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RESEARCH ARTICLE

A multi-set charged system search for truss optimization with variables of different natures; element grouping

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Abstract

Optimization problems may include variables of different natures. In structural optimization for example different variables representing cross-sectional, geometrical, topological and grouping properties of the structure may be present. Having different interpretations, the effects of these variables on the objective function are not alike and their search spaces may represent different characteristics. Thus, it is helpful to take these variables apart and to control each set separately.

Based on the above considerations, in this paper a multi set charged system search (MSCSS) is introduced for the element grouping of truss structures in a weight optimization process. The results are compared to those obtained through predefined grouping by different algorithms. The comparisons show the efficiency and the effectiveness of the proposed algorithm. Although this paper only considers size optimization of truss structures where sizing and grouping variables are present and regarded as variables of different natures, the algorithm can be extended to cover the simultaneous shape and size optimization and topology optimization of different types of structures.

Keywords

optimization; element grouping; multi set charged system search; truss structures

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1 Introduction

Meta-heuristic algorithms are more suitable than conventional methods for structural optimum design due to their capability of exploring and finding promising regions in the search space in an affordable time [1]. The traditional engineering optimization algorithms are based on nonlinear programming methods that require substantial gradient information and usually seek to improve the solution in the neighborhood of a starting point. Many real-world engineering optimization problems, however, are very complex in nature and quite difficult to solve using these algorithms [2]. However hybridization of the conventional and meta-heuristic algorithms and combining suitable features of both categories may result in competent search techniques [3].

Charged System Search (CSS) is a population based meta-heuristic algorithm which has been proposed recently by Kaveh and Talatahari [4]. In the CSS each solution candidate is considered as a charged sphere called a Charged Particle (CP). The electrical load of a CP is determined considering its fitness. Each CP exerts an electrical force on all the others according to the Coulomb and Gauss laws from electrostatics. Then the new positions of all the CPs are calculated utilizing Newtonian mechanics, based on the acceleration produced by the electrical force, the previous velocity and the previous position of each CP. Many different structural optimization problems have been successfully solved by the CSS [4–7].

It is a common practice to group the members of a structure in order to decrease the construction costs. Engineers group members together based on their past experiences, personal preferences and fabrication requirements. This is ad hoc grouping [8]. But the problem of element grouping does not seem to be simple enough to be handled manually specially for more complex structures. In recent years, some element grouping algorithms have been proposed by different researchers, Krishnamoorthy et al. [9] Togan and Dologlu [10, 11] Barbosa and Lemonge [12] Barbosa et al. [13] and Walls and Elvin [8] among others.

The element grouping task is fulfilled here utilizing the idea proposed by Barbosa et al. [13]. This method does not consider any structural characteristic to group the members and hence is not restricted to a particular type of structures and is not misled by inconvenient assumptions. In this method two sets of variables define the final cross-sectional configuration of the structure; the *pointer variables* assigning each member to a particular group, and the *type variables* assigning a cross-sectional area to each group.

In the original CSS, all of the variables are stored in a single vector for each CP. This means that the moving strategies and the control parameters are all the same for different variables. Consequently, when a CP explores the search space, there is no meaningful difference between the variables of different natures; a pointer variable would be treated like a type variable for example. It is clear that a similar change in a pointer variable does not have the same effect as a type variable on the structure; a pointer variable merely affects a single member, while a type variable affects all the members of the corresponding group. Thus, treating the variables of different natures separately appears to be of significant importance to let the algorithm to reveal its best performance.

In this paper a multi set charged system search is introduced for the element grouping of truss structures. Two sets of CPs are considered. One of the CP sets contains the pointer variables and the other contains the type variables. This offers the possibility of controlling each of the variable sets properly.

The remainder of this paper is organized as follows: In section 2, weight optimization of truss structures is stated. CSS and MSCSS are introduced briefly in section 3. Some numerical examples are studied in section 4. The concluding remarks are summarized in section 5.

2 Problem Statement

In a truss size optimization problem, the goal is to find a set of optimal cross-sectional areas for the members which minimize the weight of the structure. The magnitudes of the stresses induced in the members together with the displacements of some of the nodes of the structure are usually considered as the constraints. The connectivity information and the nodal coordinates are kept unchanged during the optimization process. For a structure with a predefined element grouping, the problem can be stated mathematically as follows:

$$\begin{aligned}
&\text{Find} X = [x_1, x_2, x_3, \ldots, x_n] \\
&\text{to minimize} Mer(X) = f(X) \times f_{\text{penalty}}(X) \\
&\text{subjected to} \sigma_{i\,\min} \le \sigma_{il} \le \sigma_{i\,\max} \\
&\delta_{k\,\min} \le \delta_{kl} \le \delta_{k\,\max} \\
&i = 1, 2, \ldots, nm \\
&k = 1, 2, \ldots, dc \\
&l = 1, 2, \ldots, lc
\end{aligned} \tag{1}$$

where X is the vector containing the design variables; in a discrete optimum design problem the variables x_i are restricted to be selected from a list of available sections; n is the number of variables (number of groups); $Mer(X)$ is the merit function; $f(X)$ is the cost function, which is taken as the weight of the structure; $f_{\text{penalty}}(X)$ is the penalty function which results from the violations of the constraints corresponding to the response of the structure [14]; σ_{il} is the stress of the ith member under lth loading condition and σ_{imin} and σ_{imax} are its lower and upper bounds, respectively; δ_{kl} is the displacement of the kth degree of freedom under the lth loading condition, δ_{kmin} and δ_{kmax} are the corresponding lower and upper limits respectively; nm is the number of members of the structure; dc is the number of displacement constraints and lc is the number of loading conditions.

The cost function is taken as the weight of the structure and can be expressed as:

$$f(X) = \sum_{i=1}^{nm} \rho_i L_i A_i \tag{2}$$

where ρ_i is the material density of member i; L_i is the length of member i; and A_i is the cross-sectional area of member i.

The penalty function is defined as [4]:

$$f_{\text{penalty}}(X) = (1 + \varepsilon_1 \upsilon)^{\varepsilon_2}, \quad \upsilon = \sum_{i=1}^{q} \upsilon_i \tag{3}$$

where q is the number of constraints. If the ith constraint is satisfied υ_i will be taken as zero, if not it will be taken as:

$$\upsilon_i = \left| 1 - \left(\frac{p_i}{p_i^*} \right) \right| \tag{4}$$

where p_i is the response of the structure and p_i^* is its bound. The parameters ε_1 and ε_2 are selected considering the exploration and the exploitation rate of the search space. The exploration and exploitation rates are the tendency of the agents to explore new areas of the search space and to use good solutions found in the previous stages, respectively. In this paper ε_1 is taken as unity and the value of the ε_2 varies linearly from 1.5 to 3 as the optimization process proceeds.

3 The Optimization Algorithm

3.1 Original CSS

Recently an efficient optimization algorithm, known as the Charged System Search, has been proposed by Kaveh and Talatahari [4]. This algorithm is based on electrostatics and Newtonian mechanics laws.

The Coulomb and Gauss laws provide the magnitude of the electric field at a point inside and outside a charged insulating solid sphere, respectively, as follows [15]:

$$E_{ij} = \begin{cases} \frac{k_e q_i}{a^3} r_{ij} & \text{if} r_{ij} < a \\ \frac{k_e q_i}{r_{ij}^2} & \text{if} r_{ij} \ge a \end{cases} \tag{5}$$

where k_e is a constant known as the Coulomb constant; r_{ij} is the separation of the centre of sphere and the selected point; q_i is the magnitude of the charge; and a is the radius of the charged sphere. Using the principle of superposition, the resulting electric force due to N charged spheres is equal to [4]:

$$F_j = k_{eq} \sum_{i=1}^{N} \left(\frac{q_i}{a^3} r_{ij} i_1 + \frac{q_i}{r_{ij}^2} i_2 \right) \frac{r_i - r_j}{\|r_i - r_j\|}$$
$$i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a$$
$$i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \tag{6}$$

Also, according to Newtonian mechanics, we have [15]:

$$\Delta r = r_{\text{new}} - r_{\text{old}} \tag{7}$$

$$\upsilon = \frac{r_{\text{new}} - r_{\text{old}}}{\Delta t} \tag{8}$$

$$a' = \frac{\upsilon_{\text{new}} - \upsilon_{\text{old}}}{\Delta t} \tag{9}$$

where r_{old} and r_{new} are the initial and final positions of the particle, respectively; υ is the velocity of the particle; and a' is the acceleration of the particle. Combining the above equations and using Newton's second law, the displacement of any object as a function of time is obtained as [15]:

$$r_{\text{new}} = \frac{1}{2} \frac{F}{M} \Delta t^2 + \upsilon_{\text{old}} \Delta t + r_{\text{old}} \tag{10}$$

Where F is the resultant force vector acting on the particle; M is the particles mass and Δt is the time interval.

Inspired by the above electrostatic and Newtonian mechanics laws, the pseudo-code of the CSS algorithm is presented as follows [6]:

Level 1: Initialization Step 1. Initialization. Initialize the parameters of the CSS algorithm. Initialize an array of charged particles (CPs) with random positions. The initial velocities of the CPs are taken as zero. Each CP has a charge of magnitude (q_i) defined considering the quality of its solution as:

$$q_i = \frac{fit\,(i) - fit_{\text{worst}}}{fit_{\text{best}} - fit_{\text{worst}}} \quad i = 1, 2, \ldots N \tag{11}$$

where fit_{best} and fit_{worst} are the best and the worst fitness of all the particles; $fit(i)$ represents the fitness of agent i. The separation distance r_{ij} between two charged particles is defined as:

$$r_{ij} = \frac{\|X_i - X_j\|}{\left\| \frac{(X_i + X_j)}{2} - X_{\text{best}} \right\| + \varepsilon} \tag{12}$$

where X_i and X_j are the positions of the ith and jth CPs, respectively; X_{best} is the position of the best current CP; and ε is a small positive to avoid singularities.

Step 2. CP ranking. Evaluate the values of the fitness function for the CPs, compare with each other and sort them in increasing order.

Step 3. CM creation. Store the number of the first CPs equal to charged memory size (CMS) and their related values of the fitness functions in the charged memory (CM).

Level 2: Search Step 1. Attracting force determination. Determine the probability of moving each CP toward the others considering the following probability function:

$$p_{ij} = \begin{cases} 1 & \frac{fit(i) - fitbest}{fit(j) - fit(i)} > rand \vee fit(i) > fit(j) \\ 0 & else \end{cases} \tag{13}$$

where $rand$ is a random number uniformly distributed in the range of (0,1).

Then calculate the attracting force vector for each CP as follows:

$$F_{ij} = q_j \sum_{i, i \neq j} \left(\frac{q_i}{a^3} r_{ij} i_1 + \frac{q_i}{r_{ij}^2} i_2 \right) p_{ij} (X_i - X_j)$$
$$j = 1, 2, \ldots, N$$
$$i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a$$
$$i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \tag{14}$$

where F_j is the resultant force affecting the jth CP.

Step 2. Solution construction. Move each CP to the new position and find its velocity using the following equations:

$$X_{j,\text{new}} = rand_{j1} k_a \frac{F_j}{m_j} \Delta t^2 + rand_{j2} k_\upsilon V_{j,\text{old}} \Delta t + X_{j,\text{old}} \tag{15}$$

$$V_{j,\text{new}} = \frac{X_{j,\text{new}} - X_{j,\text{old}}}{\Delta t} \tag{16}$$

where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range (1,0); m_j is the mass of the CPs, which is equal to q_j in this paper. The mass concept may be useful for developing a multi-objective CSS. Δt is the time step, and it is set to 1. k_a is the acceleration coefficient; k_υ is the velocity coefficient to control the influence of the previous velocity. In this paper k_υ and k_a are taken as:

$$k_a = c_1(1 + iter/iter_{\max}), \quad k_\upsilon = c_2(1 - iter/iter_{\max}) \tag{17}$$

Where c_1 and c_2 are two constants to control the exploitation and exploration of the algorithm; iter is the iteration number and $iter_{\max}$ is the maximum number of iterations.

Step 3. CP position correction. If each CP exits from the allowable search space, correct its position using the HS-based handling as described by Kaveh and Talatahari [4, 12].

Step 4. CP ranking. Evaluate and compare the values of the fitness function for the new CPs; and sort them in an increasing order.

Step 5. CM updating. If some new CP vectors are better than the worst ones in the CM, in terms of their objective function values, include the better vectors in the CM and exclude the worst ones from the CM.

Level 3: Controlling the terminating criterion Repeat the search level steps until a terminating criterion is satisfied. The terminating criterion is considered to be the number of iterations.

3.2 Multi set charged system search for element grouping

Here it is assumed that the element grouping is not predefined. Aside from the members' cross-sectional areas, the optimization technique seeks for an optimal grouping for the elements of the structure. In fact, the main advantage of MSCSS algorithm in comparison to the original CSS is its ability to group the members of the structure. MSCSS discards the predefined ad hoc grouping and searches for an optimal one. The details are explained in the remainder of this section.

Barbosa et al. [13] have proposed a genetic algorithm encoding for cardinality constraints in which two sets of variables are utilized to introduce a particular solution. The first set of variables is called the pointer variables which associate each member of the structure to a group. The second set is called the type variables which assign a particular cross-sectional area to each group. All of the variables, defining a solution candidate, are listed into a string (chromosome) which takes part in the optimization process as an individual.

A similar idea is used here with some modifications for the element grouping task. A pointer variable only determines a single member's condition. When the value of a pointer variable is changed, a member leaves a group and joins another. On the contrary, a type variable determines the cross-sectional area of a group of members. Therefore, it is clear that these two kinds of variables, when experiencing similar changes, affect the structure's configuration and behavior in different manners.

It should be noted that the work done in this paper is different in nature form that of Barbosa and Lemonge [12] and Barbosa et al [13] and the results are not comparable. In these references the problem of element grouping is not considered; they employ an ad hoc grouping and then try to reduce the number of variables to satisfy the additional "cardinality constraints".

Like any other meta-heuristic algorithm, successful application of the Charged System Search is strongly influenced by properly setting its parameters. A universally optimal parameter values set for a given meta-heuristic does not exist [16]. A proper combination of the parameters depends on the characteristics of the certain problem under consideration and the variables involved.

In order to take the different natures of the variables into account and to make it possible to use different parameter sets for each variable type a multi set charged system search is introduced here. Two different sets of CPs are considered. Each of the CPs of the first set is a vector representing the pointer variables of a solution candidate. For each CP in the first set there exists an associated CP in the second set which is a vector representing the type variables of the same solution candidate. The two CPs in a pair share a fitness value which is used for quality evaluation. Each of the CP sets explores its own search space independently. Figure 1 shows an example of a pair of CPs representing a solution candidate in a ten-bar truss example with a set of 32 discrete available sections. The members are to be grouped in four groups. A pair like this determines a solution candidate. The pointer variable CP has as many variables as the number of members of the structure. The value of each of these variables determines the group which the corresponding member belongs to. The type variable CP has as many variables as the number of groups of the structure. The value of each of these variables determines the cross-sectional area of the corresponding group (or a number which is associated with a particular section in a list when considering discrete variable problems).

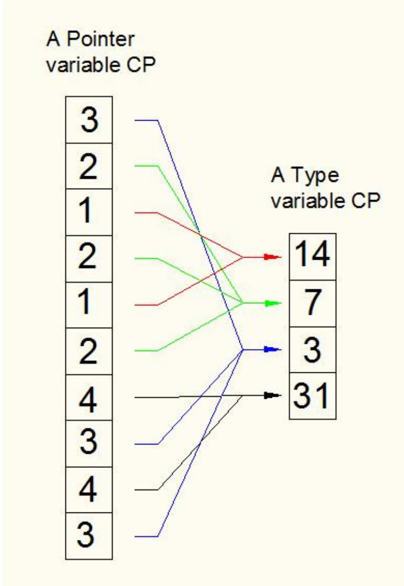

Fig. 1. A pair of CPs representing a solution candidate in the MSCSS

The difference between the two charged particle sets is imposed through using different values for the particle size and the parameters c_1 and c_2 to control the exploration and exploitation rates effectively. These parameters are determined by try and error here although they can also be determined adaptively. For the sake of simplicity the parameter $c_1$is assumed to be equal to unity for both CP sets in all examples; the parameter c_2 for the pointer variables' CP set is assumed to be twice that of the type variables' CP set. This assumption is also proven to be useful through a try and error process. All of other aspects of MSCSS are similar to those of the original CSS.

4 Design Examples

Four examples are studied in this section and the results are compared to the previously obtained results. Except for the first example, a ten-bar truss, in which the results of the grouped structures are compared to the previously obtained ungrouped structures, the remaining comparisons are carried out between the results of the MSCSS and those obtained through ad hoc grouping.

4.1 A Ten-bar Truss

Figure 2 shows a ten-bar truss which has been investigated without element grouping by Wu and Chow [17], Rajeev and Krishnamoorthy [18], Ringertz [19] and Li et al. [20] among others. The material density is 0.1 lb/in^3 (2767.990 kg/m^3) and the modulus of elasticity is 10,000 ksi (68,950 MPa). The members are subjected to stress limitations of 25 ksi (172.375 MPa). All nodes in both directions are subjected to displacement limitations of 2.0 in (5.08 cm). Nodes 2 and 4 are subjected to a downward load of P = 10^5 lbs (445 kN).

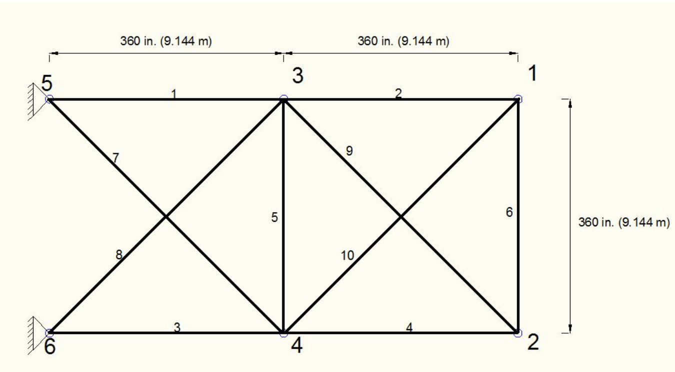

Fig. 2. A ten-bar truss

Here, the goal is to group the members of the structure into 4 groups while minimizing its weight. Two optimization cases are considered. For case 1, the discrete variables are selected from the set D={1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97,5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50} (in^2) or {10.46, 11.62, 12.85, 13.75, 15.37, 16.91, 16.98, 18.59, 18.92, 19.95 20.21, 21.82, 22.40, 22.92, 23.44, 24.79, 24.99, 25.05, 26.99, 27.24, 28.99, 29.63, 30.99, 32.09, 33.06 , 359.86, 46.61, 51.46, 74.25, 87.16, 89.74, 91.68, 100.07, 103.30, 109.11, 121.37, 128.48, 142.03, 147.84, 171.09, 193.68, 216.28} (cm^2). For case 2, the discrete variables are selected from the set D = {0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5}(in^2)or{0.65, 3.23, 6.46, 9.68, 12.91, 16.14, 19.37, 22.60, 25.82, 29.05, 32.28, 35.51, 38.74, 41.96, 45.19, 48.42, 51.65, 54.88, 58.10, 61.33, 64.56, 67.79, 71.02, 74.25, 77.47, 80.70, 83.93, 87.16, 90.39, 93.61, 96.84, 100.07, 103.30, 106.53, 109.75, 112.98, 116.21, 119.44, 122.67, 125.89, 129.12, 132.35, 135.58, 138.81, 142.03, 145.26, 148.49, 151.72, 154.95, 158.17, 161.40, 164.63, 167.86, 171.09, 174.31177.54, 180.77, 184.00, 187.23, 190.45, 193.68, 196.91, 200.14, 203.37} (cm^2).

Table 1 and 2 represent a comparison between the results obtained by different researchers. It is found that Wu's results do not satisfy the constraints of this problem [20]. The results obtained by MSCSS are only 0.6 and 2.2 percents heavier than the best ungrouped results in cases 1 and 2 respectively, while only using four different sections.

Figures 3 and 4 show the convergence curve of the best results obtained by the MSCSS for cases 1 and 2, respectively.

Table 3 represents the values of the parameters used in the optimization of the ten-bar truss.

Tab. 1. Comparison of optimal designs for the 10-bar planar truss structure (case 1)

Variables (in^2)	Wu and Chow [17]	Rajeev and Krish-namoorthy [18]	Li et al. [20]	MSCSS in^2 (cm^2)
A_1	26.50	33.50	30.00	30.00 (193.68)
A_2	1.62	1.62	1.62	1.62 (10.46)
A_3	16.00	22.00	22.90	22.00 (142.03)
A_4	14.20	15.50	13.50	22.00 (142.03)
A_5	1.80	1.62	1.62	1.62 (10.46)
A_6	1.62	1.62	1.62	1.62 (10.46)
A_7	5.12	14.20	7.97	7.97 (51.46)
A_8	16.00	19.90	26.50	22.00 (142.03)
A_9	18.80	19.90	22.00	22.00 (142.03)
A_{10}	2.38	2.62	1.80	1.62 (10.46)
Weight(lb)	4376.20	5613.84	5531.98	5567.3 (2525.2 kg)

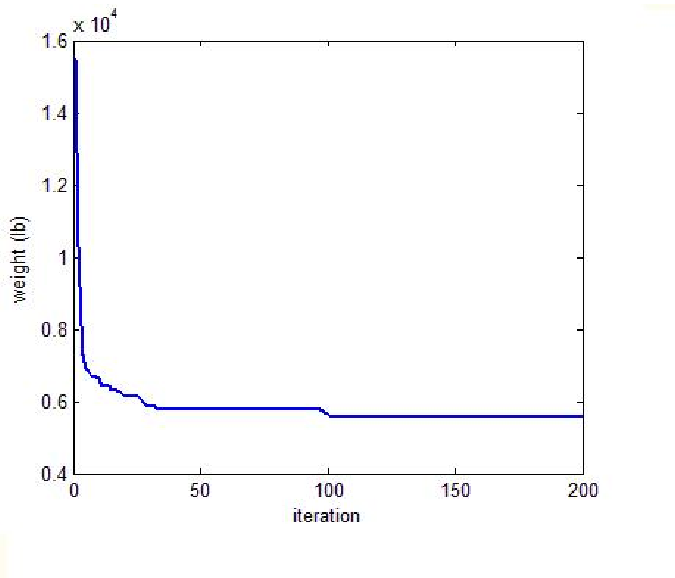

Fig. 3. Convergence curve of the best result obtained for the 10-bar truss by the MSCSS (case 1)

Tab. 2. Comparison of optimal designs for the 10-bar planar truss structure (case 2).

Variables (in^2)	Wu and Chow [17]	Ringertz [19]	Li et al. [20]	MSCSS in^2 (cm^2)
A_1	30.50	30.50	31.50	31.50 (203.36)
A_2	0.50	0.10	0.10	0.10 (0.65)
A_3	16.50	23.00	24.50	20.50 (132.35)
A_4	15.00	15.50	15.50	20.50 (132.35)
A_5	0.10	0.10	0.10	0.10 (0.65)
A_6	0.10	0.50	0.50	0.10 (0.65)
A_7	0.50	7.50	7.50	9.00 (58.10)
A_8	18.00	21.0	20.50	20.50 (132.35)
A_9	19.50	21.5	20.50	20.50 (132.35)
A_{10}	0.50	0.10	0.10	0.10 (0.65)
Weight(lb)	4217.30	5059.9	5073.51	5171.5 (2345.7 kg)

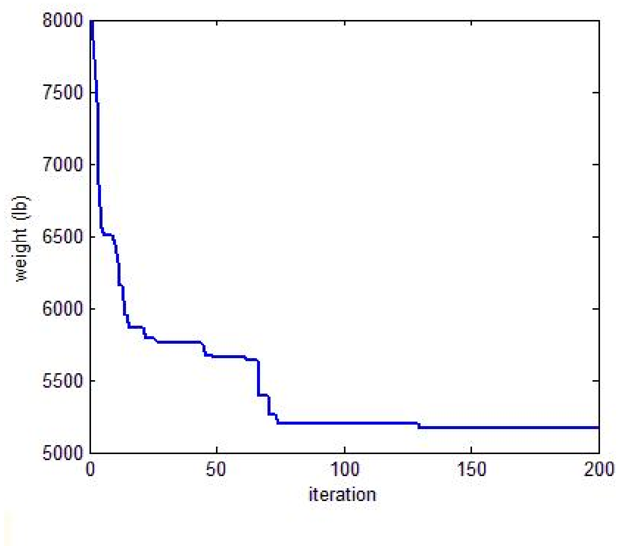

Fig. 4. Convergence curve of the best result obtained for the 10-bar truss by the MSCSS (case 2)

4.2 An Eighteen-bar Cantilever Planar Truss

The eighteen-bar planar truss shown in Figure 5 has been investigated by Imai and Schmit [21] and Lee and Geem [22] as a pure size optimization problem. The material density is 0.1 lb/in^3 (2767.990 kg/m^3) and the modulus of elasticity is 10,000 ksi (68,950 MPa). The members are subjected to stress limitations of 20 ksi (137.89 MPa). An Euler bucking compressive stress limitation is also imposed on the members under

Tab. 3. The values of the parameters used in the optimization of the ten-bar truss

		Number of particles	c1	c2	Particle size (a)
Case 1	Pointers' CP set	50	1	9	0.3
	Types' CP set	50	1	4.5	1
Case 2	Pointers' CP set	50	1	9	0.3
	Types' CP set	50	1	4.5	1

compression according to the following equation:

$$\sigma_i^E = \frac{-k_i A_i E}{L_i^2} \tag{18}$$

where E is the modulus of elasticity and k_i is a constant which is determined considering the shape of the section. L_i is the member length and A_i is the cross-sectional area. In this example, the buckling constant is taken to be $k = 4$. The loading condition consists of a set of vertical downward point loads $P = 20$ kips (89 kN) acting on all the upper nodes.

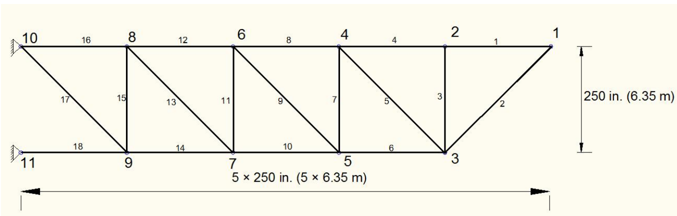

Fig. 5. An eighteen-bar cantilever planar truss

This is a continuous optimization problem with a minimum cross-sectional area of A=0.1 in^2 (645.16 cm^2). Table 4 represents the ad hoc element grouping together with the element grouping obtained by the MSCSS, and Table 5 compares the optimal results.

Tab. 4. Ad hoc element grouping together with the grouping obtained by MSCSS for the 18-bar planar truss

Group number	Ad hoc grouping	Grouping obtained by MSCSS
1	A1 , A4 ,A8 ,A12 ,A16	A1 , A3 ,A4 ,A5 ,A8 , A9 , A12 ,A13
2	A2 , A6 , A10 , A14 , A18	A2 , A6 ,A7 ,A16 ,A17
3	A3 , A7 ,A11 ,A15	A10 , A11 ,A15
4	A5 ,A9 ,A13 ,A17	A14 , A18

It can be seen that the result obtained by the MSCSS is 22.2 percent lighter that the best previously obtained result. This is mainly because of the effectiveness of element grouping algorithm offered by the MSCSS.

Figure 6 shows the convergence curve of the optimization process performed by the MSCSS for the eighteen bar truss, and Table 6 represents the values of the parameters used.

Tab. 5. Comparison of optimal designs for the 18-bar planar truss structure

Group number (in^2)	Imai and Schmit [21]	Lee and Geem [22]	MSCSS in^2 (cm^2)
1	9.998	9.980	6.022 (38.85)
2	21.65	21.63	10.146 (65.46)
3	12.50	12.49	13.706 (88.43)
4	7.072	7.057	21.886 (141.21)
Weight (lb)	6430.0	6421.88	4992.18 (2264.36 kg)

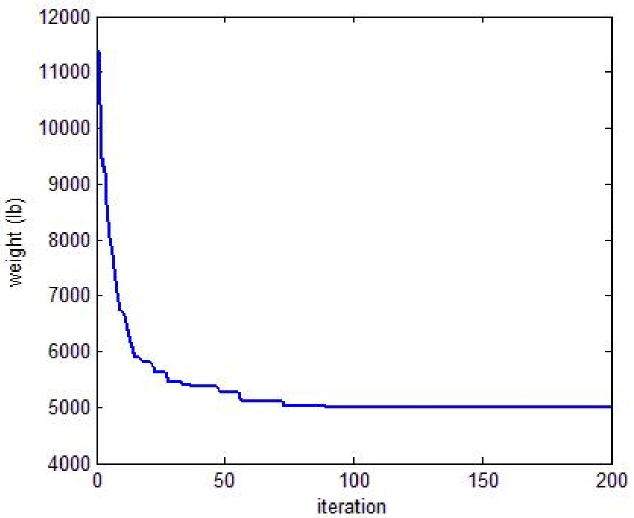

Fig. 6. Convergence curve of the best result obtained for the 18-bar truss by the MSCSS

Tab. 6. The values of the parameters used in the optimization of the 18-bar truss

	Number of particles	c1	c2	Particle size (a)
Pointers' CP set	50	1	9	0.3
Types' CP set	50	1	4.5	1

4.3 A 52-bar Planar Truss

Figure 7 shows a 52-bar planar truss which has been previously analyzed with an ad hoc grouping by Wu and Chow [17], Lee and Geem [23], Li et al [20] and Kaveh and Talatahari [24]. The material density is 7860.0 kg/m^3 and the modulus of elasticity is 2.07×10^5 MPa. The members are subjected to stress limitations of 180 MPa. Point loads Px = 100 kN, Py = 200 kN are acting on the upper nodes of the structure as depicted in the figure. The discrete variables are selected from the American Institute of Steel Construction (AISC) Code, which is shown in Table 7 [25].

Table 8 represents the ad hoc element grouping which has been previously used by all researchers together with the optimal element grouping found by the MSCSS. The only restriction imposed on the element grouping task by the MSCSS is to maintain the symmetry with respect to y axis. Table 9 represents a comparison between the results obtained through ad hoc grouping and the result obtained by the MSCSS.

Tab. 7. The available cross-sectional areas of the ASIC code [25]

No.	in^2	mm^2	No.	in^2	mm^2
1	0.111	71.613	33	3.840	2477.423
2	0.141	90.96786	34	3.870	2496.778
3	0.196	126.4518	35	3.880	2503.229
4	0.250	161.2905	36	4.180	2696.778
5	0.307	198.0648	37	4.220	2722.584
6	0.391	252.2584	38	4.490	2896.778
7	0.442	285.1617	39	4.590	2961.294
8	0.563	363.2263	40	4.800	3096.778
9	0.602	388.3876	41	4.970	3206.456
10	0.766	494.1942	42	5.120	3303.23
11	0.785	506.4523	43	5.740	3703.231
12	0.994	641.2912	44	7.220	4658.071
13	1.000	645.1622	45	7.970	5141.942
14	1.228	792.2591	46	8.530	5503.233
15	1.266	816.7753	47	9.300	6000.008
16	1.457	940.0013	48	10.850	7000.009
17	1.563	1008.388	49	11.500	7419.365
18	1.620	1045.163	50	13.500	8709.689
19	1.800	1161.292	51	13.900	8967.754
20	1.990	1283.873	52	14.200	9161.303
21	2.130	1374.195	53	15.500	10000.01
22	2.380	1535.486	54	16.000	10322.59
23	2.620	1690.325	55	16.900	10903.24
24	2.630	1696.776	56	18.800	12129.05
25	2.880	1858.067	57	19.900	12838.73
26	2.930	1890.325	58	22.000	14193.57
27	3.090	1993.551	59	22.900	14774.21
28	1.130	729.0332	60	24.500	15806.47
29	3.380	2180.648	61	26.500	17096.8
30	3.470	2238.713	62	28.000	18064.54
31	3.550	2290.326	63	30.000	19354.86
32	3.630	2341.939	64	33.500	21612.93

It can be seen that the result obtained by the MSCSS is considerably better than the best result obtained through ad hoc grouping while using 10 groups instead of 12 groups. Table 10 represents the values of the parameters utilized for the optimization of the 52-bar planar truss. Fig. 8 shows the convergence curve of the best result obtained by the MSCSS for the 52-bar planar

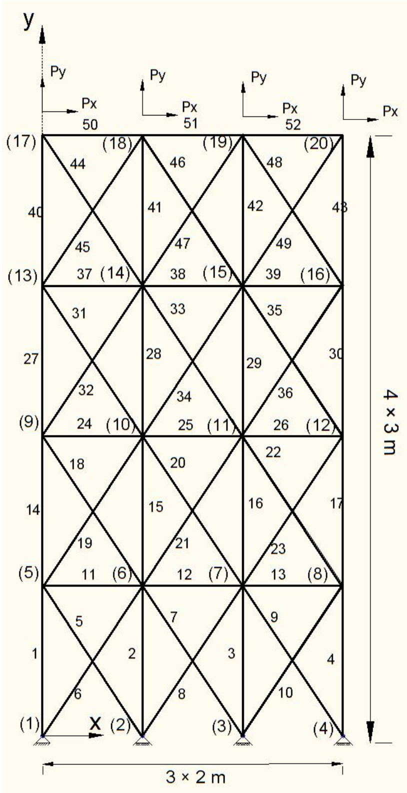

Fig. 7. A 52-bar planar truss

Tab. 8. Ad hoc element grouping together with the grouping obtained by MSCSS for the 52-bar truss

Group number	Ad hoc grouping	Grouping obtained by MSCSS
1	A1–A4	A2, A3, A5, A7, A8, A10
2	A5–A10	A18, A23, A37, A38, A39, A51
3	A11–A13	A12, A15, A16, A25, A41, A42, A50, A52
4	A14–A17	A11, A13, A33, A34
5	A18–A23	A19, A22, A24, A26, A31, A36, A44, A49
6	A24–A26	A28, A29, A45, A46, A47, A48
7	A27–A30	A32, A35, A40, A43
8	A31–A36	A20, A21
9	A37–A39	A6, A9, A27, A30
10	A40–A43	A1, A4, A14, A17
11	A44–A49	-
12	A50–A52	-

Tab. 9. Comparison of optimal designs for the 52-bar planar truss structure

Variables (mm^2)	Wu and Chow [17]	Lee and Geem [23]	Li et al [20]	Kaveh and Talatahari [24]	MSCSS
1	4658.055	4658.055	4658.055	4658.055	252.26
2	1161.288	1161.288	1161.288	1161.288	363.23
3	645.160	506.451	363.225	494.193	506.45
4	3303.219	3303.219	3303.219	3303.219	641.29
5	1045.159	940.000	940.000	1008.385	1045.16
6	494.193	494.193	494.193	285.161	1283.87
7	2477.414	2290.318	2238.705	2290.318	1374.19
8	1045.159	1008.385	1008.385	1008.385	1993.55
9	285.161	2290.318	388.386	388.386	2696.78
10	1696.771	1535.481	1283.868	1283.868	4658.07
11	1045.159	1045.159	1161.288	1161.288	-
12	641.289	506.451	792.256	506.451	-
Weight (kg)	1970.142	1906.76	1905.495	1904.83	1611.77

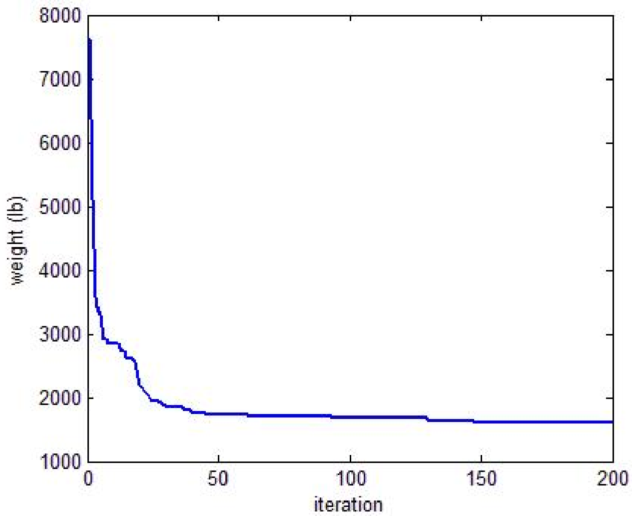

Fig. 8. Convergence curve of the best result obtained by the MSCSS for the 52-bar planar truss

Tab. 10. The values of the parameters used in the optimization of the 52-bar truss

	Number of particles	c1	c2	Particle size (a)
Pointers' CP set	100	1	9	0.3
Types' CP set	100	1	4.5	1

truss.

4.4 A 72-bar Spatial Truss

A 72-bar space truss as shown in Figure 9 has been analyzed previously by Wu and Chow [17], Li et al [20] and Kaveh and Talatahari [24] considering an ad hoc grouping. The material density is 0.1 lb/in^3 (2767.990 kg/m^3) and the modulus of elasticity is 10,000 ksi (68,950 MPa). The members are subjected to stress limitations of 25 ksi (172.375 MPa). The uppermost nodes

Tab. 11. Loading conditions for the 72-bar space truss

node	Case 1			Case 2		
	Px kips (kN)	Py kips (kN)	Pz kips (kN)	Px kips(kN)	Py kips(kN)	Pz kips (kN)
1	0.5(-22.25)	0.5(22.25)	-0.5(-22.25)	_	_	-0.5(-22.25)
2	_	_	_	_	_	-0.5(-22.25)
3	_	_	_	_	_	-0.5(-22.25)
4	_	_	_	_	_	-0.5(-22.25)

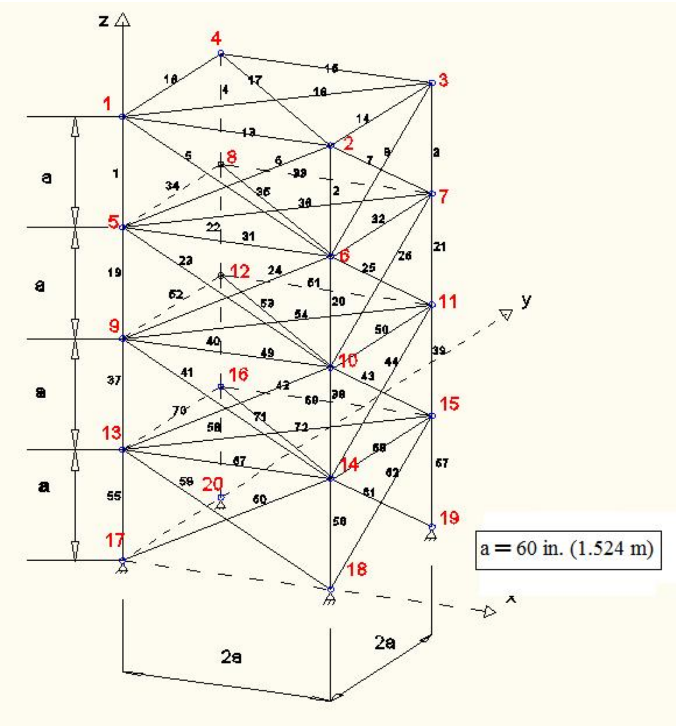

Fig. 9. A 72-bar spatial truss

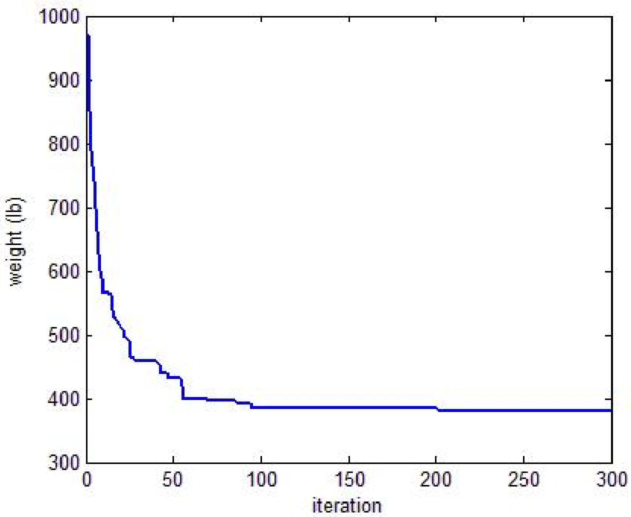

Fig. 10. Convergence curve of the best result obtained by the MSCSS for the 72-bar space truss (case 1)

are subjected to displacement limitations of 0.25 in (0.635 cm) both in x and y directions. There are two optimization cases to be implemented. Case 1: The discrete variables are selected

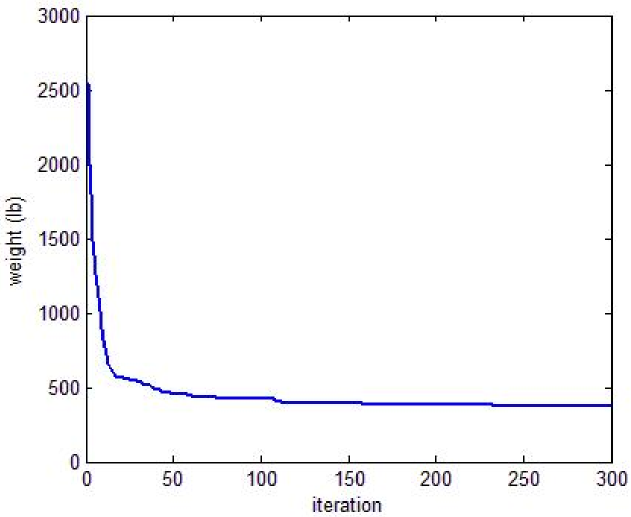

Fig. 11. Convergence curve of the best result obtained by the MSCSS for the 72-bar space truss (case 2)

from the set D = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,0.8,0.9,1.0, 1.1,1.2, 1.3,1.4, 1.5,1.6, 1.7, 1.8, 1.9, 2.0, 2.1,2.2, 2.3,2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1,3.2} (in^2) or {0.65, 1.29, 1.94, 2.58, 3.23, 3.87, 4.52, 5.16, 5.81, 6.45, 7.10,7.74, 8.39, 9.03, 9.68, 10.32, 10.97, 12.26, 12.90, 13.55, 14.19, 14.84,15.48, 16.13, 16.77, 17.42, 18.06, 18.71, 19.36, 20.00, 20.65} (cm^2). Case 2: The discrete variables are selected from Table 7. The loading conditions for both optimization cases are listed in Table 11.

Table 12 represents the ad hoc element grouping which has been previously used by all researchers together with the optimal element grouping found by the MSCSS. Table 13 and 14 represent a comparison between the results obtained through ad hoc grouping and the results obtained by the MSCSS for the 72-bar space truss for cases 1 and 2, respectively. Figure 10 and 11 represents the convergence curves of the best results obtained by the MSCSS for the 72-bar space truss in cases 1 and 2, respectively. Table 15 lists the values of the parameters used for the optimization of the 72-bar space truss.

It can be seen from Table 13 and 14 that the results obtained by the MSCSS are slightly lighter than the results obtained through ad hoc grouping. It can also be seen that in case 2 the MSCSS algorithm has used fewer number of different variables.

Tab. 12. Ad hoc element grouping together with the grouping obtained by MSCSS for the 72-bar truss

Group number	Ad hoc grouping	Grouping obtained by MSCSS	
		Element No. (case 1)	Element No. (case 2)
1	A1–A4	1 4 5 8 10 11 14 18 19 27 30 31 33 34 36 38 42 49 50 51 52 54 56 58 62 63 66 67 70 71	7 11 15 24 30 33 35 41 49 50 51 52 53 54 59 67 68 69 71 72
2	A5–A12	3 9 13 15 16 20 24 29 32 40 43 46 47 60 68 69 72	66 70
3	A13–A16	2 7 25 35	19 31 34
4	A17–A18	6 22 23 37 41 53 59	1 4 18 32 36 61 64
5	A19–A22	12 21 45 48	5 14 16 27 47
6	A23–A30	17 26 44 61 64 65	3 8 9 10 13 17 20 22 23 25 26 28 38 40 43 44 45 46 48 56 62 63
7	A31–A34	28 55 57	2 58 65
8	A35–A36	39	29 42 60 6 12 37 55 57
9	A37–A40	-	21 39
10	A41–A48	-	-
11	A49–A52	-	-
12	A53–A54	-	-
13	A55–A58	-	-
14	A59–A66	-	-
15	A67–A70	-	-
16	A71–A72	-	-

Tab. 13. Comparison of optimal designs for the 72-bar planar truss structure (case 1)

Variables (in^2)	Wu and Chow [17]	Lee and Geem [23]	Li et al. [20]	Kaveh and Talatahari [24]	MSCSS in^2 (cm^2)
1	1.5	1.9	2.1	1.9	0.2 (1.29)
2	0.7	0.5	0.6	0.5	0.3 (1.94)
3	0.1	0.1	0.1	0.1	0.5 (3.23)
4	0.1	0.1	0.1	0.1	0.7 (4.52)
5	1.3	1.4	1.4	1.3	0.8 (5.16)
6	0.5	0.6	0.5	0.5	0.9 (5.81)
7	0.2	0.1	0.1	0.1	1.5 (9.68)
8	0.1	0.1	0.1	0.1	2.3 (14.85)
9	0.5	0.6	0.5	0.6	-
10	0.5	0.5	0.5	0.5	-
11	0.1	0.1	0.1	0.1	-
12	0.2	0.1	0.1	0.1	-
13	0.2	0.2	0.2	0.2	-
14	0.5	0.5	0.5	0.6	-
15	0.5	0.4	0.3	0.4	-
16	0.7	0.6	0.7	0.6	-
Weight (lb)	400.66	387.94	388.94	385.54	379.19 (172.02 kg)

5 Concluding Remarks

In this paper a multi set charged system search is introduced to investigate structural optimization problems containing variables with different interpretations. In general, variables representing cross-sectional, geometrical, topological and grouping properties of the structure may be present in a structural opti-

Tab. 14. Comparison of optimal designs for the 72-bar planar truss structure (case 2)

Variables (in^2)	Wu and Chow [17]	Li et al [20]	Kaveh and Talatahari [22]	MSCSS in^2 (cm^2)
1	0.196	427.203	1.800	0.111 (0.72)
2	0.602	1.228	0.442	0.141 (0.91)
3	0.307	0.111	0.141	0.196 (1.26)
4	0.766	0.111	0.111	0.250 (1.61)
5	0.391	2.880	1.228	0.391 (2.52)
6	0.391	1.457	0.563	0.563 (3.63)
7	0.141	0.141	0.111	0.766 (4.95)
8	0.111	0.111	0.111	1.228 (7.93)
9	1.800	1.563	0.563	1.266 (8.17)
10	0.602	1.228	0.563	1.80 (11.62)
11	0.141	0.111	0.111	-
12	0.307	0.196	0.250	-
13	1.563	0.391	0.196	-
14	0.766	1.457	0.563	-
15	0.141	0.766	0.442	-
16	0.111	1.563	0.563	-
Weight (lb)	427.203	933.09	393.380	375.70 (170 kg)

Tab. 15. The values of the parameters used in the optimization of the ten-bar truss

		Number of particles	c1	c2	Particle size (a)
Case 1	Pointers' CP set	150	1	9	0.3
	Types' CP set	150	1	4.5	1
Case 2	Pointers' CP set	150	1	9	0.3
	Types' CP set	150	1	4.5	1

mization problem. These variables, having different effects of different orders on the structure, may be useful to be controlled separately. Here the special case of element grouping is considered but the algorithm can solve problems of other types.

MSCSS ignores the predefined grouping (ad hoc grouping) and tries to optimize the cross-sectional size and the element grouping of the structure simultaneously. The control of different variable types is imposed using different sets of optimization parameters. Here a try and error process is carried out to obtain an optimal set of parameters. It can be seen that the parameters do not show a disturbing fluctuation from one example to another.

Four illustrative examples are considered here; a ten-bar truss; an eighteen-bar cantilever truss; a 52-bar planar truss and a 72-bar spatial truss. The results demonstrate the effectiveness and efficiency of the algorithm.

Future works may concentrate on the adaptive and self-adaptive control of the optimization parameters so that the tedious task of tuning the parameters is removed. Other types of structural problems such as topology optimization problems and configuration optimization problem can also be investigated.

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