Test measurements by Eötvös torsion balance and gravimeters

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Abstract

After more than 40 years of interruption new field observations have been made by an E-54 type torsion balance (TB) in the Csepel-island. These TB measurements were accompanied by a detailed gravimetric survey of each station with LCR gravimeters. Both vertical (VG) and horizontal (HG) gravity gradients were determined at each TB station for VG interpolation and reliability tests.

Vertical gradient of gravity cannot be measured directly by the Eötvös TB. However, we successfully interpolated VG differences in the network of TB measurements following the idea originally due to Haalck. Reliability tests by comparing HG and VG gravimetric and TB measurements were also performed. Our recent paper discusses first results of these TB and gravimetric measurements which are scheduled to be continued in the future as well.

Keywords

gravity gradients · gravimeter · Eötvös torsion balance

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1 Introduction

The purpose of the research presented in this paper is to use Hungarian torsion balance (TB) measurements in improved gravity field determination for Hungary in addition to a number of (about 300000) gravity data determined by gravimetry. There are about 60000 torsion balance stations in Hungary that have been measured primarily for prospecting of raw materials. Our previous studies e.g. Tóth, 2007 [7] and geodetic processing of torsion balance measurements made by Loránd Eötvös himself showed that these measurements are suitable for gravity field determination. Also, it is possible to generate all functional of the gravity field by combining TB data with gravimetry. In order to verify these theoretical results in practice and to compare the two (gravimetric and gradiometric) gravity field determination techniques several tests have been made.

2 Determination of the horizontal gradient of gravity

We measured a test network of 300 m x 300 m spacing both by the balance and gravimeters in a flat terrain about 60 km south from Budapest, at the southern part of the Csepel-island. Distribution of points on the test area can be seen on Fig. 1. The

Fig. 1. Location of the torsion balance stations on the test area

horizontal gradients of gravity HG determined by gravimeters and torsion balance are denoted by g_s and W_{zs} respectively:

$$g_s = \frac{\partial g}{\partial u} = \sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2} = \sqrt{g_x^2 + g_y^2},$$

$$W_{zs} = \frac{\partial^2 W}{\partial z \partial u} = \sqrt{\left(\frac{\partial^2 W}{\partial z \partial x}\right)^2 + \left(\frac{\partial^2 W}{\partial z \partial y}\right)^2} = \sqrt{W_{zx}^2 + W_{zy}^2},$$

where g_x, g_y and W_{zx}, W_{zy} are the $N-S$ and $E-W$ components of the horizontal gradients of gravity (the positive x-axis points towards North and positive y-axis points towards East) and in an arbitrary direction of u according to the usual notation [9]:

$$W_{zu} = \frac{\partial^2 W}{\partial z \partial u} = \frac{\partial g}{\partial u} = g_u \quad .$$

Gravimetric measurements were performed with double star method (P-N-P-E-P-S-P-W-P-...) each of the four compass points (N, E, S, W) were located at 5 m distance from the centre point P (Fig. 2). Δg values between these points were divided by 5 m to yield components of HG in μGal/m. Standard errors

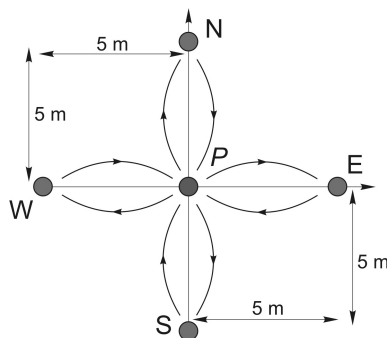**Fig. 2.** Gravimetric HG measurements by double star method

Tab. 1. Horizontal gravity gradients on the test area (g_s from measurements of gravimeters, W_{zs} from torsion balance observations in Eötvös Unit*) and ΔH_{max} are the maximum height differences all around the points up to the horizontal distance of 100 m.

Point	g_s [E]*	W_{zs} [E]*	diff. [E]*	remarks	ΔH_{max} [m]
E238	6.5 ± 8.4	1.7 ± 0.2	4.8		1.95
E208	8.4 ± 2.4	3.3 ± 0.6	5.1		2.16
11	6.4 ± 3.2	–			1.83
12	10.6 ± 5.6	5.2 ± 0.6	5.4		2.98
13	10.2 ± 12.6	2.3 ± 0.2	7.9		1.63
14	1.1 ± 2.2	0.4 ± 1.2	0.7		0.61
15	1.5 ± 2.8	2.8 ± 0.6	-1.3		1.35
22	8.7 ± 5.4	–			
22	5.8 ± 5.7	1.6 ± 0.2	4.2	repeated	2.51
23	6.3 ± 5.7	1.4 ± 0.6	4.9		1.52
24	4.5 ± 4.3	0.8 ± 0.5	3.7		1.49
25	3.8 ± 8.4	2.7 ± 0.5	1.1		2.63
26	2.5 ± 3.6	1.7 ± 0.5	0.8		2.31
33	4.8 ± 3.3	1.8 ± 0.2	3.0		
33	4.1 ± 3.2	–		repeated	1.60
34	0.5 ± 4.8	2.4 ± 1.0	-1.9		1.24
36	5.0 ± 3.9	2.2 ± 0.5	2.8		1.44
44	5.5 ± 3.6	0.7 ± 0.3	4.8		1.96
45	4.9 ± 5.7	1.2 ± 0.4	3.7		0.52
54	2.4 ± 5.0	1.3 ± 0.6	1.1		1.38
mean std.: $\pm$ **5.0**	mean diff.: **3.0**				

* 1E = 1 Eötvös Unit = 10^{-9} s^{-2}

of g_s were calculated from repeated measurements according to the formula

$$\sigma_{g_s} = \sqrt{\sigma_{g_x}^2 + \sigma_{g_y}^2}.$$

We used LCR gravimeters equipped with electronic levels and capacitance beam position indicator (CPI) electronics, and readings were taken by interpolation method on a digital multimeter with external R-C filter [3].

The mean standard deviation of our gravimetric measurements by LCR gravimeters for the horizontal gradients of gravity was ± 5 μGal (1 μGal $=10^{-8}$ m/s^2). This is better than the usual accuracy of gravity field measurements. The lower error can be attributed to the fast relocation of the instrument between measurement points with negligible adverse mechanical effects, and also to the fact that only minimal dial turning was necessary before taking readings. Gravity measurements on certain sites after several days with different weather circumstances were repeated (see Table 1 points 22 and 33). Under favourable conditions (point 33) we found a good agreement, whereas for entirely different circumstances (point 22) the discrepancies were higher.

We make a remark, however, for the rating of these results. The external conditions were very unfavorable for the TB measurements. The temperature in the observation hut increased rapidly by 7-8°C during the measurement and in certain days reached even 40°C whereas the vertical temperature gradient was 7-8°C/m. The latter figure presumably caused adverse air turbulence inside torsion wire protecting tubes.

Observations by the E-54 type torsion balance in 5 azimuths have been made by taking visual readings and after this repeated readings in the first two azimuths concluded the measuring series. First readings were taken 40-50 minutes after releasing the arrester of torsion wires, and hereinafter the readings were taken in a regular 40-minute pattern. The rotation of the beam was provided by mechanical clockwork. It follows from the construction of the double balance that each output quantity comes from two – partly independent – observations. The final result was the arithmetic mean referring for these two balances. The standard deviations of horizontal gravity gradients determined by the torsion balance were estimated from the differences and the effect of the observer's mass was also investigated (Figs. 3 and 4). This mass effect during a 2-minute observation time was found to be about 0.4-0.6 E on average. These results were obtained by using an automated CCD reading system.

The mean difference between gravimeter and torsion balance measurements is 3 μGal/m and this figure is smaller than the standard deviation of gravimeter measurements.

The last column of Table 1 contains maximum height differences ΔH_{max} of the topographic survey carried out in 8 azimuths around each measurement site up to the maximum distance of 100 m. The maximum ΔH for the cca 3 km^2 test area is
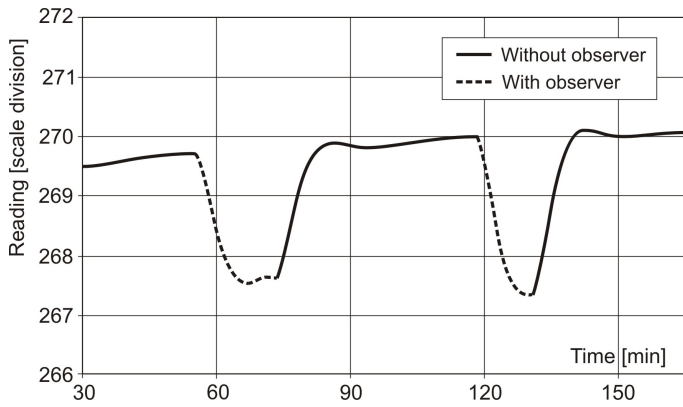
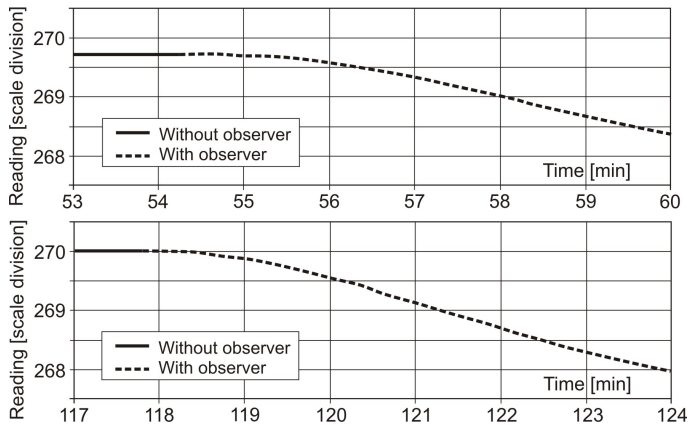**Fig. 3.** Effect of observer's mass on the torsion balance readings

Fig. 4. Magnified parts of the effect of observer's mass on the torsion balance readings

3.67 m. Effect of the nearby terrain was negligible; the ground was carefully leveled around each site.

From the comparison of TB and gravimetric determination of the horizontal gravity gradient it can be concluded that by LCR gravimeters equipped with electronic levels and CPI electronics and readings taken on a digital multimeter it is impossible to have a reliable determination of HG. It is still best determined by the torsion balance.

3 Determination of gravity differences Δg by gravimetry and the torsion balance

Gravity differences Δg between points were determined by LCR gravimeters mentioned in the previous section. Measurements were performed between 16 points at 4 different epochs and measuring series – simultaneously with 2 LCR gravimeters. These gravimeters were transported by a car between measurement points and base stations of measuring series. (Since all network points were mounted in plough-land, gravimeters were constantly put under adverse mechanical stress - shock by the transportation.)

The measurements were processed with tidal, instrument height, barometric and drift corrections. (Non-modeled effects – thermal and mechanical stress – were reckoned in the drift correction.)

The gravity differences Δg shown in Table 2 are raw (non-adjusted) values. It can be recognized from data in the differences (diff.) column that there is no scale bias between the two gravimeters.

Tab. 2. Unadjusted gravity differences of the test network

connection	distance	Δg LCR-220	Δg LCR-1919	diff.	mean
	[m]	[mGal]			
E238-12	451.961	0.318	0.342	− 0.024	0.330
12–11	298.924	− 0.369	− 0.371	+ 0.002	− 0.370
11–13	599.540	1.105	1.085	+ 0.020	1.095
13–14	298.737	0.593	0.578	+ 0.015	0.586
14–15	298.043	0.528	0.529	− 0.001	0.528
15–E208	599.259	0.636	0.649	− 0.013	0.642
E238–E208	1437.610	2.834	2.797	+ 0.037	2.816
23–24	299.136	0.314	0.324	− 0.010	0.319
24–25	300.904	0.264	0.233	+ 0.031	0.248
25–26	300.060	0.230	0.287	− 0.057	0.258
36–34	602.220	− 0.645	− 0.664	+ 0.019	− 0.654
34–33	299.760	− 0.190	− 0.195	+ 0.005	− 0.192
33–22	356.443	− 0.812	− 0.780	− 0.032	− 0.796
12–23	424.380	1.295	1.256	+ 0.039	1.276
23–33	298.010	0.407	0.438	− 0.031	0.422
54–44	424.265	− 0.364	− 0.328	− 0.036	− 0.346

Gravity differences Δg between stations can be evaluated upon the assumption that distances and height differences are small [10]. In this case horizontal gradients measured at two sites can be replaced by their mean values along the connecting line. Moreover, the effect of vertical gravity gradient on Δg was not considered. Hence the following approximate formula for obtaining the gravity difference Δg between two points A and B can be written:

$$\Delta g_{AB} = \frac{(W_{zs})_A + (W_{zs})_B}{2} t_{AB} \quad .$$

Here $(W_{zs})_A$ and $(W_{zs})_B$ are the components of gravity gradients along the direction of AB measured at the two points, respectively, while t_{AB} denotes the distance between the two points. The result is expressed in mGal (1 mGal $=10^{-5}$ m/s^2).

When gravity differences Δg between the test network points were computed as above, it was observed that the variation of horizontal gravity gradients are not linear between the points. Hence, we followed another approach for determining the figures shown in Table 3. First, a digital terrain model (DTM) with grid spacing of 5m × 5m was constructed from 1:10000 scale topographic maps and a topographic survey carried out with total stations. Second, a mass model and a reference mass model with average height of the area were constructed from triangular prisms with uniform 2000 kg/m^3 density. Third, gravity gradients were determined by forward gravity field modeling using formulas by Holstein, 2003 [5] for a grid with 50m × 50m spacing, and subsequently these gradients were used for interpolation of the nonlinear change of gravity gradients between several network points in Table 3. Finally, gravity difference Δg was computed between two such points as a sum of individual contributions Δg_i.

Tab. 3. Comparison of Δg from gravimetry and from torsion balance with nonlinear correction over the network

connection	Δg (gravimeter) [mGal]	Δg (TB) [mGal]	difference [mGal]
E238-12	0.330	0.199	0.131
12–11	− 0.370	− 0.168	− 0.202
11–13	1.095	0.454	0.641
13–14	0.586	0.166	0.420
14–15	0.528	0.084	0.444
15–E208	0.642	0.323	0.319
E238–E208	2.816	1.057	1.759
23–24	0.319	0.176	0.143
24–25	0.248	0.053	0.195
25–26	0.258	0.208	0.050
36–34	− 0.654	− 0.116	− 0.538
34–33	− 0.192	− 0.218	0.026
33–22	− 0.796	− 0.313	− 0.483
12–23	1.276	0.457	0.819
23–33	0.422	0.158	0.264

To asses the accuracy of Δg from torsion balance measurements the following factors should be considered:

1–the horizontal position error is 1-2 cm
2–the standard error of torsion balance measurements is 5-6 E
3–5 cm distance (t_{AB}) error generates cca. 1.6 μGal error
4–5° azimuth error produces cca. 1.3 μGal error

We processed torsion balance observations at all of the 17 sites without drift and with assuming linear drift of the torsion wires. The accuracy depends on several factors. One of these is the visual readout error. If we estimate it to be about 0.2-0.3 scale division, it produces 0.8-1.1 E and 1.7-3 E error of gravity gradients (W_{zx}, W_{zy}) and curvature values ($W_{\Delta g} = W_{yy} - W_{xx}$, $2W_{xy}$), respectively. The error due to the nonlinear drift of torsion wires is more severe. Readings captured with a video camera showed a nonlinear initial drift of 0.7 scale division, which may readily produce an error figure twice of the above. The accuracy achieved can be estimated from the sample standard deviation of differences between the two balances of the double balance. The following standard deviations and mean differences were computed by processing 7 measurements in 5 azimuths with the linear drift model (Table 4).

Tab. 4. Mean and standard deviation of differences between the two balances at all of the 17 sites ($1E = 1Eötvös\ Unit = 10^{-9}s^{-2}$)

	standard deviation [E]	mean [E]
W_{zx}	± 5.3	**3.8**
W_{zy}	± 6.2	**-4.1**
W_{Δ}	± 4.8	-1.0
$2W_{xy}$	± 6.8	-0.4

Measurements of the two balances are not entirely indepen-

dent since the influence of certain environmental factors (e.g. temperature) on the readings is expected to be about the same. Having said that, however, the accuracy of TB measurements can be estimated from the differences: mean standard deviation of differences at 17 stations is ± 6 E (± 0.6 μGal/m) with significant variation from one site to the other (extreme: ± 1.7 E and ± 12.4 E). Absolute values of the estimated linear drift also showed major variation of 0.1 – 1.7 scale division (s.d.) during the 40 min readout interval. From inspection of Table 4 it is evident that gradients of the two balances have a significant non-zero mean. Miscalibration or a yet undiscovered factor can be the culprit. A new calibration of the instrument is planned to settle this issue.

Another source of errors is scale misreading. Site 14 may be an example of this, since a linear drift model yielded worse results than the no-drift model, and particularly at this site differences between the two balances of the double balance produced an extremely high standard deviation of ± 12.4 E and the highest drift estimates (-1.7 and -0.9 s.d.).

As it has been mentioned, the observer's mass effect is a significant source of error, producing on average 0.4-0.6 E deviation within a 2-min interval.

Several of the above errors are expected to be eliminated or mitigated by automated reading, a more realistic drift modeling, a better thermal insulation of the observing hut and perhaps thorough a suitable choice of the reading sequence.

Table 3 shows that a reasonable agreement was found at a considerable number of sites between measured and interpolated Δg if nonlinear variation of horizontal gravity gradients were taken into account. The cause of larger discrepancies of about 0.2 mGal in this table requires further investigation.

4 VG determinations in a network of torsion balance (TB) measurements

It is well-known that vertical gradients of gravity (VG) can not be determined directly from observations of the torsion balance.

Fig. 5. VG measurement

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Tab. 5. Observed VG data

site	linear approximation		quadratic approximation	
	VG [mGal/m]	STD_{VG}	VG [mGal/m]	STD_{VG}
11	− 0.3078	± **0.041**	− 0.3120	± 0.041
12	− 0.3064	± 0.008	− 0.3099	± 0.005
13	− 0.3084	± 0.011	− 0.3035	± 0.006
14	− 0.3001	± 0.005	− 0.2982	± 0.003
15	− 0.3048	± 0.013	− 0.3107	± 0.006
22	− 0.3053	± 0.008	− 0.3063	± 0.005
23	− 0.3049	± 0.009	− 0.3010	± 0.005
24	− 0.3066	± 0.006	− 0.3027	± 0.002
25	− 0.3053	± 0.012	− 0.3109	± 0.006
26	− 0.3050	± **0.017**	− 0.3012	± 0.009
34	− 0.3030	± 0.012	− 0.3016	± 0.009
E208 (35)	− 0.3065	± 0.008	− 0.3080	± 0.009
44	− 0.3092	± 0.011	− 0.3076	± 0.006
45	− 0.3120	± **0.018**	− 0.3190	± 0.009
B502	− 0.3189	± 0.012	− 0.3202	± 0.007
E238	− 0.3074	± 0.006	− 0.3083	± 0.007
mean	**− 0.3062**		**− 0.3067**	

There is a certain technique, however, whereby vertical gradients can be calculated at each TB measurement site if one or several VG values are available in the network [7]. To verify this, gravimetric vertical gradients were determined at the majority of TB sites of the test network [2]. Measurements were made by two LCR gravimeters at heights 50, 700 and 1300 mm above the markers (Fig. 5). Table 5 shows the mean VG determined by the two gravimeters for all the measured sites. Measurement accuracy was strongly degraded by the effect of variable intensity wind-forces (standard deviation (STD) of measurements affected by intense wind-forces are indicated by boldface letters).

The average VG over the test area is close to its normal (nominal) value (0.3086 mGal/m) and the deviations reach only several μGal/m for these sites.

5 Application of Haalck's method

An original idea due to Haalck, 1950 [4] enables the determination of missing W_{zz} vertical gravity gradients VG from TB measurements since third derivatives of the geopotential establishes the required link between observed and missing components according to the following formulas:

$$
\begin{aligned}
W_{zzx} &= -W_{\Delta x} - 2W_{xyy} \\
W_{zzy} &= W_{\Delta y} - 2W_{xyx} \\
W_{zzz} &= -W_{xzx} - W_{yzy}.
\end{aligned}
\tag{1}
$$

To consider the nonlinear variation of gravity gradients, we prepared grids of gravity gradients with the aid of the digital terrain model of the area with spacing of 50m × 50m. By numerical differentiation of these grids the necessary derivatives on the right hand side of (1) were computed, and at a constant height z above the mean sea level – by numerical integration of W_{zzx}, W_{zzy} using formulas of Vassiliou (1986) [8] – we obtained W_{zz}. The map of the computed W_{zz} (in E units) can be seen in

Fig. 6, together with measurement sites and topographic survey points. The correlation of computed vertical gradients VG with the DTM of the area (Fig. 7) is remarkable. Several conclusions can be drawn by testing the numerical solution procedure.
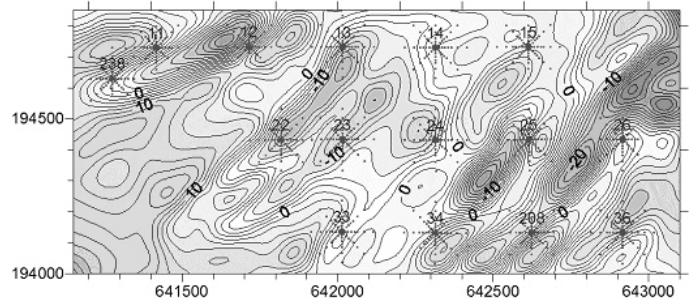
Fig. 6. Interpolated vertical gradients VG for the test area from TB measurements and DTM. Units are E (1E = 1Eötvös = 10^{-9} s^{-2}). Dots are topographic survey points for TB sites
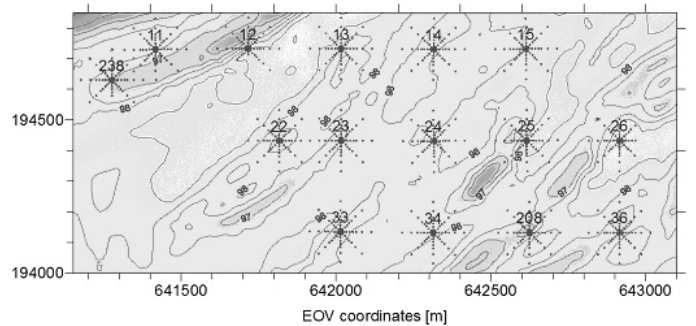
Fig. 7. DTM of the test area. Heights are in [m]. Dots are topographic survey points for TB sites

Tab. 6. Observed VG by quadratic approximation, interpolated VG by Haalck's method and their differences

site	observed VG	interpolated VG	differences
		[mGal/m]	
11	-0.3120	-0.3120	0.0000
12	-0.3099	-0.3122	-0.0023
13	-0.3035	-0.3113	-0.0078
14	-0.2982	-0.3116	-0.0134
15	-0.3107	-0.3130	-0.0023
22	-0.3063	-0.3118	-0.0055
23	-0.3010	-0.3116	-0.0106
24	-0.3027	-0.3111	-0.0084
25	-0.3109	-0.3111	-0.0002
26	-0.3012	-0.3111	-0.0099
34	-0.3016	-0.3103	-0.0087
E208 (35)	-0.3080	-0.3114	-0.0034
mean:	**-0.3055**	**-0.3115**	**STD: -0.006**

From Table 6 it appears that interpolated vertical gradients VG are considerably smaller (max./min. +27/28 E for the whole area) than those obtained by gravimeters (cf. Table 5). This fact can be attributed to the applied numerical differentiation scheme (forming central differences), the transfer function of which cuts down the signal at higher frequencies [1], or from the numerical

integration procedure. The non-linear variation of vertical gradients should also be a factor. The reconstruction of a function from given gradient fields is a well-known problem e.g. in the field of image processing and it leads to the numerical solution of the 2D Poisson equation e.g. [6]. Hence we plan to investigate several other reconstruction techniques to calculate of W_{zz} from its known horizontal gradients.

6 Conclusions

Earlier theoretical investigations and geodetic torsion balance measurements made by Lorand Eötvös showed that these measurements are good for gravity field determination as well and it is possible by combining torsion balance with gravimetry to generate all functionals of the gravity field.

From the comparison of torsion balance and gravimetric determination of the horizontal gradients of gravity it can be concluded that it is impossible to have a reliable determination by LCR gravimeters, it is still best determined by torsion balance.

Computing Δg from torsion balance measurements we realized, the variations of horizontal gradients of gravity between two points would not be supposed linear neither below of a few hundred meters distance. However, generally better agreement can be seen between the measured and the computed Δg taking into account a digital terrain model for estimating the nonlinear variation of the horizontal gravity gradients - but further investigations are necessary to study the reason of the bigger discrepancies.

Comparing the vertical gradients measured by gravimeters and computing from torsion balance measurements various differences can be seen. The reason may come from the applied numerical differentiation scheme, or from the integration procedure.

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