

Single- and multiobjective optimization of a welded stringer-stiffened cylindrical shell

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Abstract

This paper presents the single- and multiobjective optimization of a welded stringer-stiffened cylindrical steel shell.

A column fixed at the bottom and free on the top is constructed of stringer-stiffened cylindrical shell and loaded by axial compression as well as by a horizontal force acting on the top. Halved rolled I-section stringers are welded outside of the shell by longitudinal fillet welds. The shell is loaded by a compression force N_F and a horizontal force H_F . The horizontal displacement of the top (w) is limited. The stiffening is economic when the shell thickness can be decreased in such a measure that the cost savings caused by this decreasing is higher than the additional cost of stiffening material and welding. Variables are the shell thickness as well as dimension and number of stringers. We have considered three objective functions: (1) material cost, (2) cost of forming the shell elements into the cylindrical shape, assembly and welding, (3) painting cost. The original Particle Swarm Optimization (PSO) algorithm was modified to handle multiobjective optimization techniques and to find discrete values of design variables. It was built into a program system, where several singleobjective and multiobjective techniques like min-max, different versions of global criterion, weighted min-max, weighted global criterion, pure and normalized weighting techniques are available.

Keywords

stiffened cylindrical shells · fabrication cost · economy of welded structures · multiobjective optimization

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1 Introduction

In the optimum design of stiffened circular cylindrical shells the basic question is whether a stiffened thin shell or an unstiffened thick shell is more economic. In our recent research we have answered this question comparing the costs of the two structural versions each optimized for minimum cost. In these studies the shell diameter has been kept constant [1]. It was shown that an optimum diameter could be found for stiffened and unstiffened versions. Depending on the horizontal deformation limit, the stiffened shell is usually more economic than the unstiffened one.

The preliminary calculation shows that, in the case of external pressure the cost increases when the shell diameter increases and vice versa. On the other hand, in the case of bending of stringer-stiffened shells an optimum diameter can be found. Our previous calculations showed, that both the stiffened and unstiffened versions had an optimum radius, in our numerical problem $R_{opt} = 2700$ mm.

Cost difference was considerable only for radii smaller than the optimum, thus, the stiffening was economic only for these radii. For radii larger than the optimum, the difference between the thicknesses and between the costs was small, since the unstiffened shell can be realized with larger radius and not too large thickness.

It could also be seen that, for both structural versions the stress constraint was active for radii larger than R_{opt} and the deflection constraint governed for radii smaller than R_{opt} .

Recent studies [2–4] have shown that the economy of stiffened shells depends on loads (compression, bending or external pressure), design constraints (buckling, deflection of the whole structure) as well as the type and position of stiffeners (ring-stiffeners, stringers or orthogonal stiffening, stiffeners welded outside or inside of the shell).

As a part of our systematic research related to stiffened cylindrical shells, in the present study a column is investigated subject to an axial compression and a horizontal force acting on the top of the column (Fig.1). The column is fixed at the bottom and free on the top. It is shown that a shell stiffened outside with stringers can be economic, when a constraint on horizon-

3.1.2 Stringer panel buckling

$$\sigma_a + \sigma_b \leq \sigma_{crp} = \frac{f_y}{\sqrt{1 + \lambda_p^4}} \quad (11)$$

$$\lambda_p^2 = \frac{f_y}{\sigma_{EP}}; \sigma_{EP} = C_p \frac{\pi^2 E}{10.92} \left(\frac{t}{L}\right)^2 \quad (12)$$

$$C_p = \psi_p \sqrt{1 + \left(\frac{0.5\xi_p}{\psi_p}\right)^2}; Z_p = 0.9539 \frac{L^2}{Rt} \quad (13)$$

$$\xi_p = 0.702Z_p; \gamma_s = 10.92 \frac{I_{sef}}{st^3} \quad (14)$$

$$\psi_p = \frac{1 + \gamma_s}{1 + \frac{A_s}{2s_e t}}; \quad (15)$$

Since the effective shell part s_e (Fig.1) is given by DNV with a complicate iteration procedure, we use here the simpler method of ECCS [9]

$$s_e = 1.9t \sqrt{\frac{E}{f_y}} \quad (16)$$

if $s_E < s$ $s_e = s_E$

if $s_E > s$ $s_e = s$

I_{sef} is the moment of inertia of a cross section containing the stiffener and a shell part of width s_e (Fig. 1). For a stiffener of halved rolled I-section it is

$$I_{sef} = s_e t z_G^2 + \frac{t_w}{12} \left(\frac{h_1}{2}\right)^3 + \frac{h_1 t_w}{2} \left(\frac{h_1}{4} - z_G\right)^2 + b t_f \left(\frac{h_1}{2} - z_G\right)^2 \quad (17)$$

$$z_G = \frac{h_1^2 t_w / 8 + h_1 b t_f / 2}{h_1 t_w / 2 + b t_f + s_e t} \quad (18)$$

3.1.3 Horizontal displacement

$$w_h = \frac{ML^2}{3EI_{x0}} \leq w_{allow} = \frac{L}{\phi} \quad (19)$$

ϕ is selected as 1000. In our case the horizontal displacement limit is 15 mm. It causes a large rigidity at the structure.

In earlier calculations we determined that the deflection limit has a great effect on the dimensions, if they are strong enough. This is an approximate calculation. For earthquake safe design more precise calculation is needed to consider the sway, or interstorey drift limit.

The exact calculation of the moment of inertia for the horizontal displacement uses the following formulae (Fig. 1):

The distance of the center of gravity for the halved UB section is

$$z_A = \frac{h_1 t_w / 2 (h_1 / 4 + t_f / 2)}{h_1 t_w / 2 + b t_f} \quad (20)$$

The moment of inertia of the halved UB section is expressed by

$$I_x = b t_f z_A^2 + \frac{t_w}{12} \left(\frac{h_1}{2}\right)^3 + \frac{h_1 t_w}{2} \left(\frac{h_1}{4} - z_A\right)^2 \quad (21)$$

The moment of inertia of the whole stiffened shell cross-section is

$$I_{x0} = \pi R^3 t + I_x \sum \sin^2 \left(\frac{2\pi i}{n_i}\right) + \left(\frac{h_1 t_w}{2} + b t_f\right) \left(R + \frac{h_1 + t_f}{2} - z_A\right)^2 \sum_{i=1}^{n_s} \sin^2 \left(\frac{2\pi i}{n_s}\right) \quad (22)$$

In earthquake calculations the used ratio between vertical and horizontal forces is 10 in this case. This is an approximation. For more precise calculation different factors should be consider according the Eurocode 8 to determine the proper value of horizontal force.

$$M = H_F L / \gamma_M; \gamma_M = 1.5; H_F = 0.1 N_F \quad (23)$$

At calculation of the deformation we should exclude the partial safety factors, introduced at stress calculation. That is, why we divide by γ_M in Eq. (23).

Numerical data: $N_F = 6 \times 10^7$ N, $f_y = 355$ MPa, $L = 15$ m, $R = 2700$ mm, $\phi = 1000$, $E = 2.1 \times 10^5$.

3.2 The cost function

Fabrication sequence:

- 1 Fabrication of 5 shell elements of length 3 m without stiffeners. For one shell element κ axial butt welds are needed (GMAW-C) (K_{F1}). For $R < 1909$ $\kappa = 2$, for $1909 < R < 2865$ $\kappa = 3$ and for $2865 < R < 3819$ $\kappa = 4$.
- 2 The cost of forming of a shell element into the cylindrical shape is also included (K_{F0}).
- 3 Welding of the whole unstiffened shell of 5 elements with 4 circumferential butt welds (K_{F2}).
- 4 Welding of n_s stiffeners to the shell with double-sided GMAW-C fillet welds. Number of fillet welds is $2n_s$. (K_{F3}).

The material cost is

$$K_M = k_{M1} 5\rho V_1 + k_{M2} \rho n_s A_s L / 2 \quad (24)$$

$$V_1 = 3000 \times 2R\pi t; \rho = 7.85 \times 10^{-6} \text{kgmm}^{-3}.$$

$$k_F = 1.0\$/\text{min}, k_{M1} = 1.0\$/\text{kg}. \quad (25)$$

The cost of forming of a shell element into the cylindrical shape according to [4] is

$$K_{F0} = k_F \Theta e^\mu; \mu = 6.8582513 - 4.527217t^{-0.5} + 0.009541996 (2R)^{0.5} \quad (26)$$

$$K_{F1} = k_F \left[\Theta \sqrt{\kappa \rho V_1} + 1.3 \times 0.1520 \times 10^{-3} t^{1.9358} (2 \times 3000) \right] \quad (27)$$

where Θ is a difficulty factor expressing the complexity of the assembly and κ is the number of elements to be assembled

$$\kappa = 2; \quad V_1 = 2R\pi t \times 3000; \quad \Theta = 2 \quad (28)$$

$$K_{F2} = k_F \left(\Theta \sqrt{25\rho V_1} + 1.3 \times 0.1520 \times 10^{-3} t^{1.9358} \times 4 \times 2R\pi \right) \quad (29)$$

$$K_{F3} = k_F \left(\Theta \sqrt{(n_s + 1) \rho V_2} + 1.3 \times 0.3394 \times 10^{-3} a_w^2 2Ln_s \right) \quad (30)$$

The fillet weld size is $a_w = 0.3t_w$, $a_{wmin} = 3$ mm.

$$V_2 = 5V_1 + n_s A_s L / 2 \quad (31)$$

The fabrication cost is

$$K_F = 5K_{F1} + 5K_{F0} + K_{F2} + K_{F3} \quad (32)$$

The cost of painting is

$$K_P = k_P (4R\pi L + n_s A_s L / 2); \quad k_P = 14.4 \times 10^{-6} \$ / \text{mm}^2. \quad (33)$$

The total cost is

$$K = K_M + K_F + K_P \quad (34)$$

The optimum design process has the following three main phases:

- 1 preparation: selection of candidate structural versions defining the main characteristics to be changed, formulation of design constraints and cost function,
- 2 solution of the constrained function minimization problem by using efficient mathematical methods,
- 3 evaluation of results by designers, comparison of optimized versions, formulation of design rules, incorporation in expert systems.

4 The Particle Swarm Algorithm

There is a great number of methods available for single objective optimization as it was described in Farkas & Jármai [10]. Methods without derivatives like: Complex [11], Flexible Tolerance, and Hillclimb. Methods with first derivatives such as: Sequential Unconstrained Minimization Technique (SUMT), Davidon-Fletcher-Powell, etc. Methods with second derivatives such as: Newton, SQP. There are also other classes of techniques like optimality criteria methods, or the discrete methods like Backtrack, the entropy-based method (Jármai [12], Farkas et al. [13]). Multicriteria optimization is used when more objectives are important to find the compromise solution (Oszczka [14]).

The general formulation of a single-criterion non-linear programming problem is the following:

$$\text{minimize } f(x)_{x_1, x_2, \dots, x_N} \quad (35)$$

$$\text{subject to } g_j(x) \leq 0, \quad j = 1, 2, \dots, P \quad (36)$$

$$h_i(x) = 0 \quad i = P + 1, \dots, P + M \quad (37)$$

$f(x)$ is a multivariable non-linear function, $g_j(x)$ and $h_i(x)$ are non-linear inequality and equality constraints, respectively.

In the last two decades some new techniques appeared e.g. the evolutionary techniques, the genetic algorithm, Goldberg [15], the differential evolution technique (Storm [16], Storn & Price [17]), the particle swarm algorithm (Kennedy & Eberhart [18]), the ant colony technique (Dorigo et al. [19, 20]). Some other high performance techniques such as leap-frog with the analogue of potential energy minimum (Snyman [21–23]), similar to the FEM technique, have also been developed.

A number of scientists have created computer simulations of various interpretations of the movement of organisms in a bird flock or fish school (Millonas [24]). The Particle Swarm Optimization (PSO) algorithm was first introduced by Kennedy [25]. The algorithm models the exploration of a problem space by a population of individuals; the success of each individual influences their searches and those of their peers. In our implementation of the PSO, the social behaviour of birds is mimicked. Individual birds exchange information about their positions, velocities and fitness, and the behaviour of the flock is then influenced to increase the probability of migration to regions of high fitness (Kennedy & Eberhard [26]).

Particle swarm optimization has roots in two main component methodologies. Perhaps more obvious are its ties to artificial life in general, and to bird flocking, fish schooling, and swarming theory in particular. It is also related, however, to evolutionary computation, and has ties to both genetic algorithms and evolutionary programming. Particle Swarm optimizers are similar to genetic algorithms in that they have some kind of fitness measure and they start with a population of potential solutions, (none of which are likely to be optimal) and attempt to generate a population containing fitter members.

In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches. Social sharing of information among conspecifics offers an evolutionary advantage: this hypothesis was fundamental to the development of particle swarm optimization.

Millonas [24] developed his models for applications in artificial life, and articulated five basic principles of swarm intelligence. The first one is the proximity principle: the population should be able to carry out simple space and time computations. The second one is the quality principle: the population should

be able to respond to quality factors in the environment. The third one is the principle of diverse response: the population should not commit its activities along excessively narrow channels. The fourth one is the principle of stability: the population should not change its mode of behaviour every time the environment changes. The fifth one is the principle of adaptability: the population must be able to change its behaviour mode when it is worth the computational price.

Basic to the paradigm are n -dimensional space calculations carried out over a series of time steps. The population is responding to the quality factors pBest and gBest (gBest is the overall best value, pBest is the best value for a particle). The allocation of responses between pBest and gBest ensures a diversity of response. The population changes its state (mode of behaviour) only when gBest changes, thus adhering to the principle of stability. The population is adaptive because it does change when gBest changes.

The original PSO algorithm, proposed by Kennedy and Eberhardt in 1995 [26], was inspired by the modelling of the social behaviour patterns of organisms that live and interact within large groups. In particular, PSO incorporates swarming behaviours observed in flocks of birds, schools of fish, or swarms of bees. A PSO algorithm is easy to implement in most programming languages, since the core of the program can be written in a few lines of code. It has been proven to be both fast and effective, when applied to a diverse set of optimization problems. PSO algorithms are especially useful for parameter optimization in continuous, multi-dimensional search spaces.

In performing a search in the multi-dimensional space associated with the optimization problem of the form (35-37), the PSO technique assigns direction vectors and velocities to each member (particle) of the swarm at their current positions. Each particle then “moves” or “flies” through the search space according to the particle’s assigned velocity vector, which may be influenced by the directions and velocities of other particles in its neighbourhood. These localized interactions with neighbouring particles, propagate through the entire “swarm” of particles and results in the swarm as a whole moving to regions of the space closer to the solution of problem (35-37). The extent to which a particular particle influences other particles is determined by its so-called “fitness” along its trajectory of candidate solution points. The “fitness” is a measure assigned to each potential solution, and it indicates how good a particular candidate solution is relative to all other solution points. Hence, an evolutionary idea of “survival of the fittest” (in the sense of Darwinian evolution) comes into play, as well as a social behaviour component through a “follow the local leader” effect and emergent pattern formation [27].

A more precise and detailed description of the particular PSO algorithm, as applied to penalty function formulation, and used in this study now follows.

4.1 Basic PSO algorithm

Given M , k_{max} , N_{max} . Set (time) instant $k=0$, $F_i^b = F^g = F_{before}^g = \infty$. Initialize a random population (swarm) of M particles (swarm members), by assigning an initial random position x_i^0 (candidate solution), as well as a random initial velocity v_i^0 , to each particle i , $i=1,2,\dots,M$. Then compute simultaneous trajectories, one for each particle, by performing the following steps.

- 1 At instant k , compute the fitness of each individual particle i at discrete point x_i^k , by evaluating $F(x_i^k)$. With reference to the minimization (35-37), the lower the value of $F(x_i^k)$, the greater the particle’s fitness.
- 2 For $i=1,2,\dots,M$:
 - if $F(x_i^k) \leq F_i^b$ then set $F_i^b = F(x_i^k)$ and $p_i^b = x_i^k$ {best point on trajectory i }
 - if $F(x_i^k) \leq F^g$ then set $F^g = F(x_i^k)$ and $g^b = x_i^k$ {best global point}
- 3 If $F^g < F_{before}^g$ then set $N = 1$, else set $N = N + 1$.
- 4 If $N > N_{max}$ or $k > k_{max}$ then STOP and set $x^* = g^b$; else continue.
- 5 Compute new velocities and positions for instant $k+1$, using the rule:
for $i=1,2,\dots,M$:

$$v_i^{k+1} := v_i^k + c_1 r_1 (p_i^b - x_i^k) + c_2 r_2 (g^b - x_i^k) \quad (38)$$

$$x_i^{k+1} := x_i^k + v_i^{k+1} \quad (39)$$

where r_1 and r_2 are independently generated random numbers in the interval $[0,1]$, and c_1 , c_2 are parameters with appropriately chosen values.

- 6 Set $k = k + 1$ and $F_{before}^g = F^g$; go to step 2.

The technique is modified in order to be efficient in technical applications. It calculates discrete optima, uses dynamic inertia reduction and craziness for some particles [27].

The method is derivative free, and by its very nature the method is able to locate the global optimum of an objective function. Constrained problems can simply be accommodated using penalty methods.

The interactive decision support program system contains several multiobjective optimization methods. They are the followings:

- Min-max method,
- Global criterion method: type - 1,
- Global criterion method: type - 2,
- Weighted min-max method,
- Weighted global criterion method,
- Pure weighting method,
- Normalized weighting method.

Tab. 1. Different optima for the total cost, using different single objective optimization techniques

Method	h_s (mm)	n_s	t (mm)	Total cost (\$)	2^{nd} Material cost	3^{rd} Fabrication cost	4^{th} Painting cost
Flexible tolerance	257.2	25	21	79081.4	46463.6	22912.1	9705.6
Hillclimb	257.2	15	24	82361.4	49678.7	24632.6	8050.1
Davidon Fletcher Powell	257.2	17	23	81593.5	49014.2	23634.2	8945.0
Particle swarm optimization	257.2	24	21	78639.7	46283.1	22746.1	9610.6

Tab. 2. Multiobjective optima for the stringer stiffened shell

Method	h_s (mm)	n_s	t (mm)	1^{st} Total cost (\$)	2^{nd} Material cost (\$)	3^{rd} Fabrication cost (\$)	4^{th} Painting cost (\$)
1^{st} objective	257.2	24	21	78639.7	46283.1	22746.1	9610.6
2^{nd} objective	257.2	24	21	78639.7	46283.1	22746.1	9610.6
3^{rd} objective	683.5	25	15	87516.6	56295.1	17691.4	13530.0
4^{th} objective	257.2	5	26	85276.8	52840.0	24632.6	7804.1
Min-max	257.2	24	21	78629.7	46283.1	22746.1	9610.6
Weighted min-max	257.2	24	21	78629.7	46283.1	22746.1	9610.6
Global criterion 1 exp. 3	308.7	23	21	79432.3	46994.2	22599.6	9838.6
Global criterion 2 exp. 2	257.2	21	22	80336.5	47738.9	23272.2	9325.4
Weighted global	257.2	21	22	80336.5	47738.9	23272.2	9325.4
Pure weighting	257.2	25	21	79081.4	46463.6	22912.1	9705.7
Normalized weighting	257.2	24	21	78629.7	46283.1	22746.1	9610.6

5 Optimization and results

The Particle swarm optimizer has been built into an interactive decision support program system (Jármai [28]) which contains the following single objective optimization methods:

Flexible Tolerance (FT) method by Himmelblau [29],

Direct Random Search (DRS) [30],

Hillclimb (HI) method by Rosenbrock [31],

Davidon-Fletcher-Powell (DFP) method by Rao [32],

Particle Swarm Optimization (PSO), Jármai et al. 2003, [733].

The efficiencies of these methods are different. All of them use the same objective, constraints subroutines. For a problem like this, which is highly non-linear, several local minima exist. They find different ones. The advantage of Particle swarm optimization is that it can find optimum for a nonconvex problem. It has found the minimum cost structure. Table 1 shows the single objective optima.

Description of methods is available in Jármai [28]. Weighting coefficients are similar to all four objectives 0.25 each. The objective functions are as follows:

Total cost of the structure in \$, K (1^{st}),

Material cost of the structure in \$, K_m (2^{nd}),

Fabrication cost of the structure in \$, K_f (3^{rd}),

Painting cost in \$, K_p (4^{th}).

Table 2 shows the different multiobjective optima. The material cost is dominating, 55-65 % of the total cost. The other two objectives are around 35-45 %. The height of stiffener is nearly

the same at all optima; the number of stiffeners and the shell thickness is changing on an opposite way due to the necessary stiffness. The greatest conflict is between the total and the painting costs. The painting cost minimum gives the greatest shell thickness t .

The optimization is performed using the Particle Swarm mathematical algorithm. The results are summarized in Table 2.

6 Conclusions

Cylindrical shells stiffened outside by stringers are economic for axial compression and horizontal force with an active deflection constraint, but without a deflection constraint they are uneconomic. In order to decrease the welding cost, the stiffeners should have cross-sectional area as large as possible and should be welded to the shell with welds as small as possible, thus the outside halved rolled I-section stringers are advantageous for this purpose. It should be noted that cost savings cannot be achieved by stringers welded inside of the shell.

The decision support system, which contains several single and multiobjective optimization techniques, is an efficient tool for structural optimization. Using the same constraints, the optimizers can find the optima. If these optima are similar, or close to each other, the designer can be sure, that he has found, or is close to the global optimum. Discrete solutions are useful for the application of the results. The robustness of PSO is visible, when the problem is non-convex. In this case the material cost is dominant; the optima for the total and for the material cost min-

imum are identical. Using different multiobjective optimization techniques, different weighting coefficients, we can get a great number of optima to get more information about the behaviour of the structure.

The PSO technique was found to be a robust method also for multiobjective optimization. Some of the objectives are in conflict. The material cost represents more than 60 % of the total cost. Optimization for the cost of forming the shell elements into the cylindrical shape, assembly and welding means a smaller shell thickness and more and larger stiffeners. Optimization for the painting cost means a thicker shell and less and smaller stiffeners.

Appendix

Approximate functions for dimensions t_f , b and t_w of UB profile series in function of h .

t_f :

$$y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + hx^7 + ix^8$$

$$a = -26.93816005910891D0$$

$$b = 0.7030053260773679D0$$

$$c = -0.005693338027675875D0$$

$$d = 2.383106288900282D-05$$

$$e = -5.605511692214832D-08$$

$$f = 7.662794440441443D-11$$

$$g = -5.902409222905948D-14$$

$$h = 2.267417977644635D-17$$

$$i = -2.999371468428559D-21$$

b :

$$y = a + bx + \frac{c}{x} + dx^2 + \frac{e}{x^2} + fx^3 + \frac{g}{x^3} + hx^4 + \frac{i}{x^4} + jx^5 + \frac{k}{x^5}$$

$$a = -1108926.658794802D0$$

$$b = 2054.96457373585D0$$

$$c = 394347552.4221416D0$$

$$d = -2.475920494568994D0$$

$$e = -91315532919.66857D0$$

$$f = 0.001858445891156483D0$$

$$g = 13189053888762.85D0$$

$$h = -7.856977790442618D-07$$

$$i = -1073670362507492D0$$

$$j = 1.422535840934241D-10$$

$$k = 3.744384150518803D+16$$

t_w :

$$y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + hx^7 + ix^8$$

$$a = 4.598131496764401D0$$

$$b = -0.1667245062310966D0$$

$$c = 0.002662252625070477D0$$

$$d = -1.662919418563092D-05$$

$$e = 5.425706060478163D-08$$

$$f = -1.003562929221022D-10$$

$$g = 1.063362615303672D-13$$

$$h = -6.028516555302632D-17$$

$$i = 1.419727611913505D-20$$

Tab. 3. Characteristics of the selected rolled UB profiles (Profil Arbed [6])

UB Profile	h mm	b mm	t_w mm	t_f mm	A_S mm ²	$I_y \times 10^{-4}$ mm ⁴
152×89×16	152.4	88.7	4.5	7.7	2032	834
168×102×19	177.8	101.2	4.8	7.9	2426	1356
203×133×25	203.2	133.2	5.7	7.8	3187	2340
254×102×25	257.2	101.9	6.0	8.4	3204	3415
305×102×28	308.7	101.8	6.0	8.8	3588	5366
356×127×39	353.4	126.0	6.6	10.7	4977	10172
406×140×46	403.2	142.2	6.8	11.2	5864	15685
457×152×60	454.6	152.9	8.1	13.3	7623	25500
533×210×92	533.1	209.3	10.1	15.6	11740	55230
610×229×113	607.6	228.2	11.1	17.3	14390	87320
686×254×140	683.5	253.7	12.4	19.0	17840	136300
762×267×173	762.2	266.7	14.3	21.6	22040	205300
838×292×194	840.7	292.4	14.7	21.7	24680	279200
914×305×224	910.4	304.1	15.9	23.9	28560	376400
1016×305×349	1008.1	302	21.1	40	444200	722300
1016×305×393	1016	303	24.4	43.9	500200	807700

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