80x80 ground elements with 2x2 sub-elements are used. (Total number of elements is 25600.) The Poisson's ratio is 0. The load is (100 units) acting in the middle of the top edge. The penalty parameter p was run from $p = 1$ to $p = 1.5$ with smooth increasing (increment is 0.1) and later to $p = 2.5$ with increment= 0.25. The applied compliance limit is C=220000.

The possible exact analytical solution of the deterministic designs can be seen in Fig. 4 a.–c. (Lógó [10]). The numerical optimal topologies can be seen in Figs. 5 a–c., respectively. These later ones are in good agreement with the analytical solutions.

Due to the difference of the displacement boundary conditions the optimal topologies are fundamentally different.

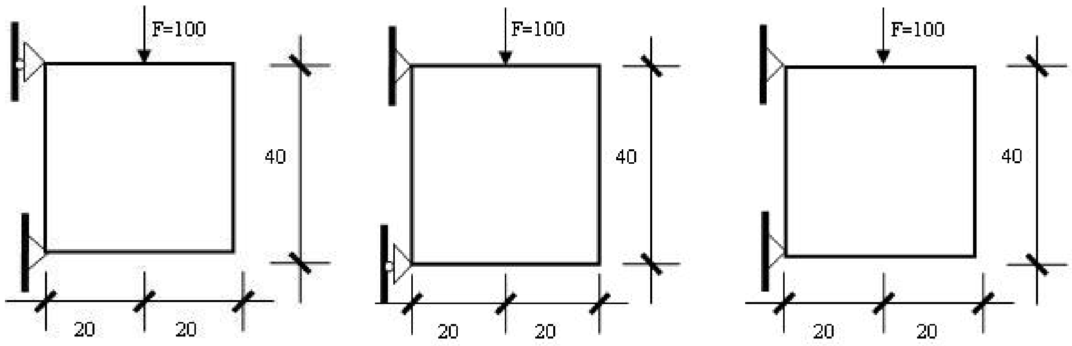
4.2 Probabilistic Design

As it was indicated earlier in the case of stochastic topology optimization the point of applications of the loads are random variables. They follow a normal distribution as it is assumed for the displacements. The simplified simulation is based on seven base points of the loads and the corresponding probabilities of each position -$w_{ij}(j = 1, ..., k)$- can be easily calculated.

The assumed expected probability is given by $q = 0.75$. The same compliance limit is applied (C=220000). The modifications and the termination criteria of the penalty parameter are the same as they are in the deterministic examples. Using the algorithm presented above the optimal topologies can be calculated. Due to the nature of the problems these topologies include the possibility of the collapse of the structure.

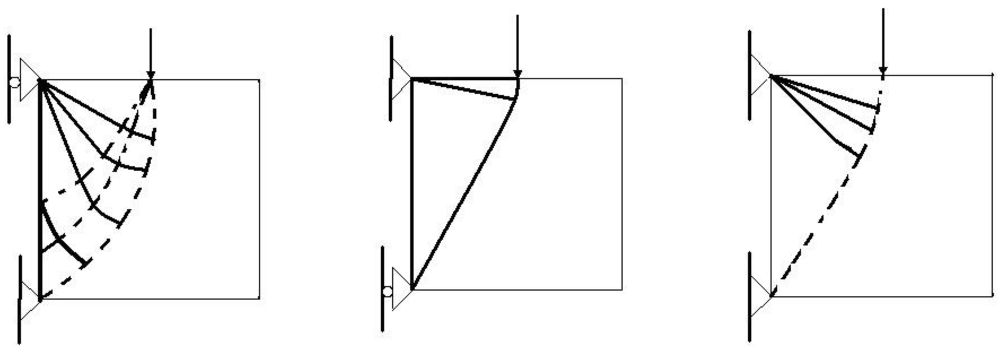
5 Conclusion

A numerical procedure was elaborated for continuum type topology optimization in the case of uncertain load positions.



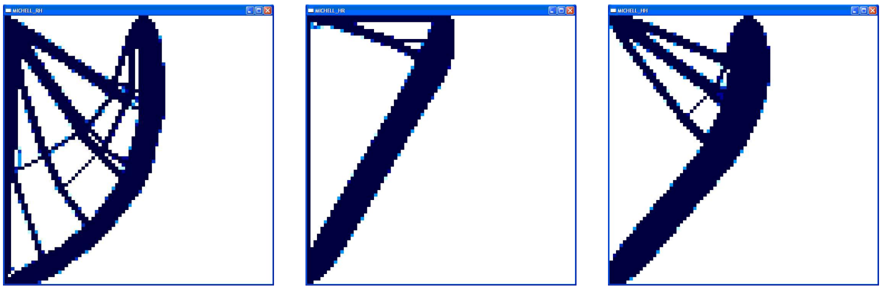
a b c

Fig. 3. Square domain with different support conditions



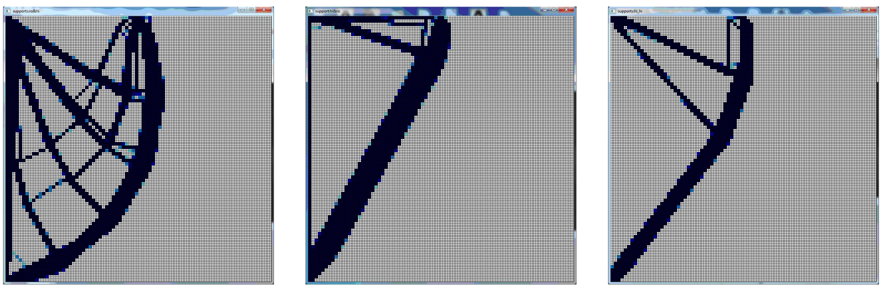
a b c

Fig. 4. Possible exact analytical solutions of the deterministic designs



a; Roller and hinge b; Hinge and roller c; Hinge and hinge

Fig. 5. Numerical solutions for square domain



a; Roller and hinge b; Hinge and roller c; Hinge and hinge

Fig. 6. Numerical solutions for square domain in case of stochastic point of applications

The parametric studies can confirm that the method is suitable for numerical calculation. The computational times are not significant. The uncertainties can modify the deterministically obtained optimal topologies. The optimal structure is thinner than the deterministic one.

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