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RESEARCH ARTICLE

Optimal design of frame structures with semi-rigid joints

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Abstract

In this paper, a genetic algorithm is proposed for discrete minimal weight design of steel planar frames with semi-rigid beam-to-column connections. The frame elements are constructed from a predetermined range of section profiles. Conventionally, the analysis of frame structures is based on the assumption that all connections are either frictionless pinned or fully rigid. Recent limit state specifications permit the concept of semi-rigid connection of the individual frame members in the structural design. In a frame with semi-rigid joints the loading will create both a bending moment and a relative rotation between the connected members. The moment and relative rotation are related through a constitutive law which depends on the joint properties. The effect, at the global analysis stage, of having semi-rigid joints instead of rigid or pinned joints will be that not only the displacements but also the distribution of the internal forces in the structure must be modified. In this study, a simplified beam-to-column connection is presented which was specified in EC3 Annex J. In order to capture the changes in the nodal force and moment distribution in terms of joint flexibility, the ANSYS finite element analysis is applied. The structural model is formulated as a combination of 3D quadratic beam elements and linear torsional springs. Present work deals with the effects of joint flexibility to the optimal design problem. The design variables – including joint properties – are discrete. Results are presented for sway frames under different load conditions.

Keywords

discrete optimization · frames · semi-rigid · genetic algorithm

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1 Introduction

It is well known that real beam-to-column connections possess some stiffness, which falls between the extreme cases of fully rigid and ideally pinned. In the engineering practice, the traditional approaches to the design of frames are concisely described as continuous framing with rigid joints and/or simple framing with pinned joints. However, the connection behaviour significantly affects the displacements and internal force distribution of framed structures.

There is a large amount of work dealing with the effect of semi-rigid joints on the optimal design of frame structures. Fully analytical and numerical solutions as well have demonstrated that in actual framed structures, pinned connections possess a certain amount of stiffness, while rigid connections possess some degree of flexibility [1, 6, 7, 10, 11]. Recently, the European Code (EC 3) for design of steel structures [4, 5] has adopted semi-rigid steel framing construction. The proposed approach to frame design, i.e. semi-continuous framing using semi-rigid joints, is then outlined; how it is to be distinguished from the traditional approaches is explained and the potential benefits (scientific and economic) for its use are raised.

It is now well recognized that assuming joints to be rigid or pinned may neither be accurate nor result be economical. Simply the fast that a joint has sufficient strength does not mean that it has sufficient stiffness to be reasonable to be modelled as rigid. Many joints, often assumed to be rigid exhibit an intermediate behaviour between the "rigid" and "pinned" states. Eurocode 3 Part 1-1 has taken this fact into account and in doing so opened the way to what is now known as "the semi-rigid approach".

In the semi-rigid approach, the behaviour of the joints is taken into account at the outset, i.e. when the components are sized at the preliminary design range, and the sizing takes account of the joint behaviour as well. The initial global analysis includes an approximate estimate of the joint characteristics (stiffness, strength and rotation capacity), and which can be refined later, as one does for the member sizes, in the final analysis. The joint is usually represented as a rotational spring at the extremity of the member (usually the beam) which characterizes the joint behaviour. Available models can represent the moment-rotation characteristic only, which is sufficient for the majority of structural joints in frames (see in EC3 Annex J.).

The aim of this study is to determine the effects of semi-rigid connection in optimal design of frame structures. The design variables are the member sections where column and beam members are distinguished. The properties of the connection spring will be changed as well during the process in a predetermined range of spring rotational stiffness.

In this study, a genetic algorithm method is applied for discrete minimal weight design of steel structures with semi-rigid connection.

Recently GA methods are very popular and have been used for sizing, shape, and topology optimization of structures. The GA methods are search algorithms that are based on the concepts of natural selection and natural genetics. The core characteristics of GAs are based on the principles of survival of the fittest and adaptation. The GA methods operate on population of set of design variables. Each design variable set defining a potential solution is called a string. Each string is made up of series of characters as binary numbers, representing the discrete variables for a particular solution. The fitness of each string is a measurement of performance of design variables defined by the objective function and constraints. GA methods consist of a series of three processes: coding and decoding design variables into strings, evaluating the fitness of each solution strings, and applying genetic operators to generate the next generation of solution strings. Most GA methods are variation of the simple GA proposed by Goldberg and Samtani [3], which consists of three basic genetic operators: reproduction, crossover, and mutation. By varying these parameters, the convergence of the problem may be altered. Much attention has been focused on finding the theoretical relationship among these parameters. Rajeev and Krishnamoorty [9] applied GA for optimal truss design and transmission tower. They presented all the computations for three successive generations. In a previous work of the first author [2] applied a GA for discrete minimal weight design problem of space trusses with plastic collapse constraints.

Hayalioglu and Degertekin [6] presented a genetic algorithm for optimum design of non-linear steel frames with semi-rigid connections subjected to displacement and stress constraints of AISC-ASD specifications. The authors [6] concluded that more economical frames can be obtained by adjusting the stiffness of the connections.

This study presents a discrete optimal design problem for steel frames with semi-rigid connection based on the recommendation of EC 3 while European cross sections are selected for frame members.

2 The Discrete Optimization Problem

Recently, several works have attended to optimal design of steel frames with semi-rigid connections. Here we will refer to some of the results e.g. papers of Hayalioglu and Degertekin [6], Jármai and Farkas [7], Xu and Grierson [10], and Xu [11]. The total cost is defined by Xu and Grierson that includes the structural cost and the connection cost as well. In this study, contrary to the papers mentioned above, the objective function will be the least weight of the structure because the total cost strongly depends on the actual price of raw materials and the actual cost of manufacturing.

2.1 Semi-rigid Frame Analysis

In general, there are two different ways to incorporate connection flexibility into computer-based frame analysis.

In this paper, the idea of Xu 12 will be adopted where the maximum bending moment of semi-rigid beams under an applied member load has been considered for the variation of the rotational stiffnesses of end connections. The minimum value of the maximum moments which can be achieved by adjusting connection stiffness was presented and proved. He demonstrated that the cross-sectional member sizes based on this minimum value of the maximum moment will correspond to the least-weight solution for any values of connection stiffness.

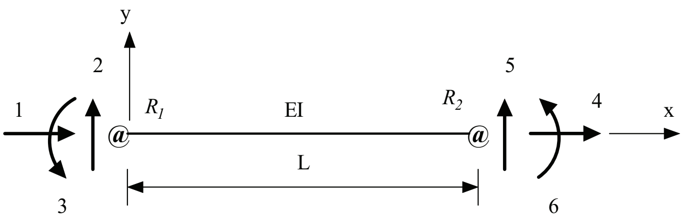

Fig. 1. Semi-rigid member

The end-fixity factor r_q defines the stiffness of the beam-to-column connection in terms of the beam moment of inertia:

$$r_q = \frac{1}{1 + \frac{3EI_z}{S_q L}}, \ldots\ldots.(q = 1, 2) \quad (1)$$

where S_q is the end-connection spring stiffness, and EI_z/L is the flexural stiffness of the attached member. For pinned connections, the rotational stiffness of the connection tends to zero and the value of the end-fixity factor is equal to zero as well. For rigid connections, the end-fixity factor is equal to ($r_q = 1$), and in case of a more realistic design, the semi-rigid connection results in a value between 1 and zero. The elastic stiffness matrix of a member i with two semi-rigid end-connections having stiffness modulus S_q ($q = 1, 2$) can be represented by the following stiffness matrix which is modified by a semi-rigid correction matrix:

$$\mathbf{K}_i = \mathbf{K}_{Si} + \mathbf{K}_{Ci} \quad (2)$$

where $\mathbf{K}_i$ is the stiffness matrix of member i with semi-rigid end-connections. The matrices $\mathbf{K}_{Si}$ and $\mathbf{K}_{Ci}$ have the following forms:

$$\mathbf{K}_{Si} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & SYM & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \quad (3)$$

$$\mathbf{K}_{Ci} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4r_2-2r_1+r_1r_2}{4-r_1r_2} & \frac{2Lr_1(1-r_2)}{4-r_1r_2} & 0 & 0 & 0 \\ 0 & \frac{6(r_1-r_2)}{L(4-r_1r_2)} & \frac{3r_1(2-r_2)}{4-r_1r_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4r_1-2r_2+r_1r_2}{4-r_1r_2} & \frac{2Lr_2(1-r_1)}{4-r_1r_2} \\ 0 & 0 & 0 & 0 & \frac{6(r_1-r_2)}{L(4-r_1r_2)} & \frac{3r_2(2-r_1)}{4-r_1r_2} \end{bmatrix} \quad (4)$$

where E is Young's modulus, and L, A, I are the length, cross-sectional area, and moment of inertia of the member, respectively. The end-fixity factors r_1 and r_2 are defined by Eq. 2.

The semi-rigid frames are more flexible than rigid steel frames. Therefore, in this study a stability analysis is required. The structural design constraints defined in the following subsections are extended by a structural stability analysis as well.

2.2 Definition of the Discrete Design Problem

The least weight design problem of frame structures with semi-rigid connections, considering only flexural behaviour, under applied loads can be defined as a discrete optimization problem in terms of the member sections, A_i and in terms of the rotational stiffnesses of end connections, S_q. The design variables A_i are selected from a discrete set of the predetermined $A_i \in B = \{B^1, B^2, ..., B^N\}$ cross-sectional areas of column elements, $A_j \in C\{C^1, C^2, ..., C^N\}$ cross-sectional areas of beam elements such that minimize the total weight, while S_q rotational stiffnesses of end connections are changing in between a given equidistance range of $S_q \in S = \{S^1, S^2, ..., S^E\}$ values.

The objective function is

$$W(A_i, A_j) \rightarrow \text{min!}, \quad (5)$$

$$i = 1, 2, ..., n \quad j = 1, 2, ..., m$$

where n is the number of column and m is the number of beam elements, q is the number of joints, N is the number of cross sectional catalogue values for columns, M is the number of cross sectional catalogue values for beam elements, and E is the number of rotational stiffness value series.

The discrete minimal weight design is subjected to size, displacement, and stress constraints. In order to satisfy the design constraints listed above, we have to determine the displacements and internal force distribution of the framed structure in terms of member cross sections and connection stiffness of joint springs. The structural model and related formulas are concerned in several papers. The detailed description of the theoretical background could be found in book of Chan and Chui [1].

In this study, for structural analysis, the ANSYS Release 9.0 finite element program is applied. The structural model is formulated as a combination of 3D quadratic beam elements and linear torsional springs. The frame is defined in x and y plane. Therefore, u_x and u_y displacements, θ_z rotation, F_x and $F_y$member forces, and M_z bending moment will be considered in the 3D coordinate system.

2.3 Displacement Constraints

The displacement constraints are

$$u_k = \bar{u}_k < 0, \quad k = 1, 2, ..., p \quad (6)$$

where u_k is the actual displacement value of the beam or column elements, $\bar{u}_k$ is its upper bound and p is the number of restricted displacements.

2.4 Bending and Axial Tension Constraints of the Columns and Beams

Constraints for normal stresses are computed from the maximal value of bending moments and from the related normal forces or from the maximal value of axial forces and related bending moments.

$$\frac{N}{f_y A} + \frac{M_z}{f_y W_z} \leq 1, \quad (7)$$

where N is the actual axial force of the beam (F_x) or column (F_y) elements, M_z is the bending moment, and f_y is the yield stress, modified by the partial safety factor.

2.5 Bending and Axial Compression Constraints of the Columns and Beams

The frames are defined in the x, y, and z global co-ordinate system where z is the bending axis. The frame members are loaded by bending and axial forces. Therefore, the overall flexural and torsional buckling constraints are formulated according to Eurocode 3. We have to satisfy the following buckling constraints about the axis z:

$$\frac{N}{\chi_z f_y A} + k_z \frac{M_z}{\chi_{LT} f_y W_z} \leq 1, \quad (8)$$

where χ_z is the overall buckling factor for the axis z, χ_T is the lateral-torsional buckling factor, k_z is a modification factor in terms of the axial force effect.

The overall buckling factor χ_z for the axis z is

$$\chi_z = \frac{1}{\phi_z + \sqrt{\phi_z^2 - \lambda_z^2}}, \quad (9)$$

where

$$\phi_z = 0.5\left[1 + \alpha_z\left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^2\right], \quad (10)$$

$$\alpha_z = \begin{cases} 0.21 & h_1/b_1 > 1.2 \\ & \text{if} \\ 0.34 & h_1/b_1 \leq 1.2 \end{cases} \quad (11)$$

The slenderness ratio of the column is

$$\bar{\lambda}_z = \frac{2H}{r_z \lambda_E},. \quad (12)$$

and the slenderness ratio of the beam is

$$\bar{\lambda}_z = \frac{1.3L}{r_z \lambda_E}. \quad (13)$$

where

$$\lambda_E = \pi \sqrt{\frac{E}{f}} \ldots \quad r_z = \sqrt{\frac{I_z}{A}}. \tag{14}$$

The lateral-torsional buckling factor χ_T is

$$\chi_T = \frac{1}{\phi_T + \sqrt{\phi_T^2 - \overline{\lambda}_T^2}}, \tag{15}$$

where

$$\phi_T = 0.5\left[1 + \alpha_T\left(\overline{\lambda}_T - 0.2\right) + \overline{\lambda}_T^2\right], \tag{16}$$

and

$$\alpha_T = \begin{cases} 0.49 & h_1/b_1 > 2 \\ & if \\ 0.34 & h_1/b_1 \leq 2 \end{cases} \tag{17}$$

The relative lateral-torsional factor is computed from the following formula:

$$\overline{\lambda}_T = \sqrt{\frac{W_z f}{M_{cr}}}, \tag{18}$$

where M_{cr} in case of columns is replaced by

$$M_{cr} = 11.132\pi^2 E \frac{I_x}{H} \sqrt{\frac{I_\omega}{I_x} + \frac{H^2 G I_t}{\pi^2 E I_x}}, \tag{19}$$

and in case of beams by

$$M_{cr} = 11.132\pi^2 E \frac{I_y}{L} \sqrt{\frac{I_\omega}{I_y} + \frac{L^2 G I_t}{\pi^2 E I_y}}. \tag{20}$$

The k_z factor is computed from the following formula replaced by the above defined variables:

$$k_z = 0.9\left[1 + 0.6\overline{\lambda}_z \frac{N}{\chi_z f A}\right]. \tag{21}$$

The buckling constraints about the x axis for the column and about the y axis for the beam elements are as follows:

$$\frac{N}{\chi_n f_y A} \leq 1, \tag{22}$$

where N is the actual axial force of the beam (F_x) or column (F_y) elements,χ_n is the overall buckling factor related to the x axis for the column and about the y axis for the beam elements.

The overall buckling factor χ_n for the axis $n = x$ of beam elements and $n = y$ for the column elements is

$$\chi_n = \frac{1}{\phi_n + \sqrt{\phi_n^2 - \lambda_n^2}}, \tag{23}$$

where

$$\phi_n = 0.5\left[1 + \alpha_n\left(\overline{\lambda}_n - 0.2\right) + \overline{\lambda}_n^2\right], \tag{24}$$

$$\alpha_n = \begin{cases} 0.21 & h_1/b_1 > 1.2 \\ & if \\ 049 & h_1/b_1 \leq 1.2 \end{cases} \tag{25}$$

The slenderness ratio of the column is

$$\overline{\lambda}_y = \frac{2H}{r_y \lambda_E}, \tag{26}$$

and the slenderness ratio of the beam is

$$\overline{\lambda}_x = \frac{1,3L}{r_x \lambda_E}. \tag{27}$$

3 The Optimization Procedure

3.1 The Applied Genetic Algorithm

The genetic algorithm (GA) is an efficient and widely applied global search procedure based on a stochastic approach. All of the recently applied genetic algorithms for structural optimization have demonstrated that genetic algorithms can be powerful design tools (see e.g. [2, 3, 8], and [9]).

The crossover operation creates variations in the solution population by producing new solution strings that consist of parts taken from selected parent solution strings. The mutation operation introduces random changes in the solution population. In GA, the mutation operation can be beneficial in reintroducing diversity in a population. In this study, a pair of parent solutions is randomly selected, with a higher probability of selection being ascribed to superior solutions.

The two parents are combined using a crossover scheme that attempts to merge the strings representing them in a suitable fashion to produce an offspring solution. Offspring can also be modified by some random mutation perturbation. The algorithm selects the fittest solution of the current solution set, i.e. those with the best objective function values. Each pair of strings reproduces two new strings using a crossover process and then dies.

3.2 The Steps of the Applied Algorithm

```
Generations = 500
PopulationSize = 500
SwapProbability = 0.1
MutationProbability = 0.1
CrossoverProbability = 0.5
Call ProblemDefinition
For Agent = 1 to PopulationSize
    Call RandomAgentGeneration (Agent)
    Call PathFollowingMethod
    Call BestFeasibleSolutionUpdate
Next Agent
For Generation = 1 to Generations
    Call PopulationOrderingByFitness (PopulationSize)
    Call FittestParentPairSelection (CrossoverProbability)
    Call Crossover (SwapProbability)
    For Each Child: Call Mutation (MutationProbability)
                    Call PathFollowingMethod
                    Call BestFeasibleSolutionUpdate
Next Generation
```

4 Numerical Examples

The effects of semi-rigid connections are observed to the optimal design of steel frames. Two examples of planar frames are studied here. In this paper, a simple-bay frame (shown in Fig. 2) and a two-bay frame were considered where the objective function is the minimal weight (volume) of the structure subjected to the sizing, displacement, and stress constraints including the member buckling as well. The design variables are discrete variables of the cross section of beam and column members. According to the structural symmetry requirements, symmetrical members are grouped into the same variables.

Tab. 1. *Catalogue values of beam section types*

Section type	h [cm]	b [cm]	t_w [cm]	t_f [cm]	A [cm2]	I_t [cm4]	I_z [cm4]	I_y [cm4]	I_ω [cm6]
IPE 80	8	4.6	0.4	0.5	7.64	0.7	80.1	8.5	119
IPE 100	10	5.5	0.4	0.6	10.32	1.2	171	15.9	354
IPE 120	12	6.4	0.4	0.6	13.21	1.7	317.8	27.7	894
IPE 140	14	7.3	0.5	0.7	16.43	2.5	541.2	44.9	1989
IPE 160	16	8.2	0.5	0.7	20.09	3.6	869.3	68.3	3977
IPE 180	18	9.1	0.5	0.8	23.95	4.8	1317	100.9	7459
IPE 200	20	10.0	0.6	0.9	28.48	7.0	1943.2	142.4	13053
IPE 220	22	11.0	0.6	0.9	33.37	9.1	2771.8	204.9	22762
IPE 240	24	12.0	0.6	1.0	39.12	12.9	3891.6	283.6	37575
IPE 270	27	13.5	0.7	1.0	45.95	15.9	5789.8	419.9	70849
IPE 300	30	15.0	0.7	1.1	53.81	20.1	8356.1	603.8	126333
IPE 330	33	16.0	0.8	1.1	62.61	28.1	11770	788.1	199877
IPE 360	36	17.0	0.8	1.3	72.73	37.3	16270	1043.5	314645
IPE 400	40	18.0	0.9	1.3	84.46	51.1	23130	1317.8	492147
IPE 450	45	19.0	0.9	1.5	98.82	66.9	33740	1675.9	794245
IPE 500	50	20.0	1.0	1.6	115.52	89.3	48200	2141.7	125425
IPE 550	55	21.0	1.1	1.7	134.42	123.2	67120	2667.6	189315
IPE 600	60	22.0	1.2	1.9	155.98	165.4	92080	3387.3	285858

The applied material is given according to the European Standard prEN (Fe E 510) steel with a modulus of elasticity of 210 000 MPa and a yield stress of 355 MPa. The Poisson factor is 0.3, and the material density is 7850 kg/m3. The cross sections are selected from the European section profiles. In the presented example the beam and column profiles are distinguished, and the cross sections have been selected from the catalogue of Table 1, and Table 2. The applied loads are $p = 5$ kN/m, and $P = 50$ kN, according to the Fig. 2.

In this study, for structural analysis and for the optimal design problem, the ANSYS Release 9.0 finite element program was applied. The structural model is formulated as a combination of 3D quadratic beam elements and linear torsional springs. The frame is defined in x, and y plane. The design constraints are formulated in 3D coordinate system using formulas (6)-(27).

Beam-to-column connections are varying from ideally-pinned to fully-rigid behaviour. The changes of the rotational stiffness of beam-to-column connections play a relevant role in the optimal design problem while the structural response is changing as well. In order to expose this effect to the optimal design, the connection stiffness ratio ($S_q L/EI_z$) related to the beam element and the end-fixity factor is applied which was introduced and defined by Xu [11, 12] first time. The end-fixity factors r_1 and r_2 are defined by Eq. (1).

For pinned connections, the rotational stiffness of the connection tends to zero and the value of the end-fixity factor is equal to zero as well. For rigid connections, the end-fixity factor is equal to ($r_q = 1$), and in case of a more realistic design, the semi-rigid connection results in a value between 1 and

Tab. 2. *Catalogue values of column section types*

Section type	h [cm]	b [cm]	t_w [cm]	t_f [cm]	A [cm2]	I_t [cm4]	I_z [cm4]	I_y [cm4]	I_ω [cm6]
HE120 A	11.4	12.0	0.5	0.8	25.34	6.0	606.2	230.9	6486
HE120 AA	10.9	12.0	0.4	0.5	18.55	2.8	413.4	158.8	4253
HE120 B	12.0	12.0	0.6	1.1	34.01	13.8	864.4	317.5	9431
HE120 M	14.0	12.6	1.3	2.1	66.41	91.7	2017.6	702.8	24880
HE140 A	13.3	14.0	0.5	0.9	31.42	8.1	1033.1	389.3	15086
HE140 AA	12.8	14.0	0.4	0.6	23.02	3.5	719.5	274.8	10226
HE140 B	14.0	14.0	0.7	1.2	42.96	20.1	1509.2	549.7	22514
HE140 M	16.0	14.6	1.3	2.2	80.56	120.0	3291.4	1144.3	54482
HE160 A	15.2	16.0	0.6	0.9	38.77	12.2	1673	615.6	31469
HE160 AA	14.8	16.0	0.4	0.7	30.36	6.3	1282.9	478.7	23794
HE160 B	16.0	16.0	0.8	1.3	54.25	31.2	2492	889.2	48038
HE160 M	18.0	16.6	1.4	2.3	97.05	162.4	5098.3	1758.8	108380
HE180 A	17.1	18.0	0.6	1.0	45.25	14.8	2510.3	924.6	60289
HE180 AA	16.7	18.0	0.5	0.8	36.53	8.3	1966.9	730.0	46427
HE180 B	18.0	18.0	0.9	1.4	65.25	42.2	3831.1	1362.8	93887
HE180 M	20.0	18.6	1.5	2.4	113.25	203.3	7483.1	2580.1	199805
HE200 A	19.0	20.0	0.6	1.0	53.83	21.0	3692.2	1335.5	108176
HE200 AA	18.6	20.0	0.5	0.8	44.13	12.7	2944.3	1068.5	84635
HE200 B	20.0	20.0	0.9	1.5	78.08	59.3	5696.2	2003.4	171413
HE200 M	22.0	20.6	1.5	2.5	131.28	259.4	10640	3651.2	347093
HE220 A	21.0	22.0	0.7	1.1	64.34	28.5	5409.7	1954.6	193506
HE220 AA	20.5	22.0	0.6	0.9	51.46	15.9	4170.2	1510.5	145809
HE220 B	22.0	22.0	1.0	1.6	91.04	76.6	8091	2843.3	295813
HE220 M	24.0	22.6	1.6	2.6	149.44	315.3	14600	5012.1	573830
HE240 A	23.0	24.0	0.8	1.2	76.84	41.6	7763.2	2768.8	328962
HE240 AA	22.4	24.0	0.6	0.9	60.38	23.0	5835.2	2077.0	240028
HE240 B	24.0	24.0	1.0	1.7	105.99	102.7	11260	3922.7	487675
HE240 M	27.0	24.8	1.8	3.2	199.59	627.9	24290	8152.6	1154493
HE260 A	25.0	26.0	0.8	1.3	86.82	52.4	10450	3667.6	517183
HE260 AA	24.4	26.0	0.6	1.0	68.97	30.3	7980.6	2788.0	383288
HE260 B	26.0	26.0	1.0	1.8	118.44	123.8	14920	5134.5	754853
HE260 M	29.0	26.8	1.8	3.3	219.64	719.0	31310	10450	1732251
HE280 A	27.0	28.0	0.8	1.3	97.26	62.1	13670	4762.6	786419
HE280 AA	26.4	28.0	0.7	1.0	78.02	36.2	10560	3664.2	591005
HE280 B	28.0	28.0	1.1	1.8	131.36	143.7	19270	6594.5	1131686
HE280 M	31.0	28.8	1.9	3.3	240.16	807.3	39550	13160.0	2524384

zero. In this examples, the rotational stiffnesses of end connections are changing in between a given equidistance range of $S_q \in S = \{1E4; 5E4; 1E5; 5E5; 1E6; 5E6; 1E7; 5E7\}$ values.

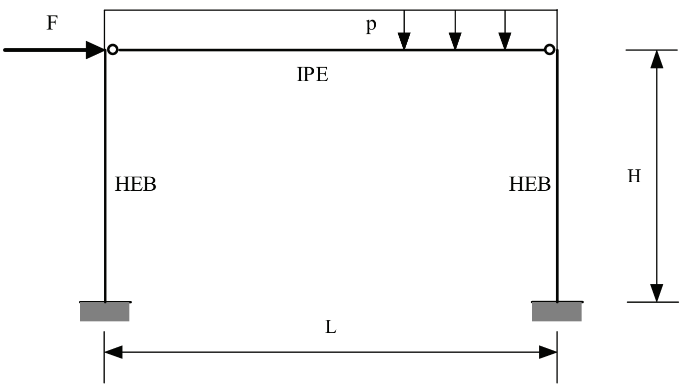

Fig. 2. Semi rigid single-bay frame

Tab. 3. *Results of the single-bay frame under symmetric loading.* (*Note: WZB1 and WZC1 – section modulus of the beam and column of optimal solution, TVOL – the total volume of the optimal solution, MAXMZB – the maximal bending moment of the beam element, MAXROTZ – the maximal rotation.)

r_q	0.00955	0.04599	0.08794	0.42209	0.59362	0.92146	0.95912	0.99155
S_q	10000	50000	1.0E+05	5.0E+05	1.0E+06	5.0E+06	1.0E+07	5.0E+07
WZB1∗	1.46E-04	1.46E-04	1.46E-04	1.09E-04	1.09E-04	7.73E-05	7.73E-05	7.73E-05
WZC1∗	7.59E-05	7.59E-05	7.59E-05	7.59E-05	7.59E-05	7.59E-05	7.59E-05	7.59E-05
TVOL∗	3.4E-02	3.4E-02	3.4E-02	3.1E-02	3.1E-02	2.8E-02	2.8E-02	2.8E-02
MAXMZB∗	39621	38281	36922	29091	26615	20904	20511	20185
MAXROTZ∗	3.84E-02	3.64E-02	3.44E-02	3.63E-02	3.18E-02	3.47E-02	3.36E-02	3.27E-02

Tab. 4. *Results of the single-bay frame under unsymmetrical loading* (Note: WZB1 and WZC1 – section modulus of the beam and column of optimal solution, TVOL – the total volume of the optimal solution, MAXFXB – the maximal axial force of the beam element, MAXFYCA and MAXFYCB - the maximal axial forces of the column elements, MAXMZB – the maximal bending moment of the beam element, MAXMZCA and MAXMZCB – the maximal bending moment of the column elements, MAXROTZ – the maximal rotation.)

r_q	4.6E-02	8.79E-02	0.42209	0.39521	0.76567	0.86728	0.9703	0.98493
S_q	50000	1.0E+05	5.0E+05	1.0E+06	5.0E+06	1.0E+07	5.0E+07	1.0E+08
WZB1	1.46E-04	0.15E-03	0.108E-03	0.19E-03	0.19E-03	0.19E-03	0.19E-03	0.19E-03
WZC1	3.1E-04	0.31E-03	0.31E-03	0.22E-03	0.17E-03	0.17E-03	0.17E-03	0.17E-03
TVOL	6.26E-02	6.26E-02	5.95E-02	5.4E-02	4.7E-02	4.7E-02	4.7E-02	4.7E-02
MAXFXB	25622	26197	29914	29276	31226	31648	32029	32080
MAXFYCA	19542	19136	17232	14179	10750	10275	9857.5	9802.8
MAXFYCB*	20458	20864	22768	25821	29250	29725	30143	30197
MAXMZB*	38207	36671	27323	34774	53671	56700	59387	59741
MAXMZCA	87721	85562	74531	62811	47259	44964	42941	42675
MAXMZCB	88615	87523	83329	70621	58741	57232	55919	55748
rotation	0.354E-01	0.32E-01	0.287E-01	0.14E-01	0.12E-01	0.11E-01	0.11E-01	0.11E-01

Tab. 5. *Results of the two-bay frame – displacements and buckling constraints* (Note: MINUX, MAXUX, MINUY, and MAXUY indicate the minimal and maximal values of displacements. The buckling constraints (BUCK) for beams and columns are considered as well.)

SET	SET 133	SET 133	SET 133	SET 108	SET 83	SET 108	SET 108	SET 108
DESIGN	feasible	feasible	feasible	feasible	feasible	feasible	feasible	feasible
MINUX	.26E-03	.117E-02	.210E-02	.706E-02	.101E-01	.891E-02	.889E-02	.883E-02
MAXUX	.26E-03	.117E-02	.210E-02	.706E-02	.101E-01	.891E-02	.889E-02	.883E-02
MINUY	.96E-01	.922E-01	.878E-01	.914E-01	.10174	.531E-01	.489E-01	.454E-01
MAXUY	.79E-30	.789E-30	.789E-30	.789E-30	.789E-30	.789E-30	.789E-30	.789E-30
BUCKC11	.19039	.24322	.29628	.57611	.74660	.66249	.65814	.65193
BUCKC12	.35076	.35096	.35149	.36038	.36810	.38620	.39033	.39421
BUCKC13	.18813	.23303	.27808	.51526	.65961	.58675	.58274	.57717
BUCKB11	.65705	.63801	.61745	.65869	.82926	.70973	.75255	.79129
BUCKB12	.65705	.63801	.61745	.65869	.77971	.47189	.45500	.44019
TVOL (OBJ)	.61E-01	.606E-01	.606E-01	.544E-01	.485E-01	.544E-01	.544E-01	.544E-01

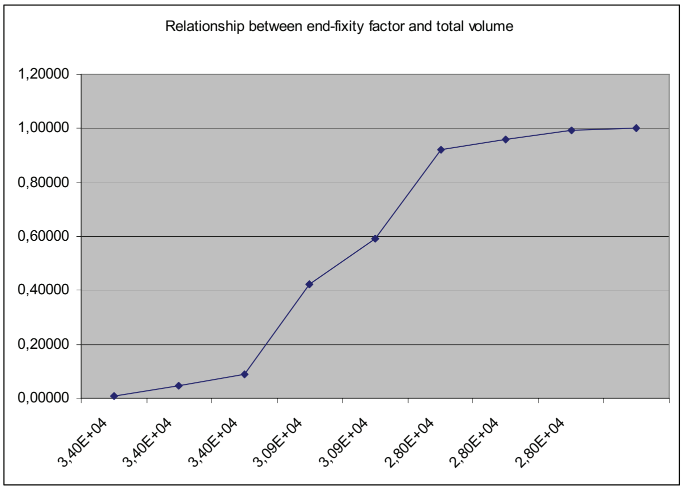

Fig. 3. Results of the single-bay frame under symmetric loading

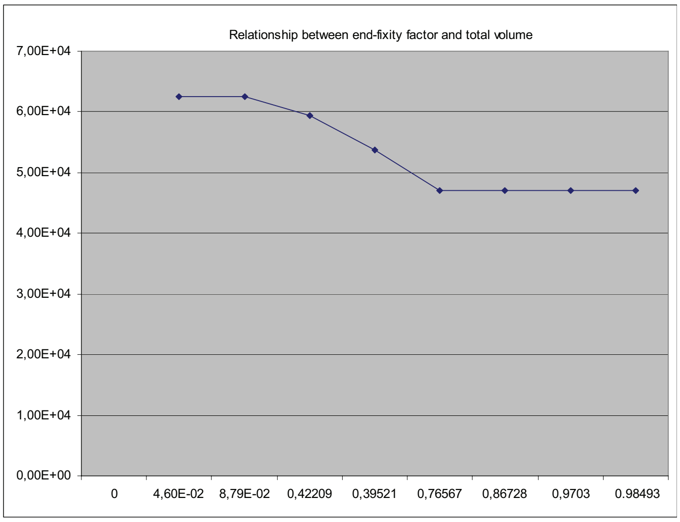

Fig. 4. Results of the single-bay frame – under unsymmetrical loading

5 Conclusions

In this paper, a genetic algorithm was applied for discrete minimal weight design of steel planar frames with semi-rigid beam-to-column connections. The frame elements are constructed from a predetermined range of section profiles. Two different catalogue values were determined for beam and col-

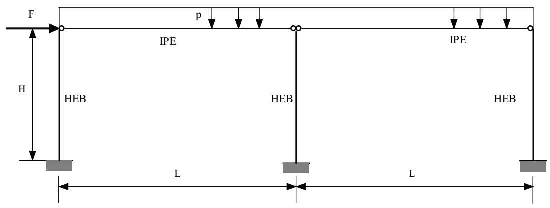

Fig. 5. Semi-rigid two-bay frame

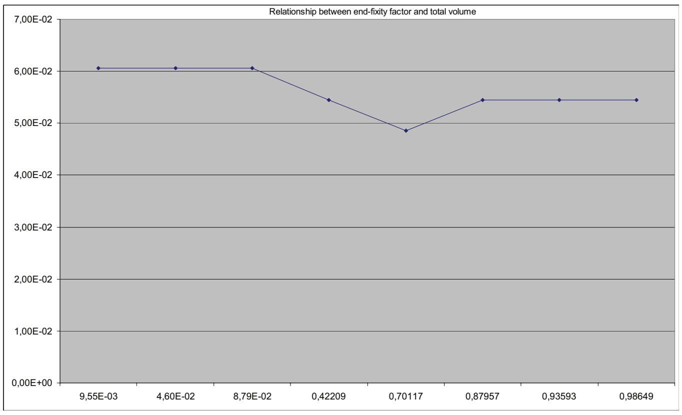

Fig. 6. Results of the two-bay frame

umn sections. In this study, both the structural analysis and the optimal design problem were solved, using ANSYS Release 9.0 finite element program.

The purpose of this study was to determine the effect of the rotational stiffness of beam-to-column connection in the optimal design while the structural response was changing. The results obtained for single-bay and two-bay frame structures are shown in Tables 3, 4, 5. The relationship between the optimal volume and the end-fixity factor is presented in Figs. 3, 4, 6. The optimal solutions highly depend on the structural geometry and on the loading conditions. For discrete optimal design of two-bay frame we obtained better solution in case of semi-rigid joints than in case of rigid or pinned connections.

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