

# Mathematical modeling and model-based optimum control of circulating fluidized bed combustors

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## Abstract

A comprehensive mathematical model of circulating fluidized bed combustors was set up and verified by measurements carried out on a 300 MW industrial boiler. The one-dimensional cell model structure was used, and equations describing the dynamics of coal combustion, gas–gas reactions, fluid dynamics of the suspension and heat transfer were implemented. Partitioning the combustion chamber into only nine cells according to the geometrical characteristics of the technology plus an empirical cyclone temperature model was found to fulfill both requirements regarding accuracy and low computational demand. The later one is a key for its applicability in the area of controller design. Based on the available model, and considering the characteristics of this technology differing definitely from those of conventional combustors, a new optimum control strategy was proposed, modeled and tested successfully.

## Keywords

Fluidized bed combustion · sustainable energy · mathematical model · advanced control · optimum control

## 1 Introduction

The Fluidized Bed Combustion (FBC) technology reached its mature for commercial applications in the early 1980s, and it became the leading clean coal combustion technology in the 90s. Throughout the decades of its history, its definite advantages became evident in the cases of low grade and problematic solid fuels. This attribute became very important by the early 2000s, and it initiated a new success story of the technology in the application area of sustainable energy production from biomass and waste-derived fuels [1, 2].

In case of a Circulating Fluidized Bed Combustor (CFBC), the combustion air is blown into the combustion chamber through dozens of nozzles located on its bottom as shown in Fig. 1. This air (the 'primary air') keeps the inventory (typically sand) in fluidized state, and the combustion of fuel particles added takes place in this suspension. Solids exiting the combustion chamber are segregated by one or more cyclones and led back into the combustion chamber. Under these conditions the combustion can take place at relatively low temperature (at about 850 °C) where neither thermal, nor prompt NO<sub>x</sub>-building can occur. Absorption of the sulfur dioxide released from some coals during combustion is promoted by milled limestone added to the suspension.

All stages of research, development and design of fluidized bed combustors may be effectively supported by programmed mathematical models describing not only some parts or subprocesses of the combustor, but its entirety. The purpose of three-dimensional (3D) FBC models [3–6] is to investigate spatial inhomogeneities in detail. Commercial CFD tools are used generally, which are very effective in solving also large scale and complex problems, but in such cases the simulation of instationary changes is out of their scopes and computational possibilities nowadays. A valuable review article on the challenges and issues can be found in [4]. The numerical demand will be dramatically reduced when one horizontal direction of inhomogeneities will be considered only. This approach was widely used in the second half of the last decade [7, 8], and it supplies very useful information regarding temperature profile and gas–solid flow including the cluster formation of the solid phase along the

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walls on the basis of new ideas and improved computational possibilities of the recent age [9]. A further known direction of efforts for lowering the computation time was the introduction of the so called 1.5D modeling approach on the turn of the century [10,11], in which case the a.m. dominant horizontal inhomogeneity was investigated only, on a simplified way. The next step is the one-dimensional modeling [12–14], of course.

Detailed steady state investigation was the purpose of all models discussed above, and because of it, they are not capable for simulating instationary changes within realistic computational time frames. However, process dynamics may be a very important issue in some cases. Designing the control system is one area where models can be used very effectively, but modeling also the instationary behaviors is essential in this case. This is the explicit goal of some works [15, 16], where the one-dimensional cell model structure was used. However, the computational times of these models are not published. If a programmed model requires long time for simulating some dynamic changes (e.g. model run time > simulated time), the use of the model is limited for off-line simulations only. In the opposite case, however, (when the programmed model runs much faster than reality) the model can be used on-line, for example, as a predictor implemented inside the controller for supporting its decisions. The known FBC models capable for this [17, 18] are semi-empirical models, which do not contain the detailed mathematical descriptions of the physical, chemical phenomena of the system, they are based on a few revealing equations parametrized by some measured values instead.

In this paper, a verified mathematical model will be presented the programmed version of which is capable for being integrated into an on-line controller due to its low computational time demand.

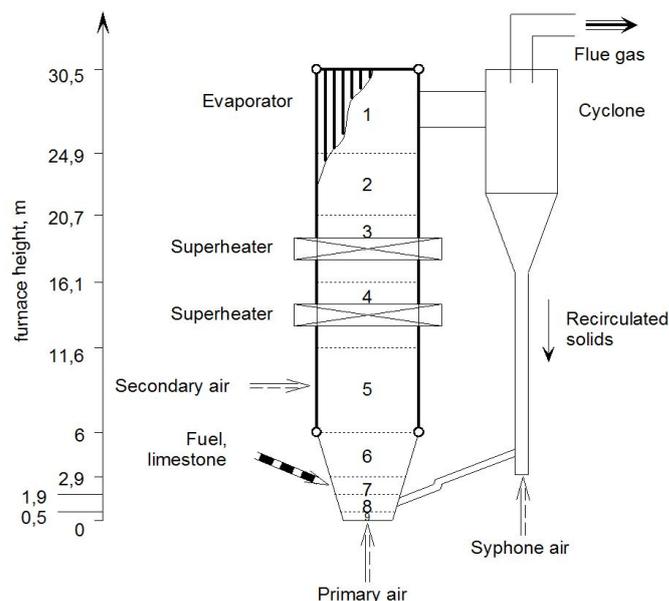
The amount of information published about the control strategies applied in industrial fluidized bed combustors is definitely very limited. Some unpublished supplier information is available, and only a few studies give details on control methods really applied in power plants [12, 15, 19]. All these structures are built up from the classical SISO (single input, single output) control blocks and they are based on the known solutions of the traditional combustion control methods. Although the control theory offers a very wide variety of advanced control methods for numerous process types, the applications of such methods in the power industry is far below the possibilities and the level of use in another industry branch, the chemical industry [20–22].

In this paper, a new control method will be presented, which considers the special characteristics of the FBC technique, and which is based on the programmed version of the verified mathematical model. The proposed structure is a multivariable optimum control, and its possible variants both upstairs and downstairs are outlined as well.

## 2 Mathematical model

A comprehensive mathematical model for the simulation of both the steady state and dynamic behaviors of circulating fluidized bed combustors will be presented. This model was developed with the special purpose of delivering an effective tool for model based control design. Because of this aspect, the generation of a numerically highly effective model was a crucial issue. As a result of this approach, the model to be presented was permanently undertaken to a critical evaluation regarding all the phenomena to be built in, which in turn resulted a model built up optimally according to the given preconditions. As the model fulfills them, and as it was successfully verified on measured data as described in the next section, the set of phenomena built in can be considered as the optimal set describing the real process.

The one-dimensional cell model structure is used, because this approach was found to be very effective regarding computational time on the one hand, and it may consider the dominant spatial inhomogeneity on the other hand. According to this, the combustion chamber is divided into cells along its vertical axes, and these cells are considered as spatially homogeneous ones. Due to the flexible structure of the model, the distribution of the furnace into cells can be chosen freely, the actually used distribution is shown on Fig. 1.



**Fig. 1.** Layout of the Circulating Fluidized Bed Combustor and its partitioning into cells according to the model concept.

The model is built up of mathematical equations, which can be divided into two major groups: the balance equations and constitutive relationships.

*Balance equations* (also called conservation laws; see Tab. 1) of dynamic systems describe the conservation of mass or energy. Most of them are formulated and solved in this model locally for each individual cell, while others globally, for the whole combustion chamber. Solid mass balances (for total solids and for discrete char size classes) belong to the later group, because the

resulting actual total mass will be distributed among the cells according to the constitutive equation describing the solid distribution along the combustor height. A similar mass balance of solids is formulated for each cell in order to obtain the internal solid refluxes crossing the borders between the cells. In spite of the bidirectional solid flow within the combustion chamber, the gas flow is considered as a pure upwards one with no reflux. According to this, the balance of the total gas in the cells is simpler, however, the changes due to production and consumption of chemical reactions must be considered as well. In the actual state of the model development, five flue gas components is considered ( $O_2$ ,  $CO$ ,  $CO_2$ ,  $NO$ ,  $NH_3$ ), the concentrations of which is calculated according to another balance equation. A key point in simulating fluidized bed combustors is to determine the temperature profile in the combustion chamber, although some published models handle it as an input variable. The cell internal temperature (which is considered as the solid temperature) can be calculated by solving the energy balance, which takes into account all the heat flows and internal sources. This balance is formulated for each cell as well, that's why also the temperature profile can be achieved along the vertical axis.

*Constitutive equations* supply the needed values for the balance equations discussed above by describing the necessary physical or chemical phenomena (Tab. 2). Drying and devolatilization are considered to happen immediately after a fuel particle enters the hot combustion chamber. The char combustion kinetics is calculated by means of the Arrhenius formulation using the kinetic data of Field [23]. The 'shrinking particle' model was implemented in a discretized manner, in which case the whole set of fuel particles is distributed into a number of discrete classes of diameters. The use of 3 such classes was found to be satisfactory, the selected class limit diameters are 120, 400 and 780  $\mu m$  for the investigated case. The proceeding of the combustion process of a single particle can also be considered as its discrete steps from upper diameter classes down to lower ones. In case of the chemical reactions one must proceed with the greatest care. A large number of chemical reactions is published together with reaction rates, but only a few of them practically occur under the actual conditions. After a careful selection, a number of such reactions were built in into the model, together with their temperature-dependent Arrhenius-type reaction rates and catalyst effects.

As shown on Fig. 1, the modeled construction of CFBC is not equipped with any external fluidized bed heat exchanger, thus, the segregated solids coming from the cyclone are directly fed back into the lower part of the combustion chamber. Because of this strong thermal link between the related points of the combustor, its mathematical description is of an evident importance. Because no appropriate reference could be used for the investigated case, the set of available on-site measured data were used for setting up the required relationship between cyclone inlet and outlet temperatures. A double linear description was found to be satisfactory (relative error < 1%), where both parameters

of the linear relationship were found to be linear functions of the fluidizing air flow  $\dot{V}_A$ . (See Fig. 2;  $p_1 = -0.0097 \text{ s/m}^3$ ;  $p_2 = 1.472$ ;  $p_3 = 10.07 \text{ K}\cdot\text{s/m}^3$ ;  $p_4 = -465.1 \text{ K}$ .)

The model for fluid dynamics is based on the calculations of bed voidage and solid mass fluxes in the combustion chamber according to [24]. The heat transfer coefficient  $\alpha$  between bed inventory and heat exchanger surfaces was the most significant parameter to find throughout the process of fitting the model to the measured data. Its final value was found to be  $67.5 \text{ W}/(\text{m}^2\cdot\text{K})$  for wall heating surfaces and  $195 \text{ W}/(\text{m}^2\cdot\text{K})$  for submerged heat exchangers, which values lie definitely within the ranges numerous authors and handbooks give for the investigated case [25, 26].

The programmed version of the mathematical model described above was generated in order to make it capable for numerical investigations and simulation runs. The Matlab-Simulink<sup>©</sup> environment was used, and certain considerations were taken in order to optimize its accuracy versus computational demand. It was recognized, for example, that keeping the overall solid mass balance on its unchanged initial value results a significant decrease in computational time, while its effect on the accuracy is negligible throughout the whole set of the available measured data. The time demand of the programmed mathematical model appears to be far below the simulated time, it runs one to two orders of magnitude faster than the simulated reality on a commercial office computer.

**Tab. 1.** The set of balance equations included in the mathematical model

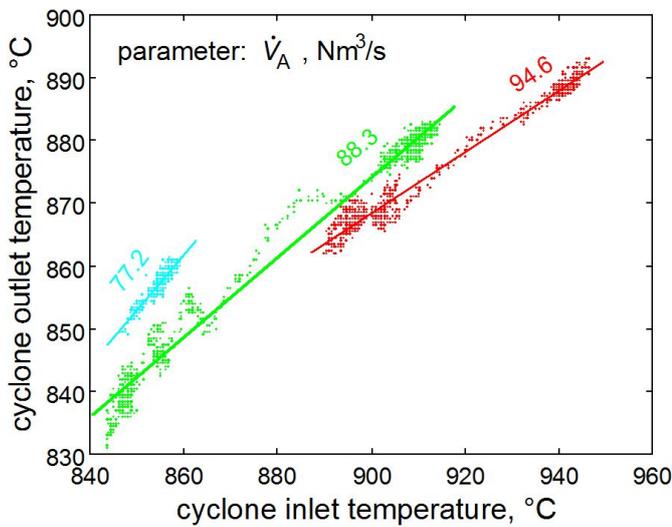
<ul style="list-style-type: none"> <li>• Globally formulated balances (for the combustion chamber):</li> </ul>	
• Solid mass:	$\frac{dm_s}{dt} = \sum_{\text{bordercross}} \dot{m}_s$
• Char mass:	$\frac{d\dot{m}_{C,j}}{dt} = \sum_{\text{bordercross}} \dot{m}_{C,j} + \sum_{\text{reaction}} \dot{m}_{C,j} + \sum_{\text{shrinking}} \dot{m}_{C,j}$
<ul style="list-style-type: none"> <li>• Locally formulated balances (separately for each cell):</li> </ul>	
• Solid mass:	$\frac{dm_{S,i}}{dt} = \sum_{\text{bordercross}} \dot{m}_{S,i}$
• Flue gas:	$\frac{dN_i}{dt} = \sum_{\text{bordercross}} \dot{N}_i + \sum_{\text{reaction}} \dot{N}_i$
• Flue gas components:	$\frac{dN_{i,k}}{dt} = \sum_{\text{bordercross}} \dot{N}_{i,k} + \sum_{\text{reaction}} \dot{N}_{i,k}$
• Energy:	$c_S \cdot m_S \cdot \frac{d\vartheta_{S,i}}{dt} = \sum_{\text{bordercross}} \dot{H}_i + \sum_{\text{reaction}} \dot{Q}_i + \sum_{\text{transfer}} \dot{Q}_i$

### 3 Model verification

Experiments were conducted on a 300 MW fluidized bed combustor of an industrial power plant in Europe by the experts of the University of Oulu, Finland. The investigated facility is equipped with two cyclones, return lag and no external heat exchanger, it is 30.5 m high with a horizontal cross section of 19.7 m x 6.75 m. The lower conical part is thermally insulated, while evaporators and superheaters are placed in the upper part as shown on Fig. 1, which scheme also indicates some fur-

**Tab. 2.** The set of constitutive equations included in the mathematical model. (Each of them will be formulated for each cell, however the indices 'i' referring to them will be omitted here.)

•Drying and devolatilization of the fuel particles: $\dot{m}_{DD} = -\dot{m}_{F,in} \cdot (\gamma_{HOH} + \gamma_V)$
•Char combustion kinetics: $\dot{m}_{CC,j} A_{C,j} \cdot \frac{p_{O_2}}{K_{diff,j}} + \frac{1}{K_{kin,j}}$
•Char shrinking through combustion: $\dot{m}_{step,j} = \dot{m}_{CC,j} \cdot \frac{d_j^3 - d_{j-1}^3}{d_j^3 - d_{j-1}^3}$
•Homogeneous chemical reactions: $\dot{N}_r = k \cdot c_{k1} \cdot c_{k2} \cdot c_{catal,r}$
•Cyclone thermal characteristics: $\vartheta_{c,out} = (p_1 \cdot \dot{V}_{air} + p_2) \cdot \vartheta_{C,in} + (p_3 \cdot \dot{V}_{air} + p_4)$
•Solids distribution along the vertical axis: $\varepsilon(h) = \begin{cases} \varepsilon_D & \text{if } 0 < h \leq h_D \\ \varepsilon_\infty + (\varepsilon_D - \varepsilon_\infty) \cdot e^{-a \cdot (h_{max} - h_D)} & \text{if } h_D < h < h_{max} \end{cases}$
•Vertical upwards solids flow: $\dot{m}_{S,up}(h) = \dot{m}_\infty + (\dot{m}_D - \dot{m}_\infty) \cdot e^{-a \cdot (h - h_{max})}$
•Heat transfer towards the water-steam cycle: $\dot{Q} = \alpha \cdot A_w \cdot (\vartheta_S - \vartheta_w)$



**Fig. 2.** Cyclone outlet temperatures  $\vartheta_{C,out}$  versus cyclone inlet temperature  $\vartheta_{C,in}$  at different flow rates of total combustion air  $\dot{V}_A$  (significant effect) and fuel feed (no effect).

ther design data. It is fuelled by peat (8.55 MJ/kg, 53% water, 25% carbon, 1.7% ash, 0.1% sulfur; 31.6% volatile), its thermal efficiency is above 90%, and it generates 356 t/h 45 bar steam.

About half of the available measured data was used for fitting the model, while the rest for checking the correspondence. In the first step of fitting the model to the investigated real boiler, the model parameters were set according to the real plant data. This period of data mining in the boiler documentation and verbal communications was followed by a systematic search for optimal values of free or undefined parameters of the system. The most significant phenomena involved in this period are heat transfer as mentioned above and fluid dynamics.

During final model verification, the extended input data series of the experiments were introduced to the programmed model as well, and its responses were compared to those of the experiments.

Three inputs were systematically varied during the experiments: fuel feed rate and primary and secondary air flows as shown on the upper part of Fig. 3. Throughout these system-

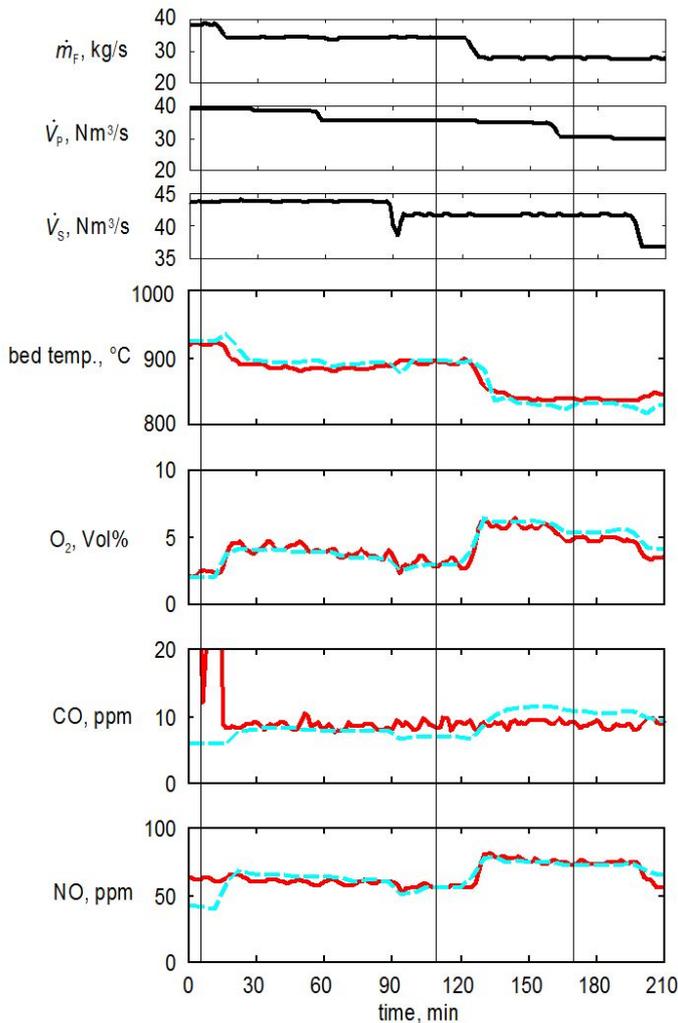
atic changes, a definitely wide range was scanned within the allowed load limits of the actual plant. Simulated and measured time functions of a few selected output variables are drawn on the same diagram, which allows an easy comparison.

A three-minute wide filter with uniform weighting coefficients was applied on the measured time functions for better visibility. The variance of the noises burdening the original measured data sets were not negligible in some cases, for CO, e.g., it was about  $\pm 8$  ppm, which lies within the range of the average differences between measurements and model calculations.

Besides plotting time functions of the modeled variables, the cell model structure allows also the investigation of the spatial changes of those variables at selected time points. The next figure gives an interesting example on this (Fig. 4). The selected time points were indicated by vertical bars on the time functions of Fig. 3. These results indicating the dominant spatial inhomogeneities within the combustion chamber are of high importance because of their significant roles in the development of the flue gas final emission levels. Minimizing the emissions is a key research issue today, especially in case of problematic, waste-derived fuels, which efforts can be efficiently supported by this type of combustion models. Similarly, the separate introduction of the combustion air on the primary- and secondary levels is a technological tool for this, the results of which can be observed (or even further investigated and enhanced) on the basis of this modeling approach.

#### 4 Control strategy

A new combustion control strategy for fluidized bed combustors will be presented in this section. The basic goal of combustion control is always identical, regardless of the actual combustion technique or control strategy, namely, setting the air flow rate so that optimal combustion conditions can be assured among the actual fuel feed, fuel composition and other circumstances. Despite this strong similarity, significant differences appear between the FBCs and other combustion technologies, since in case of Fluidized Bed Combustion,



**Fig. 3.** Comparison of predicted and experimental data. Identical input time series (upper three diagrams) were applied to both the programmed mathematical model and industrial facility (dashed and continuous lines, respectively).

- the combustion air must assure not only optimal combustion, but also appropriate fluidization;
- the combustion air must be set with respect to the actual fuel inventory, not to the actual fuel feed rate;
- the air distribution between primary and secondary air is a supplementary task of high importance.

The proposed approach in setting up the control strategy follows the traditional way of formulating a cost function ([27], also called: target function) to be minimized, but the significant differences listed above will be considered as well. While the traditional combustion control has one control variable only (the air flow), the new one is two-dimensional, since the optimal flow rates of both primary and secondary air must be controlled. While the traditional combustion control considers a few losses only (basically: incomplete combustion loss because of too low air flow and heat loss by exhaust gas because of too high air flow), the new one must consider also some others of significant influences. The list of aspects we propose to build in into the new cost function is the following:

- Satisfactory fluidization must be assured in the lower section of the combustion chamber (below the secondary air inlet).
- Satisfactory fluidization must be assured in the upper section of the combustion chamber (above the secondary air inlet).
- The characteristic bed temperature must not be too high.
- Total CO emission must not be too high (or: it must not exceed its threshold).
- Total NO emission must not be too high (or: it must not exceed its threshold).

This list of terms was found to be sufficient in practical cases. Limiting the bed temperature also from below was found to be unnecessary for example, because other terms of the list assure that this deviation can not happen. As a mathematical representation of this set of terms, exponential functions are proposed because of their easy handling both numerically and analytically, and also because of their abilities for being parametrized so that different limiting shapes can be realized from a nearly linear manner up to a practically sharp threshold. According to this, the proposed form of the cost function  $K$  (which is to be minimized by the combustion control) is the following:

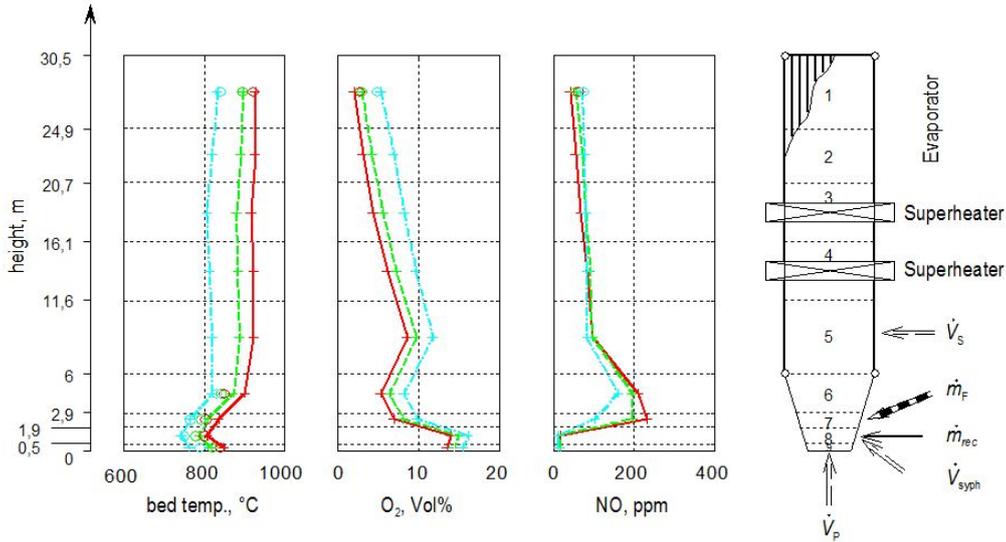
$$\begin{aligned}
 K = & \exp(a_1 \cdot \dot{V}_P + b_1) \\
 & + \exp(a_2 \cdot (\dot{V}_P + \dot{V}_S) + b_2) \\
 & + \exp(a_3 \cdot \vartheta + b_3) \\
 & + \exp(a_4 \cdot C_{CO} \cdot (\dot{V}_P + \dot{V}_S) + b_4) \\
 & + \exp(a_5 \cdot C_{NO} \cdot (\dot{V}_P + \dot{V}_S) + b_5).
 \end{aligned} \tag{1}$$

The proposed control concept allows other cost functions as well, of course. Its parameters should be set according to the actual local needs dictated by the technology (temperature, fluidization, e.g.), economical circumstances (prices of losses, e.g.) and authority prescriptions (emission limits, e.g.). In the actual case, the next set of parameters was found to be the best, and this one was used throughout the further investigations:

$$\begin{aligned}
 a_1 = & -0,24 \text{ s/m}^3; \quad a_2 = -0,11 \text{ s/m}^3; \quad a_3 = 0,018 \text{ 1/K}; \quad a_4 = \\
 & 0,15 \text{ s/(m}^3 \cdot \text{ppm)}; \quad a_5 = 0,015 \text{ s/(m}^3 \cdot \text{ppm)} \\
 b_1 = & 5,4; \quad b_2 = 7,91; \quad b_3 = -19,4; \quad b_4 = -3; \quad b_5 = -3.
 \end{aligned}$$

## 5 Controller design and test

The task formulated in the previous section (to minimize the cost function) should be realized by an appropriately designed controller. Furthermore, we believe that better control performance can be reached on the basis of better knowledge of the process to be controlled. While in case of the traditional PID controller the whole information about the process is represented by only three numbers, advanced control theories use more detailed models. It is advisable in the actual case to benefit the existence of a validated mathematical model, of course, however, the control task is not the most general one. The minimum of a calculated variable should be found in this case, not a



**Fig. 4.** Spatial inhomogeneities in the combustion chamber at different loads (Line types: continuous:  $t=5$  min; dashed:  $t=110$  min; dashdotted:  $t=175$  min)

given set-point of the controlled variable should be followed, as generally. Different approaches can be followed while designing a controller configuration to solve the model-based optimum control problem outlined above, some of them are listed here:

- Off-line optimum seeking algorithms can be run on the programmed model and cost function while simulating a high variety of operating conditions. The found optimal settings can then be stored in a real time (on-line) controller.
- An on-line optimum seeking algorithm can be realized in the real time controller.
- The above closed loop optimum seeking controller can be supported by initial guesses coming from either the off-line optimum search (first bullet above) or learned previous results of the on-line search (second bullet).
- Further model-based on-line optimum seeking procedures can be developed based on the results of the advanced control theory.

The second approach will be discussed here in detail, since this one can be considered as a basis for further enhancements either up or down.

The idea of the proposed control structure is simple, it traces back the optimum control task to an ordinary control task. According to this, the gradient of the actual value of the cost function should be controlled to zero (Fig. 5).

All process variables of the fluidized bed combustor involved in the cost function will be continuously measured, of course. Their actual values will be forwarded to the block that calculates the actual scalar value of the cost function, the minimum of which should be found and set by the remaining elements of the control structure. Its gradient should be estimated in the next block. The space of search is two-dimensional spanned by the manipulated variables  $\dot{V}_P$  and  $\dot{V}_S$ , but in practice it often seems

(Markers: +: simulated; O: measured)

to be better handleable to use another space defined by the coordinate transformation  $\dot{V}_A = \dot{V}_P + \dot{V}_S$ ,  $r = \dot{V}_P / \dot{V}_A$ , where  $\dot{V}_A$  is total air and  $r$  is air distribution.

In the gradient estimator, a known identification method will be used first. A two-dimensional, discrete-time ARX model will be identified on-line, which standard method delivers the model parameters in the following form:

$$A(q) \cdot \Delta y(t) = B_1(q) \cdot \Delta u_1(t) + B_2(q) \cdot \Delta u_2(t) + e(t), \quad (2)$$

where  $q$  is the time shift operator,  $y(t)$  is the process output (which is the  $K$  value in the actual case), and  $\Delta u_1$  and  $\Delta u_2$  are the process inputs ( $\dot{V}_P$  and  $\dot{V}_S$  in the actual case). This procedure needs to know also the perturbation signal, which will be defined and added to the inputs by the controller block. The results of the identification procedure are in this case the coefficients of the polynomials  $A(q)$ ,  $B_1(q)$ , and  $B_2(q)$ . The final output of this block (the gradient estimates) can be calculated according to

$$\frac{\partial K}{\partial \dot{V}_A} = \frac{B_1(q)}{A(q)} \Big|_{q=1}, \quad (3)$$

$$\frac{\partial K}{\partial r} = \frac{B_2(q)}{A(q)} \Big|_{q=1}. \quad (4)$$

The controller block in the proposed control structure (Fig. 5) can be any traditional controller. The set-point is zero, and the process variable to be controlled is the estimated gradient delivered by the block described above. In the actual, first implementation of the scheme, a rather simple, conservative control law was built in: the (two-dimensional) controller step is always proportional to the negative gradient [28] received. A flat sawtooth signal of very low amplitude compared to the effective outputs added to a random binary signal (of low amplitude as well) was chosen as perturbation signal needed for the ARX identification. It is generated within the control block, it is added to the calculated control output, and it is forwarded extra to the ARX

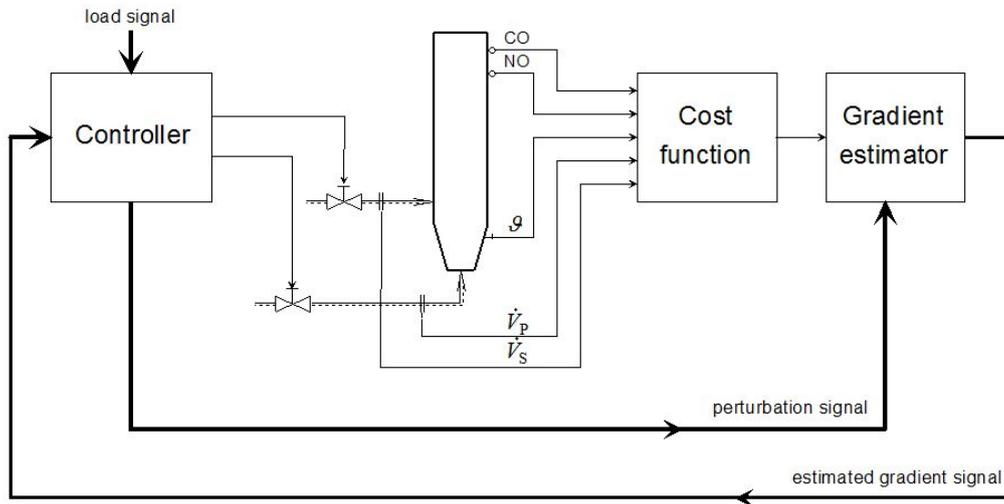


Fig. 5. The proposed structure of combustion control using Extremum Control developed for Fluidized Bed Combustors.

identifier located in the gradient estimator block. The load signal of the block is introduced to the controller block for further developments only, as an additional information for learning the optimum values once found.

The control strategy was realized in the simulation environment Matlab-Simulink<sup>®</sup>. The model described above was used throughout the simulations tests, and also for plotting the surface of the cost function over the  $\dot{V}_A - r$  space. (The surface of this function is not visible for the on-line controller, of course.) Fig. 6 shows the paths of some simulated searches started from different initial guesses far from the optimum. The results are satisfactory, the controller succeeded in shifting the fluidized bed combustion system close to its optimum in all cases.

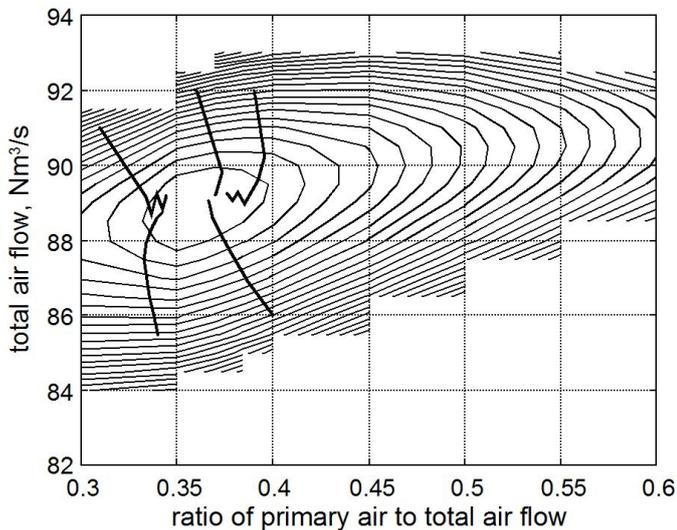


Fig. 6. Trajectories of the Extremum Control from different starting points.

Based on the current results, further tests of the new control strategy are planned after implementing it on an industrial-sized fluidized bed combustor. The model-based control development allows also a number of further investigations. The optimal primary- and secondary air flows as functions of the actual boiler load were calculated for example, and the resulted functions were rather similar to the ones found experimentally to be

the best [12, 15]. Further research will focus on the application of the programmed mathematical model in other model-based control procedures offered by the advanced control theory according to the bulleted list at the beginning of this section.

## 6 Conclusions

Several models of fluidized bed combustors are known, however, only a few of them deal with the instationary characteristics of the process. In case of control design, such a dynamic process model must fulfill two requirements, it must be accurate and fast running at the same time.

The mathematical model presented in this paper fulfills both of them. Its accuracy was verified against measurements, and its computational time demand is far below the simulated time. The built in equations describing the set of phenomena found to be adequate for both above purposes are coal combustion, gas-gas reactions, fluid dynamics of the suspension, and heat transfer, plus an empirical cyclone temperature model. Mass and heat balance equations were formulated for different solids and gas components. The one-dimensional cell model structure was used. The combustion chamber was divided into nine cells on such a way that they follow all the significant geometrical changes and characteristics of the technology.

The FBC technology differs basically from other combustor types, one example is the double role of air flow: combustion and fluidization. In spite of such differences, most known control strategies use the traditional approach also in this case. In this paper, the task of combustion control is formulated by means of a cost function. A general form of this equation was proposed the exponential terms of which define sufficiently the optimal operating point within the two-dimensional space spanned by the two air flows. The parameters of these terms should be set according to the actual local circumstances such as supplier prescriptions and financial ambiance. A control structure was also proposed for finding the minimum of this cost function on-line. This structure was programmed, and it was joined to the verified mathematical FBC model. The tests of this

control strategy pointed out its proper operation.

Further research will focus on the development and test of some other control concepts, where the existing, very effective mathematical model will be used by the controller as an on-line predictor. The mathematical model described in this paper was verified by means of measurements, while the control strategy was tested on the verified model by means of simulations only. Both control strategies (the one described and tested in this paper, and the next one under development) are planned to be implemented and tested on an industrial sized fluidized bed combustor.

### Nomenclature

$A$   $m^2$  surface, cross-section  
 $a$   $1/m$  decay constant for fall off of solid density  
 $c$   $mol/m^3$  molar gas concentration  
 $h$   $m$  height (above the air distributor)  
 $\dot{H}$   $W$  enthalpy flow  
 $\dot{m}$   $kg/s$  mass flow  
 $N$   $mol$  amount of substance  
 $p_{1...4}$  coefficients of the empirical cyclone model  
 $p_{O_2}$   $Pa$  partial pressure of oxygen  
 $\dot{Q}$   $W$  heat flow  
 $\dot{V}$   $m^3/s$  volume flow  
 $t$   $s$  time  
 $\alpha$   $W/(m^2 \cdot K)$  heat transfer coefficient  
 $\gamma_{HOH}$   $kg/kg$  water in fuel  
 $\gamma_V$   $kg/kg$  volatile in fuel  
 $\vartheta$   $K$  temperature  
 $\varepsilon$   $m^3/m^3$  volume fraction of solids

### Subscripts

bordercross material or energy flow crossing the borders of the investigated cell  
 $C$  char; cyclone  
 $CC$  char combustion  
 $D$  dense phase  
 $DD$  drying and devolatilization  
 $F$  fuel  
 $i$  cell (volume element in the cell model)  
 $j$  class of particle sizes in the shrinking particle model  
 $k$  gas component ( $k \in O_2, CO, CO_2, NO, NH_3$ )  
catal catalyst  
 $r$  chemical reaction identifier  
reaction material production or consumption due to chemical reactions  
rec recirculating solid flow

$S$  solid

shrinking material flow from or to a particle size class

syph syphone air

transfer heat flow via heat transfer

$V$  volatile

$W$  wall

### References

- 1 Hupa M, Coda Zebetta E, *Current State of the Technology in Fluidized Bed Combustion*, 50. Meeting of the International Energy Agency – Fluidized Bed Conversion (IEA-FBC) (Toronto, Kanada, May 21, 2005), 2005. [www.iea-fbc.org](http://www.iea-fbc.org).
- 2 Winter F, Szentannai P, *IEA Fluidized Bed Conversion Programme - Status Report 2010*, Bundesministerium für Verkehr, Innovation und Technologie, Radetzkystraße 2, 1030 Wien; Abteilung für Energie- und Umwelttechnologien, 2010. <http://www.nachhaltigwirtschaften.at>.
- 3 Knöbig J, Werther J, *Dreidimensionale Modellierung der Kohleverbrennung in großtechnischen Feuerungen mit zirkulierender Wirbelschicht*, Entwicklungslinien der Energie- und Kraftwerkstechnik (Siegen, Sep 10, 1996), 1996, pp. 191–207. VDI Berichte 1280.
- 4 Lettieri P, Mazzei L, *Challenges and Issues on the CFD Modeling of Fluidized Beds: a Review*, The Journal of Computational Multiphase Flows **1** (Jun 2009), no. 2, 83–131.
- 5 Wischniewski R, Ratschow L, Hartge EU, Werther J, *Reactive gas-solids flows in large volumes–3D modeling of industrial circulating fluidized bed combustors*, Particuology **8** (Feb 2010), no. 1, 67–77.
- 6 Rozainee M, Ngo SP, Salema AA, Tan KG, *Computational fluid dynamics modeling of rice husk combustion in a fluidised bed combustor*, Powder Technology **203** (Nov 10, 2010), no. 2, 331–347.
- 7 Boemer A, Schmidt A, Renz U, *Modellierung der Fluidodynamik blasenbildender Wirbelschichten*, Wirbelschichtfeuerungen: Erfahrungen und Perspektiven (Düsseldorf, Feb 18, 1997), 1997, pp. 249–263. VDI Berichte 1314.
- 8 Artlich S, Mackens W, Werther J, *Zweidimensionale Simulation der Kohleverbrennung in Druckwirbelschichtfeuerungen*, Lie algebras and their representations (Seoul, Sep 10, 1996), 1996, pp. 253–267. VDI Berichte 1280.
- 9 Zhou W, Zhao C S, Duan L B, Qu C R, Chen X P, *Two-dimensional computational fluid dynamics simulation of coal combustion in a circulating fluidized bed combustor*, Chemical Engineering Journal (2010), Available online 2 October 2010.
- 10 Hannes J, Renz U, van den Bleek CM, *Mathematische Modellierung von zirkulierenden Wirbelschichtfeuerungen industriellen Maßstabs*, VGB Kraftwerkstechnik (1998), no. 2, 69–76.
- 11 Werner A, Brändle B, Linzer W, *Simulation von zirkulierenden Wirbelschichtfeuerungen: ein Vergleich von Rechnung und Meßdaten*, Wirbelschichtfeuerung und -vergasung: Erfahrungen und Perspektiven (Cottbus, 2000), Mar 28, 2000, pp. 41–52. ISBN 3-18-091535-8, VDI Berichte 1535.
- 12 Edelmann H, *Modellierung der Dynamik und des Regelverhaltens für einen Dampferzeuger mit zirkulierender Wirbelschichtfeuerung*, VDI Verlag, Düsseldorf, 1992. Fortschrittsbericht, Reihe 6, Nr. 275, Dissertation Universität GH Siegen.
- 13 Glasmacher-Remberg Ch, Nemet A, Fett FN, *Towards a more general process model for power plants with atmospheric or pressurized fluidized bed combustion*, ASME **2** (1997), 1139–1149.
- 14 Hadavand A, Jalali AA, Famouri P, *An innovative bed temperature-oriented modeling and robust control of a circulating fluidized bed combustor*, Chemical Engineering Journal **140** (Jul 2008), no. 1-3, 497–508.

- 15 **Bunzemeier A**, *Mathematisches Modell zur regeldynamischen Analyse eines Dampferzeugers mit zirkulierender Wirbelschichtfeuerung*, VDI Verlag, Düsseldorf, 1992. Fortschrittsbericht, Reihe 6, Nr. 273.
- 16 **Galgano A, Salatino P, Crescitelli S, Scala F, Maffettone P L**, *A model of the dynamics of a fluidized bed combustor burning biomass*, *Combustion and Flame* **140** (Mar 2005), no. 4, 371–384.
- 17 **Lunze J, Wolff A**, *Robuste Regelung einer Wirbelschichtverbrennungsanlage für Klärschlamm*, *Automatisierungstechnik* **40** (1996), no. 11, 522–532.
- 18 **Havlena V, Pachner D**, *Model-Based Predictive Control of a Circulating Fluidized Bed Combustor*, ProcessEng, Vienna, 2010. *Power Plant Applications of Advanced Control Techniques*, ISBN 978-3-902655-11-0, p. 500.
- 19 **Czinder J**, *Control of Power Plants*, Műegyetemi kiadó, Budapest, 2000. In Hungarian.
- 20 **Dittmar R, Pfeiffer B-M**, *Modellbasierte Prädiktive Regelung in der industriellen Praxis*, *Automatisierungstechnik* **54** (2006), no. 12, 590–601.
- 21 **Szentannai P, Winter F**, *Modern Control Techniques for Fluidized Bed Combustors*, 59. Meeting of the International Energy Agency – Fluidized Bed Conversion (IEA-FBC) (Czestochowa, Poland, Oct 7, 2009), 2009. [www.iea-fbc.org](http://www.iea-fbc.org).
- 22 **Szentannai P (Ed.)**, *Power Plant Applications of Advanced Control Techniques*, ProcessEng, Vienna, 2010. ISBN 978-3-902655-11-0, p. 500.
- 23 **Field MA, Gill DW, Morgan BB, Hawskey PGW**, *Combustion of Pulverized Coal*, Leatherhead, UK, 1967. The British Coal Utilization Research Association (BCURA).
- 24 **Kunii D, Levenspiel O**, *Entrainment of Solids from Fluidized Beds, I. Hold-Up of Solids in the Freeboard, II. Operation of fast Fluidized Beds*, *Powder Technology* **61** (1990), 193–206.
- 25 **Hamdan M A, Al-qaq A M**, *The prediction of heat transfer coefficient in circulating fluidized bed combustors*, *Energy Conversion and Management* **49** (Nov 2008), no. 11, 3274–3277.
- 26 **Gnanapragasam N V, Reddy B V**, *Numerical modeling of axial bed-to-wall heat transfer in a circulating fluidized bed combustor*, *International Journal of Heat and Mass Transfer*, **52** (Mar 2009), no. 7-8, 1657–1666.
- 27 **Tettamanti T, Varga I**, *Distributed traffic control system based on model predictive control*, *Periodica Polytechnica Civil Engineering* **54** (2010), no. 1, 3–9, DOI 10.3311/pp.ci.2010-1.01.
- 28 **Csébfalvi A**, *A hybrid meta-heuristic method for continuous engineering optimization*, *Periodica Polytechnica Civil Engineering* **53** (2009), no. 2, 93–100, DOI 10.3311/pp.ci.2009-2.05.