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RESEARCH ARTICLE

Spectral analysis of CHAMP kinematic velocities determined by applying smoothing cubic splines

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Abstract

Kinematic velocities are needed when gravity field recovery ought to be done from kinematic positions by the use of the energy integral. [4] *has tested smoothing cubic splines applying on CHAMP kinematic orbit. The solution has been found potent when a reference gravity field is used during the processing. However the possibility of an error emerges that the reference field 'leaks' into the estimated velocity, which would harmfully affect the gravity field model solution in the subsequent step. The use of smoothing splines demands strict investigations in the Fourier-domain. Tests in the Fourier domain are shown in the recent paper.*

Keywords

numerical differentiation · kinematic velocity · spectral analysis · transfer function

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1 Introduction

The use of measurements from the CHAMP, GRACE and GOCE [1], which are the first satellites with primary mission of global gravity field determination, has provided a new challenge for the geodetic community on data processing and validating the results. This makes every single detailed analysis related to data processing having central interest. In this paper we concentrate on a specific aspect of kinematic velocity determination: how numerical differentiation tricks affect the spectral characteristics of the resulting velocities. This study focuses on the CHAMP satellite, and it should be kept in mind, since all the conclusions are case-sensitive.

In [4] the need of kinematic velocity determination has been introduced, which is not repeated here. Only certain aspects, which are important for the recent analysis, are addressed here again. Since dynamic (and reduced-dynamic) orbits contain a priori physical information, namely a global gravity model is used for integrating the orbits, these orbits are very smooth. They provide a realistic orbital characteristic on the long and middle wavelength. However, due to the theoretical and numerical limitations of the global gravity models, on short wavelength no information is included. We have also a geometrical type of orbit, i.e. kinematic orbit, which is fully based on observations, therefore it is contaminated by observation errors. The positions are observed at very small sampling rate, i.e. 30 seconds. It is an unprecedented spatial resolution of global satellite-only gravity models; no certain global information on these short-wavelengths has ever been available before. So kinematic orbits have both signal and error on short-wavelengths, and no information on this frequency band is available in the reduced-dynamic orbits. Therefore the difference of the two orbits is a good visualization of the kinematic short-wavelength signal and errors, and the spectral characteristics of it should reflect the committed smoothing.

The method has not been mentioned yet: the analysed method is the smoothing by cubic spline functions In [4] this method (among other 5) has provided an impressive result for determination of kinematic velocity. The smoothing could effectively be applied only on data with a nearly white spectrum. This could be achieved only by introducing a reference orbit, a fully dynamic orbit using an a priori known gravity model. As it is always noted: no gravity model should be involved in an exact processing of gravity data. In this paper the effect of the use of reference models will be analysed and discussed in terms of spectral characteristics.

2 The data

This study is based on tests on one day of CHAMP kinematic orbit, day 200 of 2002 The day has been chosen by chance, and the data of this day by visual screening was found to be 'typical' among the 'good' data, what means consistent data during the day with small data gaps only. The same day was analysed in [4].

As it was mentioned in the previous section, we employ a reference orbit for the velocity determination. The reference orbit is used in a remove-restore manner. By removing the reference position from the kinematic positions, the residuals have been used for the differentiations (having an amplitude of some cm – instead of some thousands of km). The obtained residual velocities have been added to the reference velocities to get the kinematic velocities.

For reference orbits purely dynamic and reduced-dynamic orbits have been determined. These were: dynamic orbit using EIGEN-1S model [7], dynamic orbit using TEG-4 model (both up to degree and order of 120), reduced-dynamic orbit based EIGEN-2 gravity model [8]. In [4] the dynamic and reduced-dynamic positions were compared to the kinematic positions; the RMS of the positions were 1.1233 m, 1.1091 m and 0.0311 m for the dynamic EIGEN-1S, dynamic TEG-4 and reduced-dynamic EIGEN-2 orbits, respectively. A kind of comparison also has been done for the velocities: the EIGEN-1S and the TEG-4 velocities differ from the reduced-dynamic velocities with an RMS of 1.5826 mm/s and 1.3562 mm/s, respectively.

Figs. 1-3 show the Power Spectral Density (PSD) of different sets of velocities. As one can see in the figures, different velocities show very similar spectral characteristics with the exception of the velocity when no reference has been used for the estimation. In this case the velocity becomes considerably noisy at $0.001-0.002$ Hz, equivalently at about 600-800 s. All the others show differences at the shortest wavelengths, about at $0.008-0.010$ Hz, or 100-125 s.

In Figs. 1-3 the following notations were used for the different sets of velocities:

- 'dynamic velocity (EIGEN-1)': dynamic velocity, with EIGEN-1S model used for precise orbit determination (POD).
- 'dynamic velocity (TEG)': dynamic velocity, with TEG-4 model used for POD.
- 'reduced-dynamic velocity (EIGEN-2)': reduced-dynamic velocity, with EIGEN-2 model used for POD.

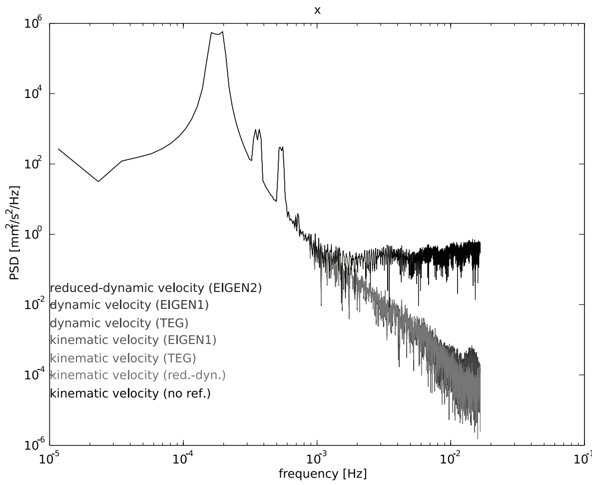

Fig. 1. PSD of kinematic and dynamic velocities along x axis of CTS (Conventional Terrestrial System). Abscissa: frequency [Hz], ordinate: PSD [mm^2/s^2/Hz], .

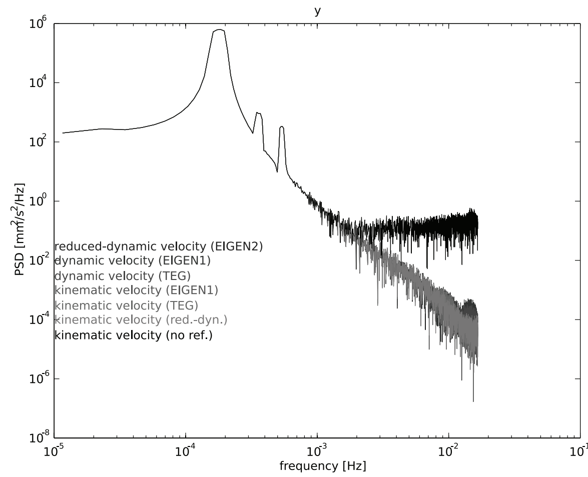

Fig. 2. PSD of kinematic and dynamic velocities along y axis of CTS. Abscissa: frequency [Hz], ordinate: PSD [mm^2/s^2/Hz], .

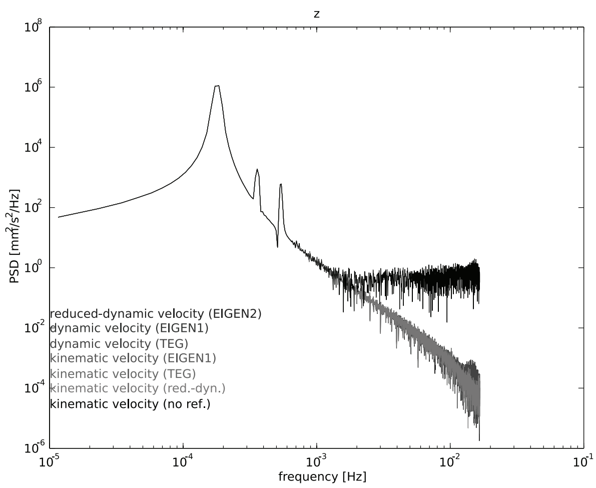

Fig. 3. PSD of kinematic and dynamic velocities along z axis of CTS. Abscissa: frequency [Hz], ordinate: PSD [mm^2/s^2/Hz], .

- 'kinematic velocity (EIGEN-1)': kinematic velocity, using 'dynamic velocity (EIGEN-1)' as reference for the velocity determination.

– 'kinematic velocity (TEG)': kinematic velocity, using 'dynamic velocity (TEG)' as reference for the velocity determination.

– 'kinematic velocity (red.-dyn.)': kinematic velocity, using 'red-dynamic velocity (EIGEN-2)' as reference for the velocity determination.

– 'kinematic velocity (no ref.)': kinematic velocity, derived from the full position signal, no reference orbit applied.

3 Applied Tools

In order to visualize the spectral behaviour of velocities, we determine the so-called 'transfer function'. The transfer function, tf, is defined as a ratio of two different estimates of the same stochastic process, x and y, in the frequency domain:

$$tf = \frac{P_{xy}}{P_{yy}}. \quad (1)$$

In the equation P_{yy} is the Power Spectral Density (PSD) of y, and P_{xy} stands for Cross Spectral Density (CSD) of x and y. Let us assume y being a signal, a stochastic variable, which is estimated or measured by x. In this case y is the input signal and x is the output. A perfect estimation of y would imply an identity of the two signals, therefore tf would be 1 over the whole spectrum.

If x contains noise at a frequency f_n, then the noise generates a value different from 1 value at f_n in tf. Nota bene: Assuming y being the noise-free signal, values larger than 1 in the transfer function show the noise of x.

The temporal resolution of the estimated/observed stochastic variable is determined by the sampling interval, according to the Nyquist rule. The frequency where the signal starts to vanish significantly is the so-called 'cut-off frequency'. Nota bene: Assuming y containing the full signal of a stochastic variable over the whole spectrum, the cut-off frequency of x can be detected where the transfer function starts to drop from 1 consistently (but not monotonously).

If the kinematic velocity is the estimate x, then a possible choice for the input signal y, is the reduced-dynamic velocity. The reduced-dynamic velocity is a good approximation of the noise-free velocity, since the reduced-dynamic velocities are known to be smoother than the kinematic ones (cf. Section 1). The other criterion of an y, to have power over the whole spectrum, can not be met.

4 Tests with Transfer Functions

4.1 Kinematic/(reduced-)dynamic Transfer Functions

In Figs. 4-7 we have transfer functions between kinematic velocities and the related reference velocities, i.e. those velocities, which were added in the restore step. These are the EIGEN-1S (kinematic/dynamic), the EIGEN-2 (kinematic/reduced-dynamic) and the TEG-4 (kinematic/dynamic) cases. Also the no-reference-orbit-used kinematic velocity is shown, compared to the reduced-dynamic velocity (based on EIGEN-2). In Figs 4-7 the upper frames show the transfer function, and the lower frames show the phase difference between the transfers. Where the phase of the two time series, x and y, differ, the two signals get independent of each other. In Figs. 4-7 the highest-frequencies show two uncorrelated estimates of the velocity, which means dominance of noises and/or lack of signal.

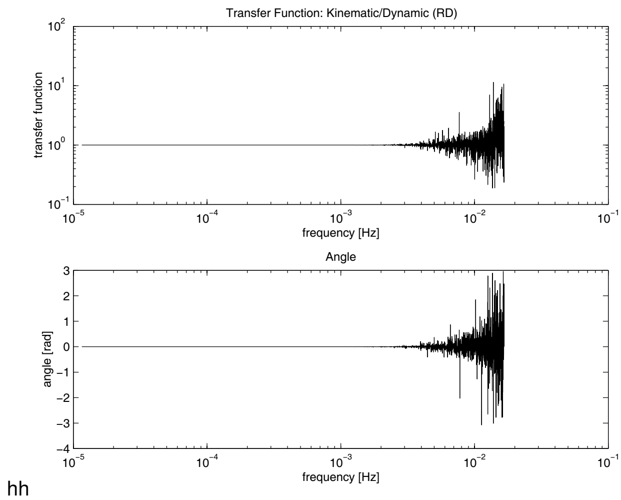

Fig. 4. Kinematic versus dynamic velocity transfer functions for EIGEN-2 reduced-dynamic orbit.

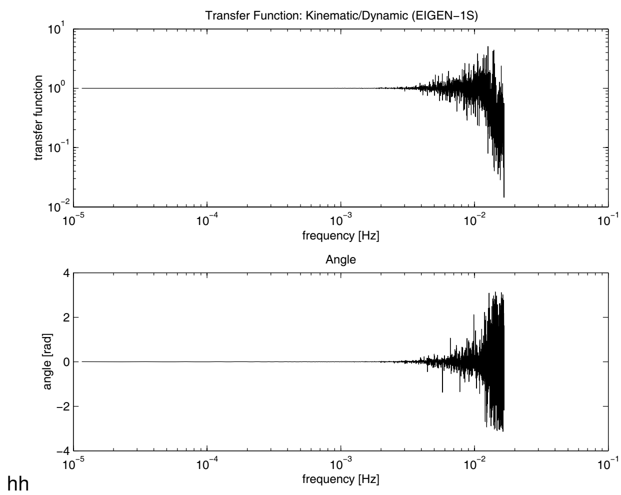

Fig. 5. Kinematic versus dynamic velocity transfer functions for EIGEN-1S dynamic orbit.

Fig. 7 shows that the no-reference-orbit kinematic velocity provides information on the high-frequencies, while the reduced-dynamic velocity does not. (It turns out as noise in the transfer function. Considering the inverse procedure, from the point of view of the reduced-dynamic velocities: determining the reduced-dynamic/kinematic (no-reference) transfer function one gets the inverse of the curve above. That would define a cut-off frequency somewhere at 0.001-0.002 Hz, what means, that no signal of reduced-dynamic velocity is available over that frequency.)

The other three transfer functions in Figs. 4-6 show similar

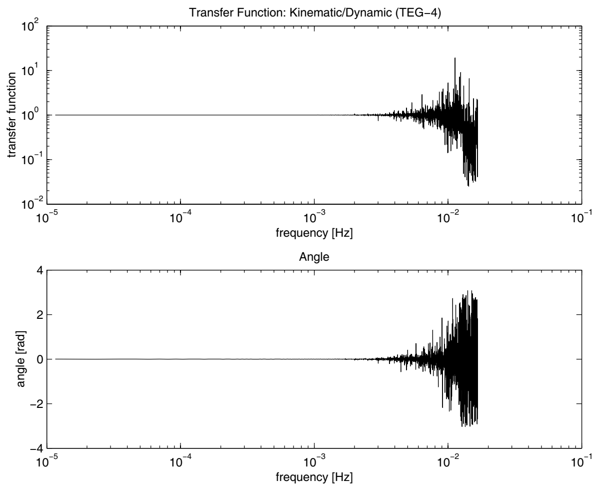

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Fig. 6. Kinematic versus dynamic velocity transfer functions for TEG-4 dynamic orbit.

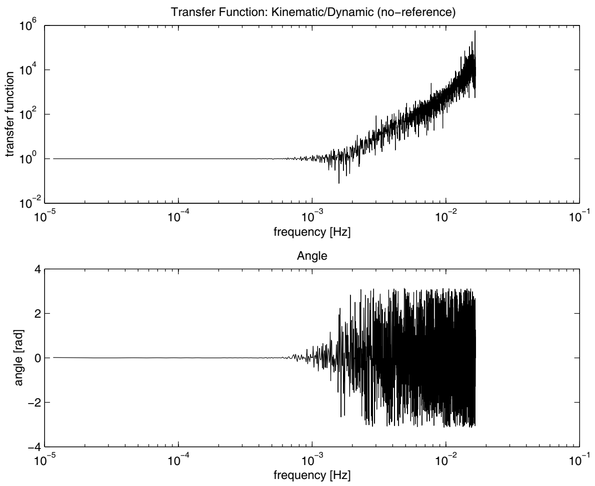

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Fig. 7. Kinematic versus dynamic velocity transfer functions for no-reference-orbit-used kinematic / EIGEN-2 reduced-dynamic orbit.

characteristics: a cut-off frequency can be found somewhere at $0.008 - 0.012$ Hz, 80-120 s, though it is not well-determined. Especially in Fig. 4, the reduced-dynamic EIGEN-2 case (note: the cut-off frequency of kinematic/reduced-dynamic transfers has already been found to be uncertain (see above in this section) due to the non-stochastic characteristics of the reduced-dynamic data). At this frequency the data becomes considerably noisy too; in the case of Fig. 4, this effect appears at higher frequencies. The figure shows that no relevant information below $80 - 120s$ temporal resolution is contained in the kinematic velocity signal. The size of signal drop is smallest in the reduced-dynamic EIGEN-2 case, however we should remember that in this case the position residuals are also much smaller than in the case of the dynamic models due to the stochastic pulses of the reduced-dynamic POD. Figs. 4-7 show that smoothing on smaller position residuals can be done with less loss of high-frequency information.

4.2 Dynamic/dynamic Transfer Functions

Can the spectral characteristic of the reference orbit be identified in the kinematic velocity? In Figs. 8-10 dynamic models are compared with each other. The cut-off frequency and the noises seem to be characteristically close to the dynamic/kinematic transfer functions (cf. Figs. 4-6). Cut-off frequencies in Fig. 8 and Fig. 10 show that the dynamic EIGEN-1S velocities contain more signal at high-frequencies than the dynamic TEG-4 and the reduced-dynamic EIGEN-2 orbits. The different (reduced-)dynamic models and their effect on the kinematic velocities case by case are discussed below.

Reduced-dynamic EIGEN-2 velocity: the signal shows a drastic drop at about 0.012 Hz frequency (Fig. 8). The kinematic velocity using this model for reference shows no definitive drop (Fig. 4).

Dynamic EIGEN-1S velocity: contains information at highest frequencies as well (cf. Fig. 8 and Fig. 10). The kinematic velocity using this model for reference drops considerably at 0.01 Hz (Fig. 5).

Dynamic TEG-4 velocity: no relevant information above 0.012 Hz (cf. Fig. 10). The kinematic velocity using this model for reference shows a drop at a little bit higher frequency (Fig. 6).

Summary: The three different (reduced-)dynamic orbits show different high-frequency information content (Figs. 8-10), but the loss of high-frequency kinematic velocities seems to occur at the same frequency in all cases (Figs. 4-6). Especially the dynamic EIGEN-1S and TEG-4 cases demonstrate that the additional high-frequency content of the dynamic orbit (cf. Fig. 10) does not generate artificial high-frequency signal on the kinematic velocity (cf. Fig. 6), but shows a drop very similar to the EIGEN-1S case (Fig. 4).

Conclusion: (1) no high-frequency information of the reference orbit leaks into the kinematic velocity; (2) the numerical derivation results in an information loss above 0.012 Hz, that is 80 s. For CHAMP it corresponds to 600 km in the orbit, which corresponds to about degree 70 in spherical harmonic sense.

4.3 Kinematic/kinematic Transfer Functions

Transfer functions between the kinematic orbits are shown in Figs. 11-14. In these figures no cut-off frequencies can be defined. The noise dominates above 0.012 Hz, obviously in particular in the phase differences. This suggests spectral consistency of kinematic velocities derived by different reference orbits up to 0.012 Hz.

5 Discussion and Summary

What is the amplitude of kinematic velocities above 0.012 Hz? The amplitude differences can be seen in Fig. 15. We show the PSD of the kinematic velocity (reference orbit: EIGEN-2 reduced-dynamic), and the PSD of velocity residuals of the other kinematic orbits compared to this orbit (i.e. residual = kinematic

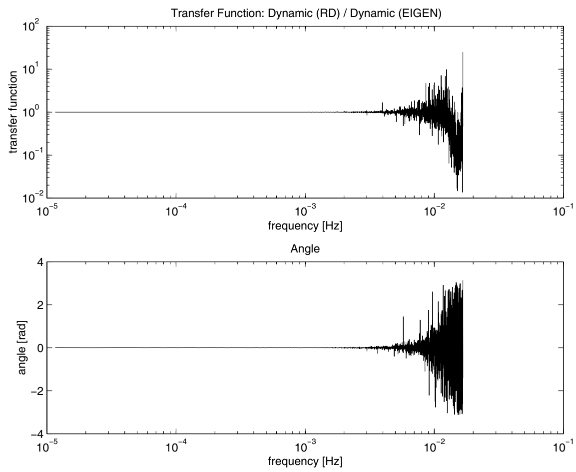

Fig. 8. Dynamic/dynamic velocity transfer functions for EIGEN-2 reduced-dynamic orbit / EIGEN-1S dynamic orbit.

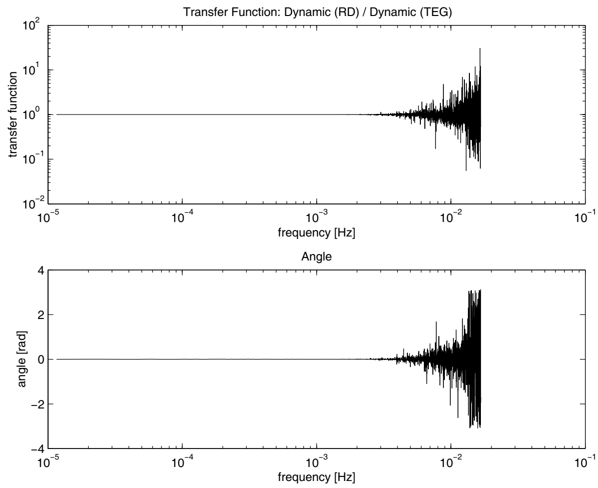

Fig. 9. Dynamic/dynamic velocity transfer functions for EIGEN-2 reduced-dynamic orbit / TEG-4 dynamic orbit.

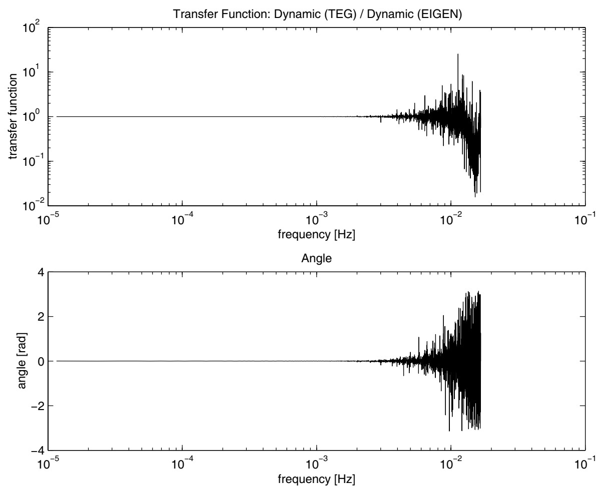

Fig. 10. Dynamic/dynamic velocity transfer functions for TEG-4 dynamic orbit / EIGEN-1S dynamic orbit.

orbit - kinematic based on reduced-dynamic EIGEN-2). The figure shows that the noise gets close to the signal at 0.009 Hz, and reaches it close to 0.012 Hz (80 s). This region we have some

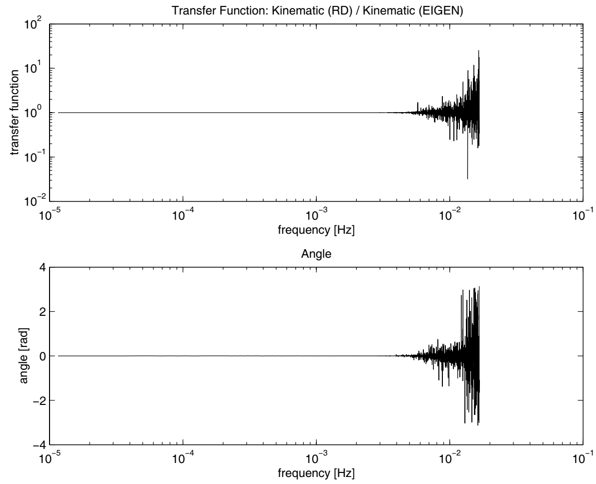

Fig. 11. Kinematic/kinematic velocity transfer functions based on EIGEN-2 reduced-dynamic orbit / EIGEN-1S dynamic orbit.

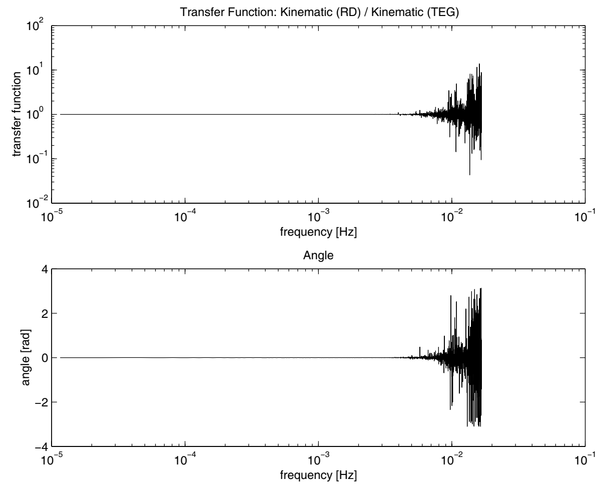

Fig. 12. Kinematic/kinematic velocity transfer functions based on EIGEN-2 reduced-dynamic orbit / TEG-4 dynamic orbit.

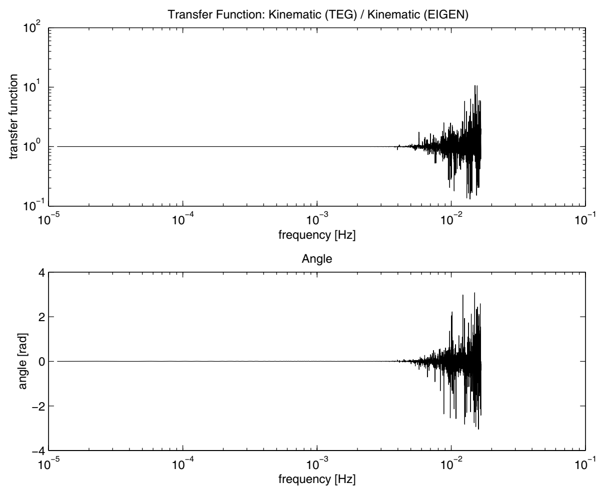

Fig. 13. Kinematic/kinematic velocity transfer functions based on TEG-4 dynamic orbit / EIGEN-1S dynamic orbit.

10 to the power −3 $mm^2/s^2/Hz$ signal, which is at this frequency equivalent to 0.003-0.01 mm/s. This is consistent in magnitude with the RMS differences of the kinematic velocities in Table 7

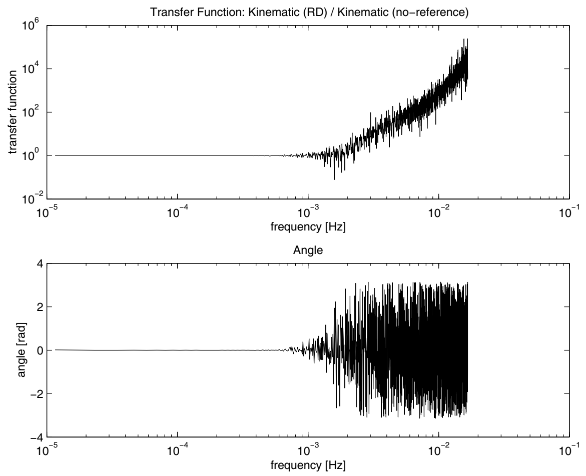

Fig. 14. Kinematic/kinematic velocity transfer functions based on EIGEN-2 reduced-dynamic orbit / no-reference-orbit.

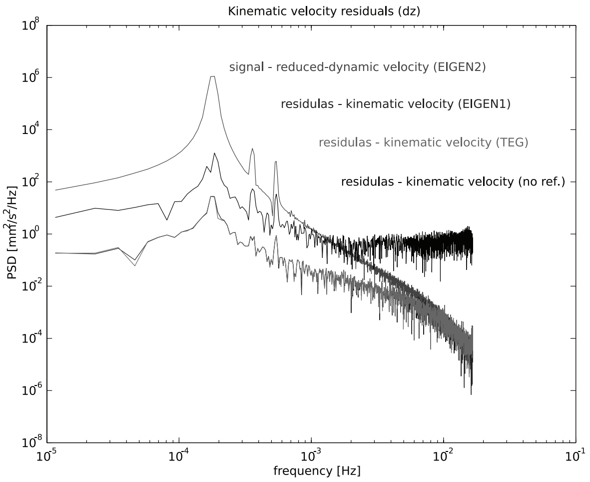

Fig. 15. PSD of kinematic velocity residuals. (Abscissa: frequency [Hz], ordinate: PSD [mm^2/s^2/Hz]).

of [4].

With the spectral analyses we have provided an empirical estimate for the cut-off frequency of CHAMP velocities by comparison of velocity sets based on slightly different origin. Since a crucial cut-off frequency has been found at 0.012 Hz, it should suggest information on the resolution of the CHAMP orbit (i.e. this estimate refers not only to the velocity, but also implicitly to the position itself). This cut-off frequency is equivalent to 80 s, which for the CHAMP means a 600 km run on the orbit. Even though the spatial resolution of an observed gravity field cannot be assigned to the spectral characteristics of the observing satellite's orbit, these two are definitely tied. A 600 km spatial resolution roughly corresponds to a degree of 70 in spherical harmonic sense. According to that, in this study we found that our use of smoothing splines is messing with gravity information (both noise and signal) over degree 70.

What data loss is expected at this degree? An important experience on CHAMP gravity fields is that no useful signal over about degree 70 is available [2, 3, 5, 8]. It is in accordance with our result. Therefore we can conclude that smoothing of CHAMP kinematic positions with the parametrization used in this study does not affect negatively the gravity signal. So it was applied for the TUM-1S CHAMP-only gravity model [5].

Finally its applicability for the CHAMP should be discussed. There has been two gravity field solutions performed in similar manner except for the determination of the kinematic velocity. The other solution used a simple interpolation technique, the Newton-Gregory interpolation, which provided the TUM-2Sp model [3]. Finally the latter solution was found to be better [3]. It just means that in case of the CHAMP the kinematic orbit noise was quite random, no need of smoothing of systematic noise was found. Therefore leaving the noise do not worsens the solution for the CHAMP. However, in the future the smoothing can be applied for other satellites with unknown orbital error characteristics, such as the GOCE, or even GRACE.

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