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RESEARCH ARTICLE

Multiple constrained sizing-shaping truss-optimization using ANGEL method

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Abstract

The aim of this study to demonstrate that the previously developed ANGEL algorithm can be efficiently used for multiple constrained sizing-shaping truss optimization problems. The applied hybrid method ANGEL, which was originally developed for simple truss optimization problems combines ant colony optimization (ACO), genetic algorithm (GA), and local search strategy (LS). ACO and GA search alternately and cooperatively in the solution space. In ANGEL, the traditional stochastic mutation operator is replaced by the local search procedure as a deterministic counterpart of the stochastic mutation. The feasibility is measured by the maximal load intensity factor computed by a third order path-following method. The powerful LS algorithm, which is based on the local linearization of the set of the constraints and the objective function, is applied to yield a better feasible or less unfeasible solution when ACO or GA obtains a solution. In order to demonstrate the efficiency of ANGEL in the given application area, a well-known example is presented under multiple constraints.

Keywords

ANGEL hybrid heuristic method; shaping-sizing truss optimization

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1 Introduction

The traditional meaning of the truss optimization is a minimal mass design in terms of the cross-sectional areas as design variables while fixed geometry is supposed [1, 2]. However, the total mass of the truss highly depends upon the join positions. The early works for simultaneous sizing-shaping truss optimization have been appeared about the seventies of the last century [3–8] where the solutions are obtained by successive iteration using gradient-based methods, although, such an optimization problem is nonconvex and nonlinear. Therefore, the solution of the implicit equilibrium equation systems and the computation of the gradients for the traditional algorithms required a prescribed move-limit for the cross-sectional and geometry design variables. Important to note, that the success of the traditional method highly depends on the initial design. However, if a good jumping-off point is found in the local environment of the optimal solution, the linearized gradient-based methods seem very accurate.

The aim of this study to demonstrate that the previously developed ANGEL algorithm [9, 12] can be efficiently used for multiple constrained sizing-shaping truss optimization problems.

The session 2 contains the general formulation of the optimization problems and the basic formulas of the computation of structural constraints, where stress, displacement, and buckling constraints are considered. The equilibrium equation systems of the geometrically linear and nonlinear structural model are based on the stationary theory of the total potential energy function as well.

In session 3, the hybrid meta-heuristic method is discussed, which is an extended version of the previously published method applied for discrete and continuous optimization problems. The naming ANGEL of the proposed method is an acronym, combines ant colony optimization (ACO), genetic algorithm (GA), and local search strategy (LS). In session 4, the linearized formulas of applied local search strategy is presented.

In session 5, throughout the Pedersen's benchmark example [6] the efficiency of the proposed method is demonstrated where four different constrained cases are considered. With the help of the statistical evaluation of the best feasible solutions of thirty independent runs, has been proven the stability and dependability of the process.

2 The optimization problem

In this paper, the single-objective continuous sizing-shaping optimization problem is considered as minimal weight design subjected to equilibrium of state variables, nodal displacements, and stress constraints.

$$W(X, Y) \to \min, \tag{1}$$

$$\{X, Y\} \in \mathbf{\Theta} \subseteq \mathbf{\Omega}, \tag{2}$$

where the design space is $\mathbf{\Omega}$ and the subspace of the feasible designs is denoted by $\mathbf{\Theta}$:

$$\mathbf{\Omega} = \left\{ \{X, Y\} \left| X_i, \in \left[X^L, X^U\right], Y_j, \in \left[Y^L, Y^U\right] \right. \right\} \tag{3}$$

$$\mathbf{\Theta} = \left\{ \{X, Y\} \left| \{X, Y\} \in \mathbf{\Omega}, G_k(X, Y) \in \left[G^L, G^U\right] \right. \right\} \tag{4}$$

where G_k, $k \in \{1, 2, \ldots, C\}$ are the implicit structural response constraints of the structure, $X = (X_1, X_2, \ldots, X_N)$, $i \in \{1, 2, \ldots, M\}$ is the vector of the continuous sizing variables, $Y = (Y_1, Y_2, \ldots, Y_M)$, $j \in \{1, 2, \ldots, N\}$ is the vector of the continuous shift variables.

In this paper, a design is represented by the set of $\{W, \lambda, X, Y, \Phi\}$, where W is the weight of the structure, $\lambda = \lambda(X, Y)$, $(0 \leq \lambda \geq 1)$ is the maximal load intensity factor, $\{X, Y\}$ is the current set of the cross-sections of member groups, and $\Phi = \Phi(X, Y)$ is the current fitness function value.

The structural response analysis is based on the geometrically nonlinear total potential energy function of elastic system, which can be described in terms of the vector of sizing and shifting design variables.

The structural state variables are the load intensity factor and the vector of nodal displacements.

$$V(X, Y, D, \lambda) = U(X, Y, D, \lambda) - D^T \lambda F. \tag{5}$$

where $U(X, Y, D, \lambda)$, the nonlinear strain energy function, is depend on the design variables and the state variables. The vector of nodal displacements and the external loads are denoted by $D \in \Re^{DOF}$, and $F \in \Re^{DOF}$, where DOF is the number of degrees of freedom.

According to the stationary theory of the total potential energy function, the equilibrium equation system of state variables is obtained by differentiation of energy function (5).

$$\frac{\partial V(X, Y, , D, \lambda)}{\partial D} = \frac{\partial U(X, Y, , D, \lambda)}{\partial D} - \lambda F = 0. \tag{6}$$

In order to measure the feasibility of the current design (e.g. measure of the satisfaction of the constraints), we have to solve the equilibrium equation system (6) in terms of the state variables. In this study, the response functions and the maximal load intensity factor λ is computed by a higher order nonlinear path-following method [13]. In order to measure the goodness of the solutions ANGEL uses a feasibility-oriented fitness function (Φ) which is based on a set of criteria introduced previously by Deb [14]:

(1) Any feasible solution is preferred to any infeasible solution,

(2) Between two feasible solutions, the one having a smaller weight is preferred,

(4) Between two infeasible solutions, the one having a larger load intensity factor is preferred.

It should be stressed that our fitness function exploits the fact, that the maximal load intensity factor λgiven by the applied path-following method is a "natural" measure of feasibility.

During the optimization process, each phases of the proposed ANGEL hybrid metaheuristic method are governed by the following fitness function $\Phi = \Phi(X, Y) \quad (0 \leq \Phi \leq 2)$:

$$\Phi = \begin{cases} 2 - \frac{W - W^L}{W^U - W^L} & \lambda = 1 \\ & if \\ \lambda & \lambda < 1 \end{cases}, \tag{7}$$

where $W^L \left(W^U\right)$ is a lower (upper) bound of the weight in the given design space: $W^L = \min\{W(X, Y) | \{X, Y\} \in \mathbf{\Omega}\}$, $W^U = \max\{W(X, Y) | \{X, Y\} \in \mathbf{\Omega}\}$. In the proposed approach, it is assumed that the heaviest (lightest) design is feasible (infeasible).

3 The ANGEL method

In the presented ANGEL algorithm (Fig. 1), the **traditional mutation operator** is replaced by the **local search procedure** as a deterministic counterpart of the stochastic mutation. That is, rather than introducing small random perturbations into the offspring solution, a gradient based deterministic local search is applied to improve the solution until a local optimum is reached. In other words, random perturbation is replaced by the "best" perturbation.

The main procedure of the proposed hybrid metaheuristic follows the repetition of these two steps:

(1) ACO with LS and

(2) GA with LS.

The hybrid algorithm is based on three operators: **random selection** (ACO + GA), **random perturbation** (ACO), and **random combination** (GA).

The initial population of the process is a totally random set. The random perturbation and random combination operators - based on normal distribution - use a tournament selection operator, to select a "more or less good" solution from the current population using the well-known discrete inverse method. The procedures use a uniform random number generator in the inverse method. We have to mention, that in our algorithm in the GA phase, an offspring not necessarily will be the member of the current population, and a parent not necessarily will die after mating. The reason is straightforward because of our algorithm uses a very simple rule: If the current design is better than the worst solution of the current population, than the better one will replace the worst solution.

The Fig. 1 contains three phases: (i) RANDOM POPULATION, (ii) ANT COLONY OPTIMIZATION, and (iii) GENETIC ALGORITHM, where Z signify the common vector of continuous sizing and shift design variables.

It should be noted, that the main framework of ANGEL is very similar to another hybrid algorithm [15] according to the same goal and common roots and basic features.

4 The local search procedure

In the presented hybrid method, a gradient based, deterministic local search (LS) is implemented to improve the solution until a local optimum or the maximal number of iterations is reached. The LS procedure is based on two linear programming (LP) models and calls a LP solver to solve them.

When the current solution of the iterative process is feasible then LS tries to find a better solution in its local neighbourhood without violating constraints:

$$\Delta W(\Delta Z) \rightarrow \min, \tag{8}$$

$$G_j(Z) + \sum_{i=1}^{N} \frac{\partial G_j(Z)}{\partial Z_i} * \Delta Z_i \in \left[G_j^L, G_j^U\right], \quad j \in \{1, 2, \ldots, M\} \tag{9}$$

$$\Delta Z_i \in \left[\Delta Z_i^L, \Delta Z_i^U\right], \quad i \in \{1, 2, \ldots, N\} \tag{10}$$

When the current solution of the iterative process is infeasible then LS tries to find a better (feasible or less infeasible) solution in its local neighbourhood:

$$\sum_{j=1}^{M} \left(\Delta G_j^L + \Delta G_j^U\right) \rightarrow \min, \tag{11}$$

$$G_j(Z) + \sum_{i=1}^{N} \frac{\partial G_i(Z)}{\partial Z_i} * \Delta Z_i \in \left[G_j^L - \Delta G_j^L, G_j^U + \Delta G_j^U\right], \quad j \in \{1, 2, \ldots, M\} \tag{12}$$

$$\Delta Z_i \in \left[\Delta Z_i^L, \Delta Z_i^U\right], \quad i \in \{1, 2, \ldots, N\} \tag{13}$$

$$\Delta W(\Delta Z) \leq \varepsilon. \tag{14}$$

5 Numerical example

The presented ANGEL method has been applied for a well-known bridge problem of the simultaneous sizing-shaping optimization. Pauli Pedersen [6] has introduced this example first time. He proposed a parabolic shape for initial layout, which is displayed on Fig. 2. The objective function is the weight of the structure subjected to stress, displacements, and stability constraints. According to the moving loads acting on the bottom joints, five load cases are considered simultaneously. The sizing variables are grouped into 13 group variables because of the structural symmetry (Tab. 1). The shift variables are the horizontal and vertical positions of the joints 5, 6, 7, 12, and 11. The initial data of the applied material properties and structural constraints are adopted from the literature (Tab. 2).

In this study, stainless steel tubular cross sections are considered as design variables. According to the thin-wall pipe structural behaviour, the following local stability constraints are proposed [16].

The stress constraint for against of Euler-buckling or peripheral shell-like buckling is given in terms of the thickness ratio.

$$\sigma_e^E = \frac{\pi E}{4L^2} \cdot \frac{0.5 - \alpha + \alpha^2}{\alpha(1-\alpha)} \cdot G_e, \tag{15}$$

$$\sigma_e^B = KE\alpha, \tag{16}$$

where $\alpha = T/D$ is the ratio of the wall-thickness and diameter of the applied G_e group elements. In the present study, since continuous design variables are considered we applied tubular cross sections with given $\alpha = 0.5$ thickness ratio.

In this paper, four different optimization problems has been solved, namely where

1 only stress constraints,

2 stress and displacement constraints,

3 stress and stability constraints, and

4 all of the constraints are considered.

The number of generations and the number of population size were 100 - 100 for each case detailed above.

The shape of the best solution to all cases (i-iv) is demonstrated on Fig. 3-6, where the shifted coordinates are signed in bracket.

The obtained best and worst values, the mean and the standard deviation of the statistical analysis of the 30 independent runs for all optimization problems are presented in Tab. 3.

The cross-sectional areas of the best results selected from the 30 independent runs are presented in Tab. 4. Finally, in Tab. 5 the joint positions and weight of the previously discussed optimal results are compared with the published results using the same material properties.

Unfortunately, only the optimal weight and related optimal solution of geometrical configurations are presented in Pedersen [6] for each case. While in first case when only stress constraints are considered our results exhibit a good accordance with Pedersen's results, but in any other case the difference is considerable which might be arisen probably because of the nonlinear structural model applied in this study and the related stability constraints.

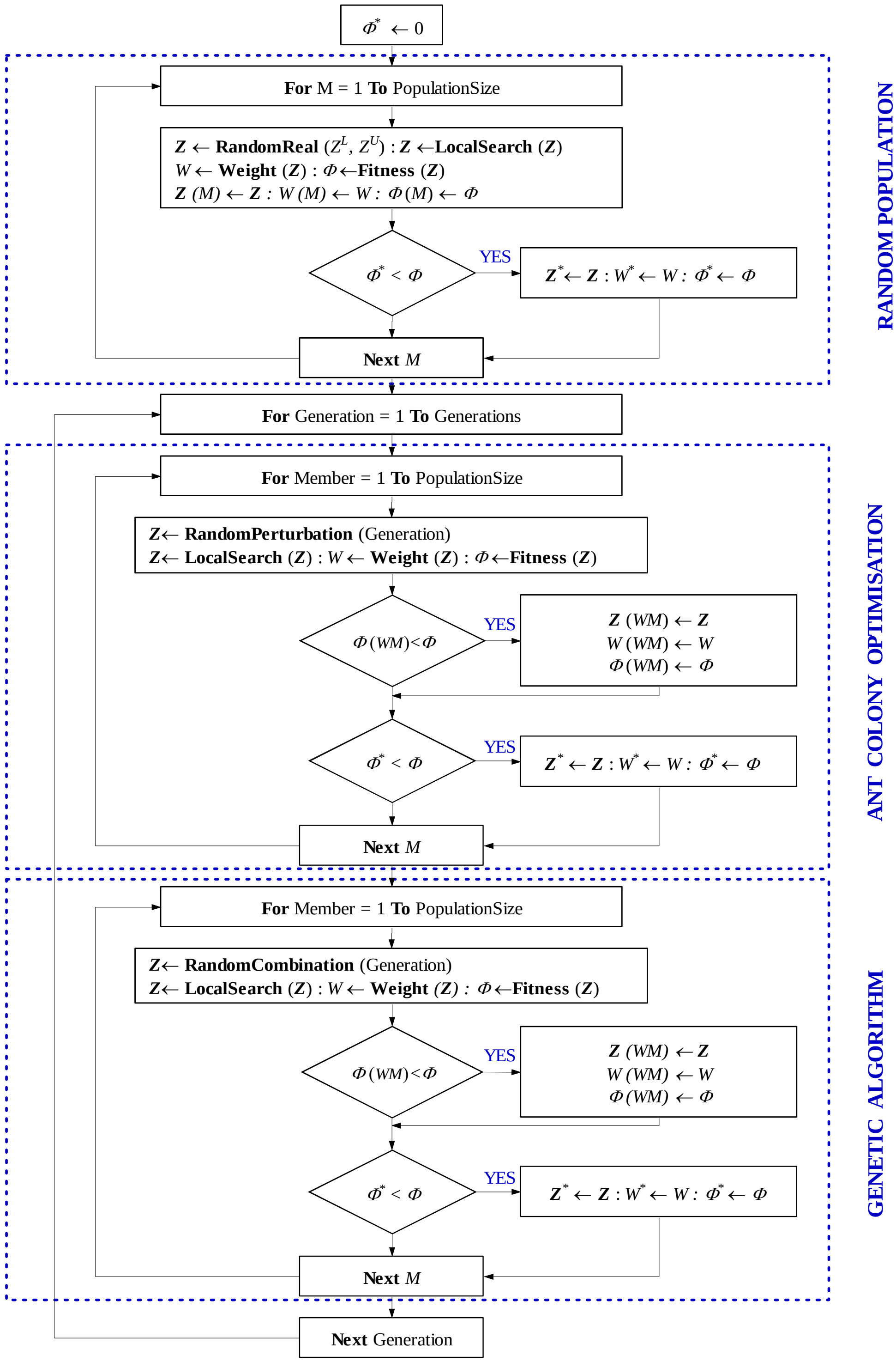

Fig. 1. The flowchart of the hybrid metaheuristic method

Tab. 1. The grouped cross sectional design variables

Group variables	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
Nodes	1-7	2-7	1-6	2-6	3-6	2-5	3-5	1-2	2-3	6-7	5-6	4-5	3-4

The results revile the fact, that the displacement constraints, according to the moving load, significantly increase the complexity of the design space. Therefore, for a fixed searching parameter set {*Generations*, *PopulationSize*} the variability of the final results will be higher, which is well demonstrated by the range or the standard deviation of the "best" solutions in Tab. 3.

Tab. 2. Data for continuous sizing-shaping optimization of the Pedersen's truss-bridge

Design variables:	
Cross-sectional variables	$G_i \in [4, 100]$ (cm^2); $e \in \{1, 2, ..., 13\}$
Geometry variables	X_5; Y_5; X_6; Y_6; Y_7
Stress constraints	$\sigma^U = 130$ MPa; $e \in \{1, 2, ..., 13\}$ $-\sigma_e^L = \max\left\{\sigma^L, \sigma_e^E, \sigma_e^B\right\}$; $\sigma^L = -104$ MPa; $e \in \{1, 2, ..., 13\}$
Side constraints for geometry variables	-250 cm $\leq X_5, X_6 \leq +250$ cm ; -200 cm $\leq Y_3, Y_5, Y_7 \leq 300q$, cm
Displacement constraints of nodes 1,2,3,8,9.	$u_k^U = \pm 1$ cm; $k \in \{1, 2, ..., 5\}$ for vertical displacements
Load cases:	
Nodal points	In direction of coordinate y
1, 2, 3, 8, 9	-300 kN
Material properties:	
Modulus of elasticity	$E = 210000$ MPa
Density of the material	$\rho = 7850$ kg/m^3

Tab. 3. Statistical analysis of 30 independent runs for truss-bridge optimization

Optimization cases	Only stress constraints	Displacement and stress constraints	Stress and stability constraints	All of the constraints
Best weight $[kg]$	1654.037	2794.259	1966.205	2866.116
Worst weight $[kg]$	1656.492	2819.659	1971.589	2934.523
Mean $[kg]$	1655.960	2803.280	1969.203	2894.924
Standard deviation	0.298	6.192	1.437	14.450

Tab. 4. Optimal design of present study for shaping-sizing problems with consideration of design different constraints

Geometry design variables	Stress constraints	Displacement and stress constraints	Buckling and stress constraints	All constrains
G_1 [cm^2]	11.1912	15.5424	10.306	18.0787
G_2 [cm^2]	10.4305	18.6539	15.985	18.8412
G_3 [cm^2]	8.1318	14.845	13.3364	17.1237
G_4 [cm^2]	11.602	13.7711	11.757	16.8234
G_5 [cm^2]	11.9618	11.4095	19.292	15.2628
G_6 [cm^2]	5.4509	9.9113	6.7076	13.0312
G_7 [cm^2]	13.6481	16.0756	16.3913	13.8369
G_8 [cm^2]	4.0217	7.063	6.0943	7.3118
G_9 [cm^2]	4.7628	10.0208	9.3416	9.4210
G_{10} [cm^2]	26.8054	48.2696	34.6539	50.4185
G_{11} [cm^2]	24.3453	43.8197	27.3987	43.7335
G_{12} [cm^2]	29.8348	50.4848	31.2037	46.2772
G_{13} [cm^2]	9.2574	10.4119	14.748	13.4586

Tab. 5. Compared results for shaping-sizing optimization of the Pedersen's truss bridge

variables	Stress constraints		Displacement and stress constraints		Buckling and stress constraints		All constrains	
	Pedersen [6]	Present study	Pedersen [6]	Present study	Pedersen [6]	Present study	Pedersen [6]	Present study
X_5 $[cm]$	-1153.00	-1178.0864	-1065.00	-1059.8235	-1138	-1162.1736	-1081	-1072.8051
X_6 $[cm]$	-633.00	-621.8893	-553.00	-554.2587	-629	-588.3484	-578	-596.2107
Y_5 $[cm]$	437.00	433.5992	505.00	517.3998	266	404.4884	437	485.9064
Y_6 $[cm]$	672.00	665.3387	780.00	792.4968	485	525.5709	653	723.4665
Y_7 $[cm]$	753.00	746.4115	864.00	889.7917	500	575.6562	739	840.7323
W $[kg]$	1656.00	1654.037	2911.00	2794.259	2905	1966.205	3315	2866.116

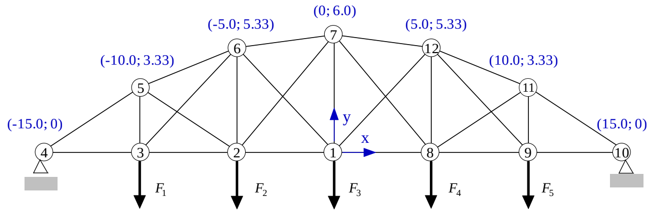

Fig. 2. The initial shape of the optimization problem

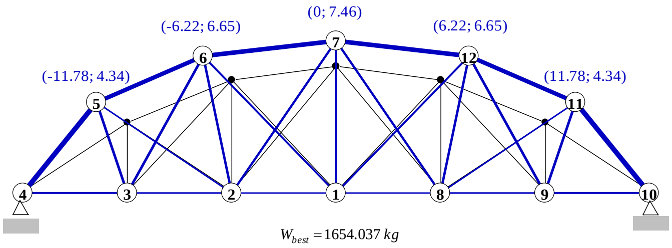

Fig. 3. Optimal design of only stress constrained optimization problem

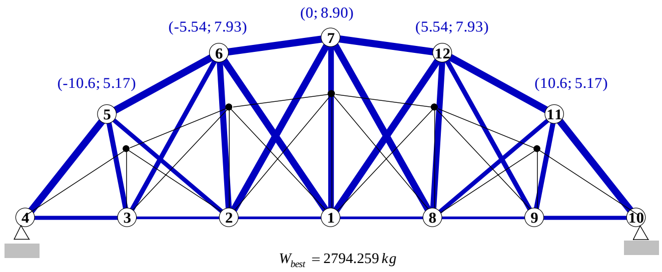

Fig. 4. Optimal design of stress and displacement constrained optimization problem

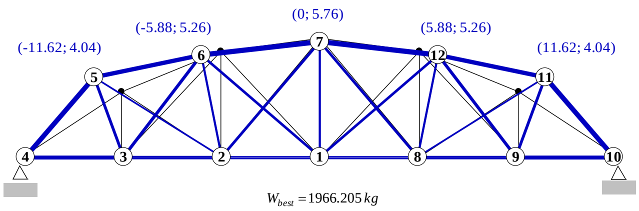

Fig. 5. Optimal design of buckling and stress constrained optimization problem

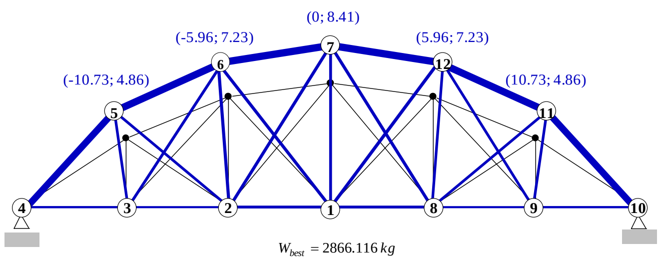

Fig. 6. Optimal design where all of the constraints are considered

6 Conclusions

In this paper, a hybrid metaheuristic method has been applied for multiple constrained truss optimization problems with continuous design variables. The proposed method combines ant colony optimization (ACO), genetic algorithm (GA), and local search strategy (LS) which seems an efficient mixture to solve simultaneous sizing-shaping truss optimization problems. The local search algorithm based on the local linearization has provided accurate results. Through a benchmark problem, which exhibits a large variety of the solutions, can be seen that the proposed hybrid algorithm seems very efficient and produces competitive results.

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